TEST OF THE ASSUMPTION OF EQUAL PHASES FOR ALL THE HELICITY AMPLITUDES IN THE PROCESS $0^-\frac{1}{2}^+ \to 1^-\frac{3}{2}^+$

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The assumption of equal phases for all the helicity amplitudes is implicit in many models. Supplementing it by the class (a) relations predicted by the quark model we find that the 19 parameters necessary to describe the decay distribution in the process $0^- \frac{1}{2}^+ \to 1^- \frac{3}{2}^+$ depend on four independent parameters only. Since the quark model predictions are well supported by experiment, we interpret our result as a test for the assumption of equal phases for the helicity amplitudes. The test is used to analyse the available experimental data.

1. Introduction

Many simple models of two-body reactions predict that all the helicity amplitudes for a reaction should have equal phases (modulo π). For instance this is the case for Regge pole models, when the exchange of a single pole, or of a single pair of exchange degenerate poles, is assumed. Consequently it is interesting to look for tests, which could be used to verify the assumption of equal phases before a definite model is introduced.

As is well known, for reactions

$$PB \to P'B', \tag{1}$$

where P, P' denote pseudoscalar mesons and B, B' spin parity $\frac{1}{2}^+$ baryons, the assumption of equal phases implies, that the polarization of B' vanishes, if B is unpolarized. For resonance production processes the corresponding tests should involve those spin density matrix elements, which can be determined from the measured decay distribution. Such a relation for the spin density matrix of the $\frac{3}{2}^+$ isobar (B^*) produced in the reaction

$$PB \to P'B^* \tag{2}$$

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has been derived by Białas and Kotański [1], and by Ringland and Thews [2]. An attempt to apply similar arguments to reaction

$$BP \to VB^*, \tag{3}$$

where V denotes a vector meson, had led, however, only to inequalities [3].

The relation found in Refs [1] and [2] is non-linear. Thus, in principle, it should be applied to events obtained at a fixed energy and at a fixed scattering angle. In practice, the experimental data are always averaged over some interval of scattering angles. One finds, however, that the results of the test depend little on the width of the angular interval, because the spin density matrix vary slowly when the scattering angle changes (*cf. e. g.* Refs [4] and [19]). Thus the customary procedure of substituting averaged matrix elements into the relation is not bad, in spite of the nonlinearity of the relation.

In this paper the problem of finding tests for the assumption of the equal phases for reaction (3) is reconsidered. The new idea is to make use of the additive quark model. The predictions of this model for the decay distributions of the VB^* systems (known as class (a) relations) have been tested in various laboratories [4]–[11] and found in very good agreement with experiment. Thus we feel that our additional assumption is justified. The results correspond to finding for reaction (3) four relations analogous to the relation found in Refs [1] and [2] for reaction (2). The formulae are given in the following Section. In Section 3 the available experimental data for reacitons (3) are analysed. Section 4 contains our conclusions.

2. Formulae

The decay distribution of the VB^* system is completely specified, when the statistical tensors, *i. e.* the averages

$$T^{20}_{M0} = -\sqrt{\frac{5\pi}{6}}\langle Y^2_M(\theta_V, \varphi_V)\rangle \tag{4}$$

$$T^{02}_{0M} = -\sqrt{\frac{5\pi}{3}}\langle Y^2_M(\theta_{B^*}, \varphi_{B^*})\rangle \tag{5}$$

$$T^{22}_{MN} = 5\pi\sqrt{\tfrac{2}{3}}\langle Y^2_M(\theta_V, \varphi_V)Y^2_N(\theta_{B^*}, \varphi_{B^*})\rangle \tag{6}$$

are given. For details about the conventions used here and for references to the original papers see Ref. [12]. In the formulae (4)–(6) the functions Y^J_M are spherical harmonics. The angles θ_V, φ_V, θ_B and φ_{B^*} are the decay angles of the vector meson and the isobar. The average is over the joint decay distribution of the BV^* system.

According to the additive quark model, the scattering amplitude for the process (3) depends on five scalar amplitudes denoted in Ref. [13] f_0, f_5, f_6, f_7 and f_8. The assumption of equal phases for all the helicity amplitudes implies that when the overall phase is chosen so that f_0 is real:

$$f_5 = f_6^* \tag{7}$$

$$f_7 = f_8^* \tag{8}$$

where the asterisk denotes complex conjugation. As seen from the formula (13) of Ref. [14] these formulae are invariant with respect to arbitrary rotations of the single particle spin reference frames around the normal to the reaction plane. Consequently the present analysis does not depend on the assumptions necessary to specify the additivity frame [15].

The formulae relating the statistical tensors to the scalar amplitudes were given in Ref. [13]. We will use single particle spin reference frames with z axes normal to the reaction plane. The orientation of the x axis can be chosen arbitrarily. Thus transversity frames, Jackson transversity frames, transverse Donohue-Høgaasen frames, or any other frames with the z axis normal to the reaction plane may be used. Using (7), (8) formulae (3)–(11) of Ref. [14] one finds

$$T^{20}_{20} = 2T^{22}_{20} = -\tfrac{2}{3} f_5 f_8 \tag{9}$$

$$T^{02}_{02} = \sqrt{2}\, T^{22}_{02} = -\frac{\sqrt{2}}{3} f_5^* f_8 \tag{10}$$

$$T^{20}_{00} + \frac{1}{\sqrt{6}} = \sqrt{2}\, T^{02}_{00} + \frac{1}{\sqrt{6}} = -2T^{22}_{00} + \frac{2}{\sqrt{6}} = \frac{2}{\sqrt{6}} (|f_5|^2 + |f_8|^2) \tag{11}$$

$$T^{22}_{22} = \frac{1}{\sqrt{6}} f_8^2 \tag{12}$$

$$T^{22}_{2-2} = \frac{1}{\sqrt{6}} f_5^2 \tag{13}$$

$$T^{22}_{11} = \frac{1}{2\sqrt{6}} f_0 f_8 \tag{14}$$

$$T^{22}_{1-1} = -\frac{1}{2\sqrt{6}} f_0 f_5 \tag{15}$$

$$f_0 = \sqrt{6 - 8(|f_5|^2 + |f_8|^2)}. \tag{16}$$

These are all the tensor components necessary to specify the decay distribution. Other either vanish, or can be expressed by the components (9)–(15). The relations between tensor components in (9)–(11) are the linear class (a) relations mentioned previously. There are, however, many more non-linear relations. According to (9)–(16) the nineteen real numbers necessary to specify the decay distribution are determined by four real numbers, *e. g.* Re f_5, Im f_5, Re f_8 and Im f_8. Thus besides the six linear relations, there are nine nonlinear relations. Four of them test the assumption of equal phases, *i. e.* relations (7) and (8), while the other five aie the nonlinear class (a) relations of the quark model [14].

In concluding this Section let us comment on the use of the Donohue-Høgaasen frame for testing the so-called class (b) and (c) relations derived from the quark model (*cf.* Refs [6]–[9] and [16]). Using the transformation properties of the amplitudes f_i

(formula (13) of Ref. [14]) it is easy to see that, if relations (9)–(16) hold, it is possible to find spin reference frames, where f_5 and f_8 are real. In this frame

$$\mathrm{Im}\, T^{20}_{20} = \mathrm{Im}\, T^{02}_{02} = 0. \tag{17}$$

By definition this is the Donohue-Høgaasen frame. Thus in this Donohue-Høgaasen frame (there is another one, where f_5 and f_8 are purely imaginary, but this does not concern us here) (7) and (8) imply

$$f_5 = f_6, f_7 = f_8. \tag{18}$$

There are just the additional assumptions necessary to derive the class (b) and class (c) relations from the quark model. Thus, if the quark model is valid, and if the assumption of equal phases is corrent the class (b) and class (c) relations should hold in the suitable Donohue-Høgaasen frame.

3. Comparison with experiment

In most experimental papers on reactions (3) only the spin density matrix elements corresponding to single statistical tensors *i. e.* the tensors with one of the upper indices equal zero are given. For such tensors the assumption of equal phases implies one new relation

$$|T^{20}_{20}| - \sqrt{2}\, |T^{02}_{02}| = 0 \tag{19}$$

valid for arbitrary spin reference frames provided spins are projected on the normal to the reaction plane.

We tried it on over 170 data points and found only one process, where it is not satisfied. Thus relation (19) is not very selective. The reason probably is that the signs have to agree anyway. The exceptional process was

$$K^- n \rightarrow K^{*0} - \Delta^- \tag{20}$$

at 3 GeV/c [10], where the second term in formula (19) is significantly larger (confidence level of about 0.2 per cent for the validity of (19)) than the first one. Since, however, the data for the same reactions at 5.5 GeV/c [11] and for the closely related reaction

$$K^- p \rightarrow K^* \Delta \tag{21}$$

at about 2.64 GeV/c [4] show no such effect, we feel that more evidence is needed to draw a definite conclusion.

More information can be obtained from the data on joint decay distributions. Instead of deriving relations we test the equivalent prediction that is possible to choose the complex amplitudes f_5 and f_8, so that the relations (9)–(16) are simultaneously satisfied. We analysed in this way the data from Refs [6]–[9] and [19].

The results of χ^2 fits are given in Table 1.

TABLE I

Test of the assumption of equal phases. There are 15 degrees of freedom in each fit thus $\chi^2 = 25$ corresponds to a confidence level about 5 per cent

Reaction	Energy	Interval of momentum transfer	χ^2	Ref.
$\pi^+p \to \varrho^0\Delta^{++}$	5 GeV/c	$-t < 0.2$ (GeV/c)^2	18.4	[6]
	8 GeV/c	$-t < 0.2$ (GeV/c)^2	11.7	[8]
	11.7 GeV/c	$-t < 0.03$ (GeV/c)^2	11.1	[19]
		$0.03 < -t < 0.05$ (GeV/c)^2	12.6	[19]
		$0.05 < -t < 0.08$ (GeV/c)^2	18.4	[19]
		$0.08 < -t < 0.12$ (GeV/c)^2	19.2	[19]
$\pi^+p \to \omega\Delta^{++}$	5 GeV/c	$-t < 0.55$ (GeV/c)^2	48.1	[6]
	8 GeV/c	$-t < 0.6$ (GeV/c)^2	18.6	[8]
$K^+p \to K^{*0}\Delta^{++}$	2.53 GeV/c	$-t < 0.023$ (GeV/c)^2	7.3	[9]
		$0.023 < -t < 0.07$ (GeV/c)^2	10.6	[9]
		$0.07 < -t < 0.14$ (GeV/c)^2	20.7	[9]
		$-t > 0.14$ (GeV/c)^2	8.2	[9]
	2.76 GeV/c	$-t < 0.027$ (GeV/c)^2	16	[9]
		$0.027 < -t < 0.08$ (GeV/c)^2	16.2	[9]
		$0.08 < -t < 0.166$ (GeV/c)^2	19.4	[9]
		$-t > 0.166$ (GeV/c)^2	15.1	[9]
	3.20 GeV/c	$-t < 0.036$ (GeV/c)^2	10.7	[9]
		$0.036 < -t < 0.109$ (GeV/c)^2	28.2	[9]
		$0.109 < -t < 0.217$ (GeV/c)^2	18.3	[9]
		$-t > 0.217$ (GeV/c)^2	15.3	[9]
$K^+p \to (k^*\Delta)^{++}$	5 GeV/c	$-t < 0.25$ (GeV/c)^2	28.7	[7]

There are 15 degrees of freedom in each fit, thus a confidence level of 5 per cent corresponds to $\chi^2 = 25$.

It is seen that all the results for

$$\pi^+p \to \varrho^0\Delta^{++} \tag{22}$$

fall below this value. We find no evidence against the assumption of equal phases.

For the process

$$\pi^+p \to \omega\Delta^{++} \tag{23}$$

data at 8 GeV/c are consistent with the assumption on equal phases, while the data at 5 GeV/c seems to rule it out. Since a drastic change in the reaction mechanism between the two energies seems unlikely, at least one of the two results should be accidental. For the moment some average of the two values of χ^2 seems to be most reliable. Since the errors quoted in the 5 GeV/c experiment are smaller, the average must be above 33. This is strong evidence against the assumption of equal phases.

The data for reaction

$$K^+p \to K^*\Delta^{++} \tag{24}$$

are again consistent with the assumption of equal phases.

4. Conclusions

A test for the assumption that all the helicity amplitudes in reactions $PB \to VB^*$ have have equal phases has been proposed. It relies on the additive quark model. A failure of the test proves that the assumption of equal phases is untenable. Consistency within errors does not necessarily prove the assumption, but it lends support to it.

When applied to single particle decay distiibutions, the test is not very stringent. For joint decay distributions, however, it is very effective: if the assumption of equal phases is correct, four real quantities must be sufficient to reproduce nineteen observable decay parameters.

An analysis of the available experimental data shows that the assumption of equal phases fails for the process $\pi^+p \to \omega\Delta^{++}$. There is also an indication that it might not be valid for the processes $K^-n \to \bar{K}^*\Delta$. For processes $\pi^+p \to \varrho^0\Delta^{++}$ and $K^+p \to K^*\Delta$ the assumption of equal phases is supported.

These results fit well into the presently accepted picture of two-body reactions. For the reactions $\pi^+p \to \varrho^0\Delta^{++}$ and $K^+p \to K^*\Delta$ the amplitudes should be predominantly real, because the first process is dominated by π exchange, and the second has exotic quantum numbers in the s-channel. No similar arguments for the other two processes are available.

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