SPIN AND PARITY ANALYSIS OF A MULTI-PRODUCTION AMPLITUDE IN THE Q REGION

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The spin and parity content of the double Regge amplitude for the reaction $K^+p \to K^{*0}\pi^+p$ at 7.3 GeV/c of Chien *et al.* is determined in the Q region. Only at the lower end of the Q band does the S wave dominate, so giving $J^P = 1^+$; at the upper end of the Q band many partial waves contribute with comparable strength. In the region where the S wave dominates the amplitude is predominantly real. Therefore, no resonance interpretation *via* duality is possible.

1. Introduction

In the three-meson systems $(\pi\pi)\pi$ and $(K\pi)\pi$ prominent enhancements with widths of several hundred MeV have been found experimentally. Their centres lie about 200 to 300 MeV above the threshold of the quasi-two body systems $\varrho\pi$, $f\pi$, $K^*_{890}\pi$ and $K\varrho$ and $K^*_{1420}\pi$. These enhancements are commonly interpreted as being related to the resonances $A_1(1080)$, $A_3(1640)$, $Q(1300)$ and $L(1775)$, respectively. It is clear that any dynamic model which restricts the transverse momenta of the final particles to small values — as is observed in nature — will lead to threshold enhancements. Attempts have been made with double peripheral models with particle [1, 2] or Regge exchanges [3, 4] and also resonance models [5]. These models reproduce the general trend of the data, but often differ in detailed data description (compare, *e. g.* [5]). For a long time the diffractive scattering at the proton-proton vertex in a Deck-like diagram was believed to be most important for the threshold enhancement. However, threshold enhancements are also obtained in processes where diffraction dissociation is forbidden [3, 6]. A nice example of this is the production of $I = 2$ meson systems, as in the reaction $\pi^- n \to (\pi^- \varrho^-)p$, *via* a (π, ϱ) and a (ϱ, ϱ) double Regge exchange graph [7]. Here, the peripheral threshold enhancement strongly decreases with increasing lab. momentum.

It is very probable that on top of the peripheral threshold enhancement more structure with smaller width is present (see, *e. g.*, [8]). For example, in the Q region at $\sim$ 1270 MeV a strange analogue of the A_1 and at 1390 MeV a strange analogue of the B meson are observed in some experiments [8, 9]; some data, however, do not show any splitting in the Q bump [10]. The resonances A_1 and Q are also claimed to exist in non-diffractive

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experiments, such as in backward production [11] and $p\bar{p}$ annihilation [12]. This evidence, however, is not completely convincing [20]. Experimentally, these resonances have unnatural parity, the assignment $J^P = 1^+$ being favoured (with the possible exception of the L resonance, which favours $J^P = 2^-$). According to the concept of duality, the threshold enhancement obtained in the double Regge model is a prediction for resonances [13]. In this context spin and parity of the threshold enhancement from the model calculation are of interest. In the double Regge model descriptions reported so far, such as for $A_1 \to \varrho\pi$, the threshold bump decays *via* an S wave [14], thus reproducing the experimentally observed $J^P = 1^+$ assignment of the A_1 resonance, in agreement with duality. Because the S wave also dominates a possible $A_1 \to \varepsilon\pi$ decay [15, 6], this decay does not include the A_1 state *via* duality [15]. In other models, such as the five point Veneziano model applied in [16] to the reaction $K^-p \to p(K^-\pi^+\pi^-)$ containing the Q region, and the resonance model [5], the experimentally observed dominating spin parity and orbital angular momentum of the decay is included right at the beginning.

We have analysed the amplitude which reproduces various aspects of the data of the reaction $K^-p \to K^*_{890}\pi p$ at 7.3 GeV [17] for its spin and parity content in the region of small $K^*_{890}\pi$ invariant mass. We work in the helicity formalism and take the spin of the K^*_{890} fully into account.

Our result is: Only at the lower end of the Q band the S wave dominates, thus giving the expected $J^P = 1^+$ for the threshold enhancement. Similarly as in the $\pi\varrho$ low-energy region [14, 18, 15] the Argand diagram does not show any rapid variation in the Q region and therefore does not give support for local duality. At the upper end of the Q band many partial waves contribute with about equal strength, so not permitting the resonance interpretation.

2. *Double Regge amplitude for $K^+p \to K^{*0}\pi^+p$ and partial wave analysis*

For the $K^+p \to K^{*0}\pi^+p$ reaction at 7.3 GeV/c which was selected by a mass cut from the data on $K^+p \to K^+\pi^-\pi^+p$ Chien *et al.* [17] have obtained a good fit to the data when considering solely the Reggeized version of the Deck graph (Fig. 1). They neglected the strangeness exchange graph where K^* emerges as middle particle. For fitting the amplitude the following distributions were taken into account:

the invariant mass of $m(K^{*0}\pi^+)$, $m(\pi^+p)$ and $m(K^{*0}p)$,
the momentum transfers $t(K^+ \to K^*)$, $t(p \to p)$ and $t(K^+ \to \pi^+)$,
the CM production angle θ_{K^*},
the Treiman-Yang angle φ_{K^*}, and
the Toller angle ω.

The amplitude of Chien *et al.* [17] is (for the notation see Fig. 1)

$$f_0^{t_1}(s_1, s_2, t_1, t_2) \sim e^{3t_2} \frac{\sqrt{\lambda(m_{K^*}^2, m_K^2, m_\pi^2)\,\lambda(s_2, m_p^2, m_\pi^2)}}{2m_{K^*}^2} \times$$

$$\times \frac{1+e^{-i\pi\alpha_\pi(t_1)}}{\sin\pi\alpha_\pi(t_1)}\left[\frac{1}{0.8}\left(s_1 - t_2 - m_K^2 - \frac{1}{2t_1}(m_{K^*}^2 - m_K^2 - t_1)(t_2 + t_1 - m_\pi^2)\right)\right]^{\alpha_\pi(t_1)} \qquad (1)$$

with $\alpha_\pi = \alpha'_\pi\ (t_1 - m_\pi^2)$; $\alpha_\pi{}' = 1.2\ \mathrm{GeV}^{-2}$. The following cuts were used: $s_2 > 2.25\ \mathrm{GeV}^2$, $0.02 \leqslant |t_2| \leqslant 1.0\ \mathrm{GeV}^2$, and $|t_1| < 1\ \mathrm{GeV}^2$. No explicit functional dependence on the Toller angle ω is included in $f_0^{t_1}$. Here, pion exchange gives a reasonable description even for a quite large momentum transfer $|t_1|$. It populates (mainly) states with $\lambda'_{\bar{1}} = \lambda'_{\overline{K^*}} = 0$, where $\lambda'_{\overline{K^*}}$ is the helicity of K in the t_1 channel CMS. Therefore, the amplitude (1) is denoted $f^{t_1}_{\lambda'_{\overline{K^*}}=0}$; it is a function of the subenergies s_1 and s_2, and the momentum transfers t_1 and t_2. The helicity amplitude $F^{s_1}_{\mu_b^-\mu_3,\lambda_1,\lambda_2,\lambda_{\bar{a}}}$ in the CMS of the s_1 channel (that is,

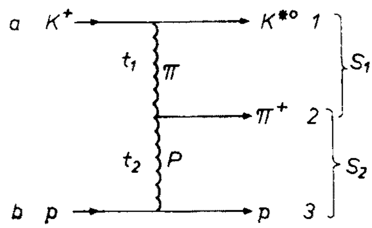

Fig. 1. Reggeized Deck graph which was solely taken into account for the analysis of the Q region. This graph also explains the notation used in the text

the CMS of the outgoing (($K^*\pi$)-system) is obtained when crossing particle 1 (K^*) and transforming the helicities of the incoming particle a (K^+) and the outgoing particle 2 (π) to the s_1 channel CMS, the undashed helicities referring to the s_1 channel CMS. The helicities of the baryons are defined in the CMS of the t_2 channel. We essentially average over the baryon helicities and therefore do not keep the labels of the baryon helicities further,

$$F^{s_1}_{\mu_b^-\mu_3\lambda_1} \equiv F^{s_1}_{\lambda_1}\ .$$

For $\lambda_a = \lambda_2 = 0$ we have [18]

$$F^{s_1}_{\lambda_1}(s_1, s_2, t_1, t_2) = \sum_{\lambda'_1} d^{J_1}_{\lambda_1\lambda'_{\bar{1}}}(\chi_1) f^{t_1}_{\lambda'_{\bar{1}}}(s_1, s_2, t_1, t_2) \tag{2}$$

so populating $\lambda_1 = 0, \pm 1$ in the s_1 channel CMS; J_1 is the spin of particle 1(K^*). The crossing angle in (2) is given by

$$\cos\chi_1 = \frac{-(t_1 + m_a^2 + m_1^2)(s_1 + m_1^2 - m_2^2) - 2m_1^2(m_a^2 - t_2 - m_1^2 + m_2^2)}{\sqrt{\lambda(t_1, m_a^2, m_1^2)\lambda(s_1, m_1^2, m_2^2)}} \tag{3}$$

and $\lambda(x, y, z)$ is the triangle function.

We obtain helicity partial waves with definite angular momentum J and parity P from Wigner projections in the Jackson frame [18, 19], where ϑ is the Jackson angle and φ is the Treiman Yang angle. Collecting the contributions to a definite normality $N = (-1)^J P = \pm 1$ for fixed total angular momentum J and its projection M along the momentum of the beam particle a, we have for the helicity partial waves ($\lambda_a = \lambda_2 = 0$)

$$a^{JMN}_{\lambda_1}(s, s_1, t_2) = \sqrt{\frac{2J+1}{4\pi}}\,\frac{1}{\sqrt{2}}\int_{-1}^{1} d\cos\vartheta \int_{0}^{2\pi} d\varphi e^{-iM\varphi} \times$$

$$\times \{d^J_{M\lambda_1}(\vartheta)F^{s_1}_{\lambda_1}(s_1, s_2, t_1, t_2) + NN_{12}d^J_{M,-\lambda_1}(\vartheta)F^{s_1}_{-\lambda_1}(s_1, s_2, t_1, t_2)\}. \tag{4}$$

Here, $\lambda_1 > 0$ and $N_{12} = n_1 \cdot n_2 = N_{K^*\pi} = -1$ (n_i is the normality of particle i). If $\lambda_1 = 0$, only states with $N = -1$ remain and for correct normalization an extra factor $1/\sqrt{2}$ is needed. The angles ϑ and φ can be expressed in terms of the invariants [18], and the ϑ and φ integrations in (4) can be transformed to t_1 and s_2 integrations. Integrating over the interval $(s_2)_{\min} \leqslant s_2 \leqslant (s_2)_{\max}$ corresponds to integrating over $0 \leqslant \varphi \leqslant \pi$ only. Using the independence of the amplitude (1) on the Toller angle ω and, therefore, also on the Treiman Yang angle φ, the helicity partial wave is

$$a_{\lambda_1}^{JM\pm}(s_1, s, t_2) = \sqrt{\frac{2J+1}{4\pi}}\sqrt{2}\int\limits_{(t_1)_{\min}}^{(t_1)_{\max}} dt_1 \int\limits_{(s_2)_{\min}}^{(s_2)_{\max}} ds_2 J(s, s_1, s_2, t_1, t_2) \cos M\varphi \times$$

$$\times \{d^J_{M\lambda_1}(\vartheta)F^{s_1}_{\lambda_1}(s_1, s_2, t_1, t_2) \pm N_{12} d^J_{M,-\lambda_1}(\vartheta)F^{s_1}_{-\lambda_1}(s_1, s_2, t_1, t_2)\} \tag{5}$$

and for $\lambda_1 = 0$

$$a_0^{JM-}(s, s_1, t_2) = 2\sqrt{\frac{2J+1}{4\pi}}\int\limits_{(t_1)_{\min}}^{(t_1)_{\max}} dt_1 \int\limits_{(s_2)_{\min}}^{(s_2)_{\max}} ds_2\, J(s_1, s, s_2, t_1, t_2) \times$$

$$\times \cos M\varphi d^J_{M0}(\vartheta) F^{s_1}_0(s, s_1, s_2, t_1, t_2). \tag{6}$$

The Jacobian is [18]

$$J(s, s_1, s_2, t_1, t_2) = \frac{\partial(\cos\vartheta, \varphi)}{\partial(s_2, t_1)} = \frac{s_1}{2\sqrt{\lambda(s_1, m_1^2, m_2^2)}} = \frac{1}{\sqrt{-\Delta_4}} \tag{7}$$

where Δ_4 is the Gram determinant formed from the momenta of particles a, b, 1 and 3. Partial waves $A^{JM}_L(s, s_1, t_2)$ with definite orbital angular momentum L are formed from helicity partial wave amplitudes $a^{JMN}_{\lambda_1}$ (s, s_1, t_2) by [19]

$$A^{JM}_{L=J-1} = \sqrt{\frac{J+1}{2J+1}}\, a_1^{JM-} + \sqrt{\frac{J}{2J+1}}\, a_0^{JM-}$$

$$A^{JM}_{L=J+1} = \sqrt{\frac{J}{2J+1}}\, a_1^{JM} - \sqrt{\frac{J+1}{2J+1}}\, a_0^{JM-}$$

$$A^{JM}_{L=J} = a_1^{JM+}. \tag{8}$$

3. *Results and discussions*

Figure 2 shows the two-fold differential cross-section $\partial^2 N/\partial s_1 \partial t_2$ (in arbitrary units) together with the most prominent partial wave cross-sections. These differential cross--sections are given for the parameters $t_2 = -0.1$ and -0.3 GeV^2. At $m_{K^*\pi} = 1050$ MeV, that is just above the $K^*\pi$ threshold, the Deck peak is to 80% an S wave, so that $J^P = 1^+$ at both t_2 values studied.

At $m_{K^*\pi} = 1240$ MeV, the lower end of the Q region, $J^P = 1^+$ contributes still about 60%, at $m_{K^*\pi} = 1400$ MeV the $J^P = 1^+S$ wave only contributes ~30 to 40% where the larger $|t_2|$ value favours higher partial waves.

Another important partial wave is $A_L^{JM} = A_P^{20}$, which increases inside the Q mass interval from 5 to 10% at $t_2 = -0.1$ GeV² and from 10 to 18% at $t_2 = -0.3$ GeV². The contribution of the A_P^{21} amplitude decreases from ~6 to ~2% with increasing $m_{K^*\pi}$

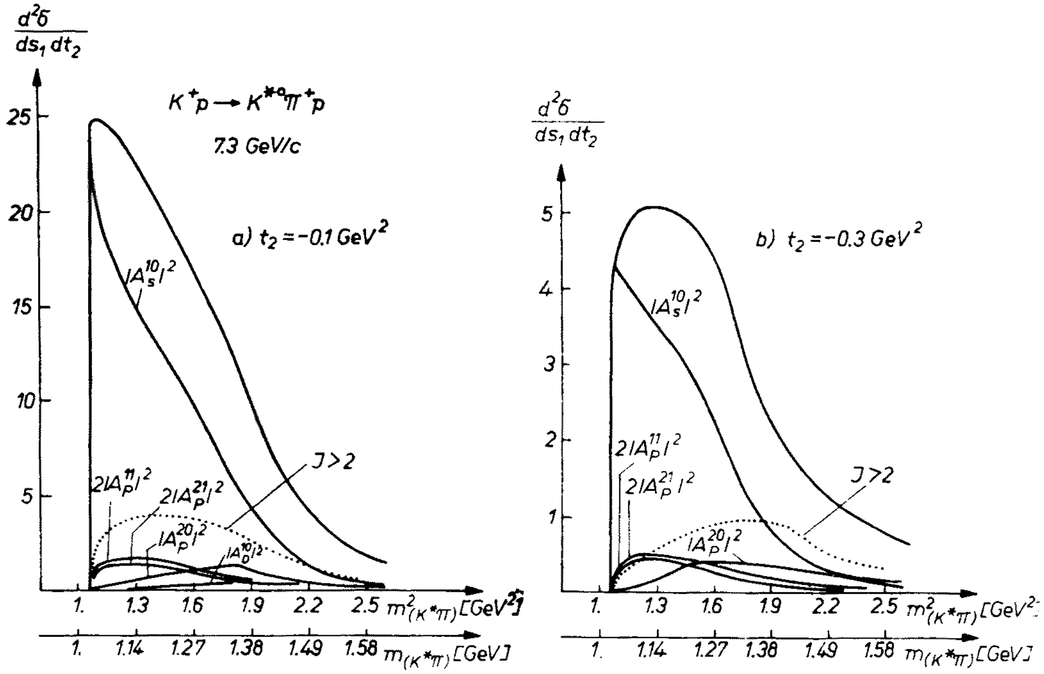

Fig. 2. Differential cross-section $\partial^2 N/\partial s_1\, \partial t_1$ of the reaction $K^+p \rightarrow K^{*0}\pi^+p$ at $t_2 = -0.1$ and -0.3 GeV² and the contributions to the cross-section from the most important partial waves

within the Q band. Partial waves with $J > 2$ contribute 20% (35%) at the lower (upper) end of the Q mass interval at $t_2 = -0.1$ GeV² and 18% (30%) at $t_2 = -0.3$ GeV².

Inside the Q band, especially towards high invariant $(K^*\pi)$ masses, no single partial wave

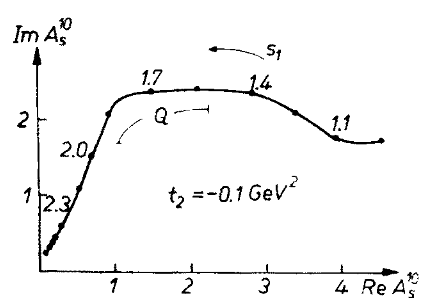

Fig. 3. Argand plot for the S wave at $t_2 = 0.1$ GeV²

dominates; therefore, a resonance interpretation is not appropriate. We still show in Figure 3 the Argand diagram of the S wave: At the lower end of the Q band, where the S wave is important, the phase of the amplitude stays almost constant; only at the upper end of the Q bump, where the S wave contributes only 1/3 to the cross-section, a rapid variation is seen; this however is probably due to the cut-off in t_1 applied, $|t_1| \leqslant 0.97$ GeV^2.

Our conclusions are:

— The Deck enhancement is the result of the strong limitations of the momentum transfers t_1 and t_2 due to double peripherality.

— Deck-type amplitudes lead to strong S waves in two-particle subsystems up to 200 MeV above threshold, quite irrespective of spin and parity of the decay products of the Deck enhancement. For a particle system containing a vector and a pseudoscalar particle this gives spin and parity 1^+, in agreement with the observed resonances in the Deck region.

— At the mass of the 1390 $(K^*\pi)$ resonance for which experimentally $J^P = 1^+$ is favoured, as is the case for the $(K^*\pi)$ resonance at 1270 MeV, the double peripheral amplitude does not give $J^P = 1^+$. Hence, it does not contain this resonance dually.

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