DOUBLE STATISTICAL TENSORS FOR RESONANCES PRODUCED BY POLARIZED PHOTONS

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Double statistical tensors for resonances produced by polarized photons are presented. The joint decay distribution in double resonance production is expressed in terms of four double statistical tensors ${}^{(\alpha)}T, \alpha = 0, 1, 2, 3$. The tensor ${}^{(o)}T$ describes the process induced by unpolarized photons, while the tensors ${}^{(i)}T$, $i = 1, 2, 3$ correspond to the processes induced by linearly or circularly polarized photons. The quark model predictions for the decay distributions of resonances photoproduced in the reaction $\gamma B \to VB^*$ are rewritten in terms of four statistical tensors ${}^{(\alpha)}T$, $\alpha = 0, 1, 2, 3$.

1. Introduction

In this paper the formalism for double statistical tensors describing the polarization of vector meson and isobar for the reaction:

$$\gamma B \to VB^* \tag{1}$$

is developed. Symbols γ, B, V and B^* denote respectively: a photon, a $1/2^+$ baryon, a vector meson and a $3/2^+$ isobar. It is assumed that the target nucleon is unpolarized.

The single statistical tensors for resonances produced by polarized photons in the reactions $\gamma B \to VB$ and $\gamma B \to PB^*$ (P denotes a pseudoscalar meson) are proposed in Ref. [1].

Experiments with polarized photons are very useful, as they can verify many more predictions of the various models than experiments with unpolarized photons. Now bubble chamber experiments with high-energy polarized photons at 2.8, 4.7 and 9.3 GeV photon energies are performed [2] for the process $\gamma p \to \varrho \Delta$. We are especially interested in testing a number of quark model predictions regarding angular decay distributions.

Quark model linear relations between the joint decay distributions of resonances photoproduced in the proces (1) are given in Ref. [3]. In Ref. [3] these relations were written in terms of the standard statistical tensors for linearly polarized photons; perpendicularly and parallelly to the scattering plane. Such a description, however, is not particularly convenient because the predictions depend on the degree of photon polarization, so now this parametrization is useless for the analysis of the experimental data with polarized photons.

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In this paper we introduce double statistical tensors describing the polarization of the vector meson and isobar in the reaction (1) showing explicitly the dependence on the polarization vector P_γ of the initial photon.

The definitions and the general properties of the double statistical tensors ${}^{(\alpha)}T$, $\alpha = 0, 1, 2, 3$ are discussed in Section 2. In Section 3 we present a method of measurement of elements of double statistical tensors ${}^{(\alpha)}T$, $\alpha = 0, 1, 2, 3$. In Section 4 we give as a special case the double statistical tensors for photons polarized perpendicularly and parallelly to the scattering plane and rewrite the quark model predictions from Ref. [3] in terms of the statistical tensors ${}^{(0)}T$ and ${}^{(1)}T$.

2. *Double statistical tensors for resonances produced by polarized photons*

According to Ref. [4] the joint density matrix of photoproduced vector meson and isobar can be written in a form showing explicitly the dependence on the polarization vector P_γ of the initial photon:

$$\varrho(V, B^*) = {}^{(0)}\varrho + \sum_{i=1}^{3} P_\gamma^{i\,(i)}\varrho, \tag{2}$$

where the length of P_γ is equal to the degree of photon polarization, while the direction of P_γ depends on the kind of polarization. Here, ${}^{(0)}\varrho$ is the joint density matrix of photoproduced isobar and vector meson when the photon is unpolarized.

Such formalism gives matrices ${}^{(\alpha)}\varrho$, $\alpha = 0, 1, 2, 3$ which do not depend on the degree of photon polarization.

We use the definitions of matrices ${}^{(\alpha)}\varrho$, $\alpha = 0, 1, 2, 3$ similarly to Ref. [5]:

$${}^{(0)}\varrho^{\lambda_V\lambda'_V}_{\lambda_\Delta\lambda_\Delta'} = \frac{1}{N}\sum T_{\lambda_V\lambda_\Delta,\lambda_\gamma\lambda_B}T^*_{\lambda_V'\lambda_\Delta',\lambda_\gamma\lambda_B}, \tag{3}$$

$${}^{(1)}\varrho^{\lambda_V\lambda_V'}_{\lambda_\Delta\lambda_\Delta'} = \frac{1}{N}\sum T_{\lambda_V\lambda_\Delta,-\lambda_\gamma\lambda_B}T^*_{\lambda_V'\lambda_\Delta',\lambda_\gamma\lambda_B}, \tag{4}$$

$${}^{(2)}\varrho^{\lambda_V\lambda_V'}_{\lambda_\Delta\lambda_\Delta'} = \frac{i}{N}\sum \lambda_\gamma T_{\lambda_V\lambda_\Delta,-\lambda_\gamma\lambda_B}T^*_{\lambda_V'\lambda_\Delta',\lambda_\gamma\lambda_B}, \tag{5}$$

$${}^{(3)}\varrho^{\lambda_V\lambda_V'}_{\lambda_\Delta\lambda_\Delta'} = \frac{1}{N}\sum \lambda_\gamma T_{\lambda_V\lambda_\Delta,\lambda_\gamma\lambda_B}T^*_{\lambda_V'\lambda_\Delta',\lambda_\gamma\lambda_B}, \tag{6}$$

where

$$N = 1/2 \sum_{\lambda_V\lambda_\Delta\lambda_\gamma\lambda_B} |T_{\lambda_V\lambda_\Delta,\lambda_\gamma\lambda_B}|^2.$$

The matrix elements of ${}^{(\alpha)}\varrho$, $\alpha = 0, 1, 2, 3$ fulfil the following relations given by the parity conservation in the helicity frame:

$${}^{(\alpha)}\varrho^{\mu\mu'}_{\lambda\lambda'} = (-1)^{\mu-\mu'+\lambda-\lambda'}\,{}^{(\alpha)}\varrho^{-\mu-\mu'}_{-\lambda-\lambda'}, \quad \alpha = 0, 1, \tag{7}$$

$${}^{(\alpha)}\varrho^{\mu\mu'}_{\lambda\lambda'} = -(-1)^{\mu-\mu'+\lambda-\lambda'}\,{}^{(\alpha)}\varrho^{-\mu-\mu'}_{-\lambda-\lambda'}, \quad \alpha = 2, 3. \tag{8}$$

For each matrix ${}^{(\alpha)}\varrho$, $\alpha = 0, 1, 2, 3$ the corresponding statistical tensor ${}^{(\alpha)}T$, $\alpha = 0, 1, 2, 3$ may be defined. The statistical tensors ${}^{(1)}T$, ${}^{(2)}T$ and ${}^{(3)}T$ include the degree of photon polarization P_γ.

The elements of double statistical tensors are defined in teims of the joint density matrix elements by the formula [6]:

$$ {}^{(\alpha)}T^{J_1J_2}_{M_1M_2} = \sum (-1)^{s_1+s_2+m+n-J_1-J_2} C(s_1-m;\, s_1m'|J_1M_1)C(s_2-n;\, s_2n'|J_2M_2)^{(\alpha)}\varrho^{mm'}_{nn'}, \quad (9) $$

where $\alpha = 0, 1, 2, 3$, s_1 and s_2 are the spins of the decaying particles and C is the Clebsch-Gordan coefficient.

Now we present the properties of double statistical tensors ${}^{(\alpha)}T$, $\alpha = 0, 1, 2, 3$.

i) Double statistical tensors ${}^{(0)}T$ and ${}^{(1)}T$

The principal properties of the tensors ${}^{(0)}T$ and ${}^{(1)}T$ are the same as for the case in which the resonances are produced in a collision of unpolarized particles [6], with one difference: there is no trace condition for ${}^{(1)}\varrho^{\mu\mu'}_{\lambda\lambda'}$, so

$$ {}^{(1)}T^{00}_{00} = \frac{1}{2\sqrt{3}} \operatorname{Tr}{}^{(1)}\varrho^{\mu\mu'}_{\lambda\lambda'}. \quad (10) $$

ii) Double statistical tensors ${}^{(2)}T$ and ${}^{(3)}T$

In the transversity frame ${}^{(2)}T^{J_1J_2}_{M_1M_2} = {}^{(3)}T^{J_1J_2}_{M_1M_2} = 0$ for M_1+M_2 even. In the helicity frame all statistical tensors with even J_1+J_2 are purely imaginary, while the tensors with odd J_1+J_2 are real. Moreover ${}^{(2)}T^{00}_{00} = {}^{(2)}T^{20}_{00} = {}^{(2)}T^{02}_{00} = {}^{(2)}T^{22}_{00} = {}^{(3)}T^{00}_{00} = {}^{(3)}T^{20}_{00} = T^{02}_{00} = {}^{(3)}T^{22}_{00} = 0$.

Formulae relating the double statistical tensors ${}^{(2)}T$ and ${}^{(3)}T$ to the density matrix elements ${}^{(2)}\varrho$ and ${}^{(3)}\varrho$ are given in Appendix.

3. *Measurements of double statistical tensors* ${}^{(\alpha)}T$, $\alpha = 0, 1, 2, 3$

The statistical tensors ${}^{(\alpha)}T$, $\alpha=0, 1, 2$ can be measured from the decay angular distribution $W^L\left(\frac{d\sigma^L}{dt}\right)\left(\frac{d\sigma^0}{dt}\right)^{-1}$ for linearly polarized photons:

$$ F_1(J_1)F_2(J_2)\, {}^{(0)}T^{J_1J_2}_{M_1M_2} = \frac{1}{2\pi} \langle Y^{J_1}_{M_1}(\theta_1, \varphi_1)Y^{J_2}_{M_2}(\theta_2, \varphi_2)\rangle_L, \quad (11) $$

$$ P_\gamma F_1(J_1)F_2(J_2)\, {}^{(1)}T^{J_1J_2}_{M_1M_2} = -\frac{1}{\pi} \langle \cos 2\Phi Y^{J_1}_{M_1}(\theta_1, \varphi_1)Y^{J_2}_{M_2}(\theta_2, \varphi_2)\rangle_L, \quad (12) $$

$$ P_\gamma F_1(J_1)F_2(J_2)\, {}^{(2)}T^{J_1J_2}_{M_1M_2} = -\frac{1}{\pi} \langle \sin 2\Phi Y^{J_1}_{M_1}(\theta_1, \varphi_1)Y^{J_2}_{M_2}(\theta_2, \varphi_2)\rangle_L \quad (13) $$

where Φ is the angle between the polarization vector of the photon and the production plane. Index "L" denotes that the expressions (11)–(13) are averaged over the decay angular distribution $W^L\left(\frac{d\sigma^L}{dt}\right)\left(\frac{d\sigma^0}{dt}\right)^{-1}$. The formulae (11)–(13) hold for any spin quantization axes.

From the formulae (11)–(13) it is seen that to evaluate experimentally *e.g.* a component of the statistical tensor ${}^{(1)}T$ we have to calculate the corresponding spherical harmonics multiplied by $\cos 2\Phi$ for each event, add up their values and divide the results by the number of events and $(-\pi F_1(J_1)F_2(J_2)P_\gamma)$.

To measure experimentally the statistical tensor ${}^{(3)}T$ the decay angular distribution W^C for circularly polarized photons must be used. Thus, in the helicity and Gottfried-Jackson frames:

$$\pm F_1(J_1)F_2(J_2)P_\gamma \operatorname{Im} {}^{(3)}T^{J_1J_2}_{M_1M_2} = \langle \operatorname{Im} Y^{J_1}_{M_1}(\theta_1, \varphi_1) \operatorname{Im} Y^{J_2}_{M_2}(\theta_2, \varphi_2)\rangle_C, \tag{14}$$

moreover

$$F_1(J_1)F_2(J_2) \operatorname{Re} {}^{(0)}T^{J_1J_2}_{M_1M_2} = \langle \operatorname{Re} Y^{J_1}_{M_1}(\theta_1, \varphi_1) \operatorname{Re} Y^{J_2}_{M_2}(\theta_2, \varphi_2)\rangle_C. \tag{15}$$

The signs "$\pm$" in formula (14) correspond to circular polarization with $\lambda_\gamma = \pm 1$. Index "C" denotes that the expressions (14)–(15) are averaged over the decay angular distribution W^C.

In all formulae (11)–(15) $F_1(J_1)$ and $F_2(J_2)$ are real coefficients [6]. For a vector meson ϱ $F_1(0) = \sqrt{\frac{3}{4\pi}}$ and $F_1(2) = -\sqrt{\frac{3}{10\pi}}$, while for an isobar Δ $F_2(0) = \sqrt{\frac{1}{\pi}}$ and $F_2(2) = -\sqrt{\frac{1}{5\pi}}$.

4. *Double statistical tensors for photons polarized perpendicularly and parallelly to the scattering plane*

As a special case we give the double statistical tensors for linearly polarized photons.

In terms of the statistical tensors the production of resonances by photons polarized perpendicularly to the scattering plane is described by the following expression (normalized to the unity):

$$\frac{{}^{(0)}T^{J_1J_2}_{M_1M_2} + P_\gamma\, {}^{(1)}T^{J_1J_2}_{M_1M_2}}{1+2\sqrt{3}\, P_\gamma\, {}^{(1)}T^{00}_{00}}, \tag{16}$$

while the case of production of resonances by photons polarized parallelly to the scattering plane is described by the quantity (normalized to the unity):

$$\frac{{}^{(0)}T^{J_1J_2}_{M_1M_2} - P_\gamma\, {}^{(1)}T^{J_1J_2}_{M_1M_2}}{1-2\sqrt{3}\, P_\gamma\, {}^{(1)}T^{00}_{00}}. \tag{17}$$

With linearly polarized photons one can measure independently fifty five double statistical tensor elements in contrast to the nineteen elements obtainable with an unpolarized beam.

We quote here the linear relations between the joint decay angular distributions of resonances produced in the process $\gamma B \to VB^*$ obtained in the quark model [3].

There are the following relations of class (a)

$$^{(0)}T^{20}_{00}+P_\gamma\,{}^{(1)}T^{20}_{00} = \sqrt{2}\,({}^{(0)}T^{02}_{00}+P_\gamma\,{}^{(1)}T^{02}_{00}), \tag{18}$$

$$^{(0)}T^{22}_{20}+P_\gamma\,{}^{(1)}T^{22}_{20} = \tfrac{1}{2}({}^{(0)}T^{20}_{20}+P_\gamma\,{}^{(1)}T^{20}_{20}), \tag{19}$$

$$^{(0)}T^{22}_{02}+P_\gamma\,{}^{(1)}T^{22}_{02} = \frac{1}{\sqrt{2}}({}^{(0)}T^{02}_{02}+P_\gamma\,{}^{(1)}T^{02}_{02}), \tag{20}$$

$$^{(0)}T^{22}_{00}+P_\gamma\,{}^{(1)}T^{22}_{00} = \frac{1}{2\sqrt{6}}(1+2\sqrt{3}\,P_\gamma\,{}^{(1)}T^{00}_{00})-\frac{1}{\sqrt{2}}({}^{(0)}T^{02}_{00}+P_\gamma\,{}^{(1)}T^{02}_{00}), \tag{21}$$

$$^{(0)}T^{22}_{20}-P_\gamma\,{}^{(1)}T^{22}_{20} = -({}^{(0)}T^{20}_{20}-P_\gamma\,{}^{(1)}T^{20}_{20}), \tag{22}$$

$$^{(0)}T^{22}_{02}-P_\gamma\,{}^{(1)}T^{22}_{02} = -\sqrt{2}\,({}^{(0)}T^{02}_{02}-P_\gamma\,{}^{(1)}T^{02}_{02}), \tag{23}$$

$$^{(0)}T^{22}_{2M} = P_\gamma\,{}^{(1)}T^{22}_{2M} \quad \text{for} \quad M = 2,\,-2, \tag{24}$$

$$^{(0)}T^{22}_{00}-P_\gamma\,{}^{(1)}T^{22}_{00} = -\frac{1}{2\sqrt{6}}(1-2\sqrt{3}\,P_\gamma\,{}^{(1)}T^{00}_{00}), \tag{25}$$

$$\sqrt{2}\,({}^{(0)}T^{02}_{00}-P_\gamma\,{}^{(1)}T^{02}_{00})+{}^{(0)}T^{20}_{00}-P_\gamma\,{}^{(1)}T^{20}_{00} =$$

$$= -\frac{1}{2\sqrt{3}}(1-2\sqrt{6}\,P_\gamma\,{}^{(1)}T^{00}_{00}). \tag{26}$$

There are the following relations of class (b):

$$^{(0)}T^{20}_{20}+P_\gamma\,{}^{(1)}T^{20}_{20} = -\sqrt{2}\,({}^{(0)}T^{02}_{02}+P_\gamma\,{}^{(1)}T^{02}_{02}), \tag{27}$$

$$^{(0)}T^{22}_{20}+P_\gamma\,{}^{(1)}T^{22}_{20} = -({}^{(0)}T^{22}_{02}+P_\gamma\,{}^{(1)}T^{22}_{02}), \tag{28}$$

$$\mathrm{Im}\,{}^{(0)}T^{22}_{2-2} = -P_\gamma\,\mathrm{Im}\,{}^{(1)}T^{22}_{2-2}, \tag{29}$$

$$\mathrm{Re}\,{}^{(0)}T^{22}_{1-1} = -P_\gamma\,\mathrm{Re}\,{}^{(1)}T^{22}_{1-1}. \tag{30}$$

There exist the following relations of class (c):

$$\mathrm{Im}\,{}^{(0)}T^{J_1J_2}_{M_1M_2} = -P_\gamma\,\mathrm{Im}\,{}^{(1)}T^{J_1J_2}_{M_1M_2} \quad \text{for} \quad J_1, M_1, J_2, M_2 =$$

$$= 2, 2, 0, 0;\ 0, 0, 2, 2;\ 2, 2, 2, 2;\ 2, 2, 2, 0;\ 2, 0, 2, 2, \tag{31}$$

$$\mathrm{Re}\,{}^{(0)}T^{22}_{11} = -P_\gamma\,\mathrm{Re}\,{}^{(1)}T^{22}_{11}, \tag{32}$$

$$^{(0)}T^{22}_{00}+P_\gamma\,{}^{(1)}T^{22}_{00}-{}^{(0)}T^{22}_{22}-P_\gamma\,{}^{(1)}T^{22}_{22}-{}^{(0)}T^{22}_{2-2}-P_\gamma\,{}^{(1)}T^{22}_{2-2} =$$

$$= \frac{1}{\sqrt{6}}(1+2\sqrt{3}\,P_\gamma\,{}^{(1)}T^{00}_{00}), \tag{33}$$

$$\mathrm{Im}\,{}^{(0)}T^{J2}_{02} = P_\gamma\,\mathrm{Im}\,{}^{(1)}T^{J0}_{02} \quad \text{for} \quad J = 0, 2, \tag{34}$$

$$ {}^{(0)}T^{22}_{11} - P_\gamma\,{}^{(1)}T^{22}_{11} = {}^{(0)}T^{22}_{1-1} - P_\gamma\,{}^{(1)}T^{22}_{1-1}. \tag{35}$$

$$ {}^{(0)}T^{02}_{00} + \sqrt{2}\,{}^{(0)}T^{20}_{00} + \sqrt{6}\,{}^{(0)}T^{02}_{02} = P_\gamma({}^{(1)}T^{02}_{00} + \sqrt{2}\,{}^{(1)}T^{20}_{00} + \sqrt{6}\,{}^{(1)}T^{02}_{02}), \tag{36}$$

$$\mathrm{Re}\,{}^{(3)}T^{02}_{01} = -2\sqrt{2}\,\mathrm{Re}\,{}^{(3)}T^{22}_{01} = \frac{2}{\sqrt{3}}(\mathrm{Re}\,{}^{(3)}T^{22}_{21} + \mathrm{Re}\,{}^{(3)}T^{22}_{2-1}), \tag{37}$$

$$\mathrm{Im}\,{}^{(3)}T^{02}_{01} = \sqrt{2}\,\mathrm{Im}\,{}^{(3)}T^{22}_{01} = \frac{1}{\sqrt{3}}(\mathrm{Im}\,{}^{(3)}T^{22}_{2-1} - \mathrm{Im}\,{}^{(3)}T^{22}_{21}), \tag{38}$$

$$\mathrm{Im}\,{}^{(3)}T^{22}_{1M} = 0 \quad \text{for} \quad M = 2, -2. \tag{39}$$

There is a set of class (a′) relations:

$$ {}^{(0)}T^{2J}_{00} = \frac{2}{\sqrt{6}}\,\mathrm{Re}\,{}^{(0)}T^{2J}_{20} \quad \text{for} \quad J = 0, 2, \tag{40}$$

$$ {}^{(0)}T^{22}_{22} + ({}^{(0)}T^{22}_{2-2})^* = \sqrt{6}\,{}^{(0)}T^{22}_{02}, \tag{41}$$

$$ {}^{(0)}T^{22}_{11} = ({}^{(0)}T^{22}_{1-1})^*, \tag{42}$$

$$\mathrm{Re}\,{}^{(3)}T^{2J}_{10} = 0 \quad \text{for} \quad J = 0, 2, \tag{43}$$

$$\mathrm{Re}\,{}^{(3)}T^{22}_{1-2} = -\mathrm{Re}\,{}^{(3)}T^{22}_{12}, \tag{44}$$

$$\sqrt{6}\,\mathrm{Re}\,{}^{(1)}T^{20}_{20} + {}^{(1)}T^{20}_{00} = 2\sqrt{2}\,{}^{(1)}T^{00}_{00}, \tag{45}$$

$$\mathrm{Im}\,{}^{(1)}T^{20}_{20} = -\,\mathrm{Im}\,{}^{(2)}T^{20}_{10}, \tag{46}$$

$$2\,\mathrm{Re}\,{}^{(1)}T^{20}_{20} - \sqrt{6}\,{}^{(1)}T^{20}_{00} = -4\,\mathrm{Re}\,{}^{(2)}T^{20}_{10}. \tag{47}$$

We note that relations (18)–(21) and (27)–(33) correspond to the case when resonances are produced by photons polarized perpendicularly to the scattering plane, while relations (22)–(26) and (34)–(36) correspond to the case of photons polarized parallelly to the scattering plane. The relations (37)–(39) and (43)–(44) are obtained for circularly polarized photons, relations (40)–(42) for an unpolarized photon beam and relations (45)–(47) for linearly polarized photons.

The discussion of linear quark model relations between the joint decay angular distributions in the process (1) can be found in Refs [3] and [7]. The analysis of nonlinear quark model relations between the decay parameters for the process (1) is contained in Ref. [8].

In this paper the formalism necessary to analyze the production of double resonances with polarized photons in terms of double statistical tensors is given. Consequently, a tool for checking the quark model relations between the joint decay angular distributions of the resonances produced in the process $\gamma B \to VB^*$ is presented.

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APPENDIX

Formulae relating the double statistical tensors ${}^{(2)}T$ and ${}^{(3)}T$ to the density matrix elements ${}^{(2)}\varrho$ and ${}^{(3)}\varrho$

$$ {}^{(\alpha)}T^{22}_{21} = \frac{1}{\sqrt{2}}\left({}^{(\alpha)}\varrho^{-11}_{-3-1} - {}^{(\alpha)}\varrho^{-11}_{13}\right), \tag{A.1} $$

$$ {}^{(\alpha)}T^{22}_{2-1} = \frac{1}{\sqrt{2}}\left({}^{(\alpha)}\varrho^{-11}_{\ 31} - {}^{(\alpha)}\varrho^{-11}_{-1-3}\right), \tag{A.2} $$

$$ {}^{(\alpha)}T^{22}_{12} = \tfrac{1}{2}\left({}^{(\alpha)}\varrho^{-10}_{-31} + {}^{(\alpha)}\varrho^{-10}_{-13} - {}^{(\alpha)}\varrho^{01}_{-31} - {}^{(\alpha)}\varrho^{01}_{-13}\right), \tag{A.3} $$

$$ {}^{(\alpha)}T^{22}_{10} = \frac{1}{2\sqrt{2}}\left({}^{(\alpha)}\varrho^{-10}_{-} - {}^{(\alpha)}\varrho^{01}_{-}\right), \tag{A.4} $$

$$ {}^{(\alpha)}T^{22}_{1-2} = \tfrac{1}{2}\left({}^{(\alpha)}\varrho^{-10}_{1-3} - {}^{(\alpha)}\varrho^{01}_{1-3} + {}^{(\alpha)}\varrho^{-10}_{3-1} - {}^{(\alpha)}\varrho^{01}_{3-1}\right), \tag{A.5} $$

$$ {}^{(\alpha)}T^{22}_{01} = \frac{1}{2\sqrt{3}}\left({}^{(\alpha)}\varrho^{-}_{-3-1} - {}^{(\alpha)}\varrho^{-}_{13}\right), \tag{A.6} $$

$$ {}^{(\alpha)}T^{20}_{10} = \frac{1}{2\sqrt{2}}\left({}^{(\alpha)}\varrho^{-10} - {}^{(\alpha)}\varrho^{01}\right), \tag{A.7} $$

$$ {}^{(\alpha)}T^{02}_{01} = \frac{1}{\sqrt{6}}\left({}^{(\alpha)}\varrho_{-3-1} - {}^{(\alpha)}\varrho_{13}\right), \tag{A.8} $$

$$ {}^{(\alpha)}T^{22}_{22} = i\sqrt{2}\,\mathrm{Im}\,{}^{(\alpha)}\varrho^{-11}_{-31} \text{ (helicity frame)}, \tag{A.9} $$

$$ {}^{(\alpha)}T^{22}_{20} = i\left(\mathrm{Im}\,{}^{(\alpha)}\varrho^{-11}_{-33} + \mathrm{Im}\,{}^{(\alpha)}\varrho^{-11}_{-11}\right) \text{ (helicity frame)}, \tag{A.10} $$

$$ {}^{(\alpha)}T^{22}_{2-2} = \sqrt{2}\,i\,\mathrm{Im}\,{}^{(\alpha)}\varrho^{-11}_{1-3} \text{ (helicity frame)}, \tag{A.11} $$

$$ {}^{(\alpha)}T^{22}_{11} = i\left(\mathrm{Im}\,{}^{(\alpha)}\varrho^{01}_{13} + \mathrm{Im}\,{}^{(\alpha)}\varrho^{0-1}_{31}\right) \text{ (helicity frame)}, \tag{A.12} $$

$$ {}^{(\alpha)}T^{22}_{02} = \frac{i}{\sqrt{3}}\,\mathrm{Im}\left({}^{(\alpha)}\varrho^{11}_{-13} + {}^{(\alpha)}\varrho^{11}_{-31} - 2\,{}^{(\alpha)}\varrho^{00}_{-13}\right) \text{ (helicity frame)}, \tag{A.13} $$

$$ {}^{(\alpha)}T^{22}_{1-1} = i\left(\mathrm{Im}\,{}^{(\alpha)}\varrho^{10}_{13} + \mathrm{Im}\,{}^{(\alpha)}\varrho^{-10}_{31}\right) \text{ (helicity frame)}, \tag{A.14} $$

$$ {}^{(\alpha)}T^{02}_{02} = \frac{i}{2}\,\mathrm{Im}\,{}^{(\alpha)}\varrho_{-11} \text{ (helicity frame)}, \qquad \text{(A.15} $$

$$ {}^{(\alpha)}T^{20}_{20} = i\sqrt{\tfrac{2}{3}}\,\mathrm{Im}\,{}^{(\alpha)}\varrho^{-31} \text{ (helicity frame)}, \tag{A.16} $$

where $\alpha = 2,3$. We use the symbols:

$$\varrho_{-}^{mm'} = \varrho_{33}^{mm'} - \varrho_{11}^{mm'} - \varrho_{-1-1}^{mm'} + \varrho_{-3-3}^{mm'}, \qquad (A.17)$$

$$\varrho_{nn'}^{-} = \varrho_{nn'}^{11} - 2\varrho_{nn'}^{00} + \varrho_{nn'}^{-1-1}, \qquad (A.18)$$

$$\varrho^{mm'} = \sum_{n} \varrho_{nn}^{mm'}, \qquad (A.19)$$

$$\varrho_{nn'} = \sum_{m} \varrho_{nn'}^{mm}. \qquad (A.20)$$

We write for the isobar indices $2n$ and $2n'$ instead of n and n'.

Formulae (A.1)–(A.8) give the double statistical tensors elements in the transversity frame as well as in the helicity frame.

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