GAMMA NEUTRINO CORRELATION IN MUON CAPTURE IN ^{19}F

By G. Hock* and Z. Oziewicz**

Institute of Theoretical Physics, University of Wrocław**

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The ratio of the two amplitudes describing completely each of the transitions $^{19}\mathrm{F}(\mathrm{g.s.}, \frac{1}{2}^+) \xrightarrow{\mu} {}^{19}\mathrm{O}(\frac{1}{2}^+)_{1,2}$ is studied. For the three transitions $^{19}\mathrm{F}(\mathrm{g.s.}, \frac{1}{2}^+) \xrightarrow{\mu} {}^{19}\mathrm{O}(\frac{3}{2}^+)_{1,2,3}$ the gamma-neutrino directional correlation and circular polarization of nuclear gamma rays are calculated. All calculations were performed in shell model intermediate coupling with configuration mixing.

1. Introduction

This paper is a direct extension of the work [1] where the capture rates of the allowed partial transitions to two levels of ^{19}O with $J^\pi = \frac{1}{2}^+$ ($E_1^* = 1.47$ MeV and $E_2^* = 3.24$ MeV) and three levels having $J^\pi = \frac{3}{2}^+$ ($E_1^* = 0.097$ MeV, $E_2^* = 2.37$ MeV and $E_3^* = 4.12$ MeV) [2] have been calculated. It is clear that the capture rates alone do not enable us to get a possible complete picture on the role of the pseudoscalar interaction as well as on the nuclear structures but there are often needed additional independent observables such as, for instance, gamma-neutrino correlations.

As in [1] we concentrate here our attention on the dependence of angular and polarization distributions in these transitions only upon the induced pseudoscalar C_P and the central force strength V_0 chosen as the nuclear model parameter.[1]

The kinematic formulae describing the muon capture processes $\frac{1}{2} \xrightarrow{\mu^-} \frac{1}{2} \xrightarrow{\gamma} j$ and $\frac{1}{2} \xrightarrow{\mu^-} \frac{3}{2} \xrightarrow{\gamma} j$ is taken from [4].

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* Institute of Nuclear Research of the Hungarian Academy of Sciences, Debrecen, Hungary.

** Address: Instytut Fizyki Teoretycznej, Uniwersytet Wrocławski, Cybulskiego 36, 50-205 Wrocław, Poland.

[1] As usual, for the weak vertex we adopt Morita and Fujii's Hamiltonian [3] and neglect all relativistic corrections to the muon wave function.

2. *Nuclear wave functions*

The nuclear shell model wave functions have been calculated in configuration mixing of the type $\Sigma_n(1d)^{3-n}(2s)^n$ while the $1d$ particles are treated in intermediate coupling [5]. For this the following nuclear parameters were chosen:

(*i*) the strength of spin-orbit force $\xi_d = -2.032$ MeV,

(*ii*) energy difference in one-particle energies of the degenerate shell for ^{17}O, $(d-s) = 1.161$ MeV,

(*iii*) the central force strength V_0 has been varied: $V_0 = 0$ (jj — coupling extreme), -30, -40 and -50 MeV,

(*iv*) the exchange parameters of central forces are the same as those of Rosenfeld,

(*v*) for the two-particle radial integrals Gaussian potential shape and oscillator basis were taken while $a/r_0 = 1$, a being the force range and r_0 the well size parameter. The evaluation of these integrals has been carried out in terms of the Talmi method.

In calculating the nuclear matrix elements the oscillator parameter r_0 was fixed as $r_0 = 1.64$ fm [5]. An average exciting energy ~2 MeV of the final states in question was taken for the neutrino energy q. The main matrix elements for the transitions considered here turned out to be small compared to those of allowed transitions in the case of some other light nuclei [6]. Thus the nuclear model dependence manifests itself rather strong, in general, contrary to the behaviour under the pseudoscalar coupling. For the transition to the level ^{19}O ($3/2^+$, $E^* = 0.097$ MeV) however, there appeared a significant model independence only of the pseudoscalar amplitude (the largest one among the other amplitudes) being of most interest in our consideration.

3. *Transition to the $J_f^\pi = \frac{1}{2}^+$ levels*

As we have $T_i \neq T_f$ there is no contribution[2] from the multipole $I = 0$, where I denotes the total momentum of the lepton field $|J_f - J_i| \leqslant I \leqslant J_f + J_i$.

Therefore each of these transitions is completely described by two amplitudes (real if time invariance in weak interaction holds) obtainable experimentally from the time dependence of the corresponding capture rates. For them we have [3, 4]

$$P_1 = (G_A - G_P)([101] + \sqrt{2}\,[121]) + 3C_A\left[011\,\frac{p}{M}\right]$$

$$M_1 = G_A\left([101] - \frac{1}{\sqrt{2}}[121]\right) - \sqrt{3/2}\,C_V\left[111\,\frac{p}{M}\right]. \tag{1}$$

We note that the induced pseudoscalar C_P enters only the amplitude P. As is well known, the ratio P_1/M_1 changes sign in the nuclear model independent approximation[3] (MIA) at $C_P \approx 20\,C_A$. By determining the sign of this ratio the doublevaluedness of C_P/C_A

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[2] Up to the small correction terms of the order $(1/M)^2$ where M is the nucleon mass.

[3] MIA neglects contributions of all nuclear matrix elements except for [101].

provided by any sole muon capture rate might be resolved for the realistic interaction, *i.e.* C_P/C_A may be fixed up to the limit *e.g.* $C_P/C_A \lesssim 20$.

For the transition to the level at $E_1^* = 1.47$ MeV P_1/M_1 was find to be practically unaffected by the nuclear model parameter V_0 so it yields nothing more than the [101] approximation (MIA in Fig. 1). There exists, however, a much more striking influence

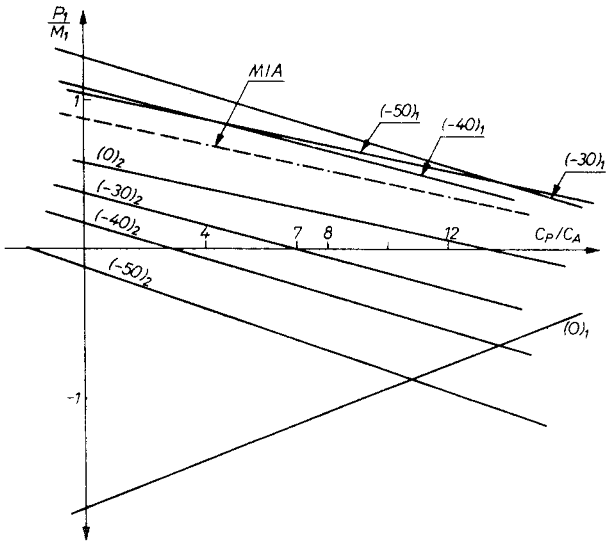

Fig. 1. The ratio P_1/M_1 *versus* C_P/C_A. The lines are marked by $(V_0)_{1,2}$ where the suffices 1 and 2 denote transitions to the levels of $^{19}O(1/2^+)$ at $E_1^* = 1.47$ MeV and $E_2^* = 3.24$ MeV, respectively

of the nucleus on P_1/M_1 in the transition to the second $1/2^+$ level at $E_2^* = 3.24$ MeV, which might be exploited for checking C_P/C_A after having the simple shell model picture cleared up convincingly.

The angular distribution and polarization measurements are valuable for the considered transitions only for checking the T nonconserving effect in muon capture. The circular polarization of nuclear gamma rays for the case when the neutrino and gamma momenta ($\boldsymbol{q}$ and $\boldsymbol{k}$ respectively) are perpendicular to the muon polarization $\boldsymbol{\sigma}$ and to one another[4] $\boldsymbol{q} \wedge \boldsymbol{k} \cdot \boldsymbol{\sigma} = 1$ demonstrates the T noninvariance in the muon capture process and has the form

$$P_\gamma = \tfrac{3}{4}\lambda \frac{\operatorname{Im} M_1 P_1^*}{2M_1^2 + P_1^2} \exp(-Rt) \tag{2}$$

for $\frac{1}{2} \xrightarrow{\mu^-} \frac{1}{2} \xrightarrow{\gamma} \frac{3}{2}$ transitions [4].

Here λ is the muon polarization on the K shell and R the conversion rate between the hyperfine (hf) levels (for the details consult [7] or [4]). The study of time noninvariant

[4] In our notation $\boldsymbol{\sigma}$, $\boldsymbol{q}$ and $\boldsymbol{k}$ are unit vectors. The formula (2) is given for left-handed neutrino emission.

correlations in muon capture may help us to determine more accurately T-conservation than in beta-decay processes where T-parity breaking may occur in the background of Coulomb phase shift only.

4. Gamma-neutrino correlation in the transitions to the $J_f = \frac{3}{2}^+$ levels

The general form of the gamma-neutrino correlation for $\frac{1}{2} \xrightarrow{\mu} \frac{3}{2}$ transitions is

$$W = 1 + \sum_{S=1}^{3} B_{S\eta} a_S P_S(\boldsymbol{k} \cdot \boldsymbol{q}). \tag{3}$$

Here the terms $B_{S\eta}$ and a_S describe the nuclear radiation and the weak process, respectively. If muon capture takes place from the $F = 0$ state ($F = J_i \pm 1/2$) all the coefficients a_S are simply kinematical constants, *viz.*

$$a_S = \sqrt{2S+1}\, C^{\frac{1}{2}\frac{1}{2}}_{\frac{1}{2}\frac{1}{2}S0}.$$

Therefore, we do not consider here the time dependence of the correlation [7] and we restrict ourself only to the case of statistically populated hf levels. The $\frac{1}{2} \xrightarrow{\mu} \frac{3}{2}$ transition is described by four independent amplitudes. Besides M_1 and P_1 defined in (1) we have still the amplitudes for $I = 2$

$$A_2 = -G_A \sqrt{2}\,[122] + \dots$$
$$V_2 = 3G_V[022] + \dots$$

We note that these amplitudes belong to the terms of second forbiddenness according to Morita and Fujii's classification [3]. The pseudoscalar is strictly forbidden to contribute to these amplitudes. Moreover, in the absence of the vector current we have $V_I = 0$. Thus, all observables including the coefficients a_S could be expressed in terms of these four independent amplitudes.

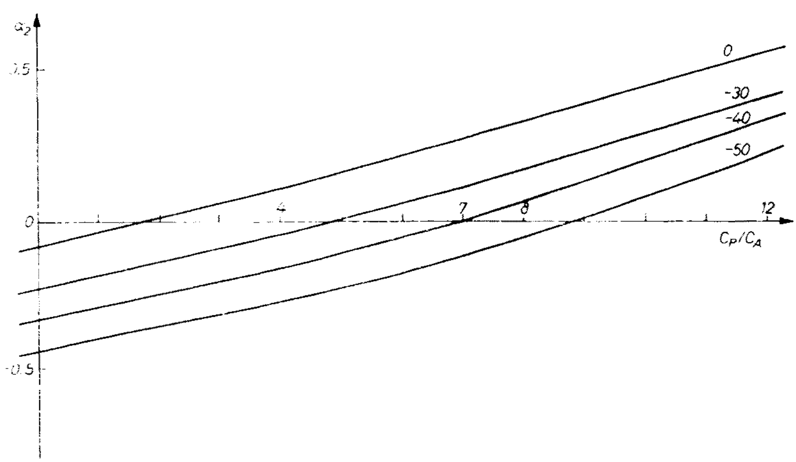

Fig. 2. The coefficient a_2 of the directional gamma-neutrino correlation. The nuclear model parameter V_0 is indicated by the numbers above the curves

4.1. The directional gamma-neutrino correlation

The directional gamma-neutrino correlation is determined by a_2 alone. For the statistically populated hf levels we have [4]

$$a_2 \equiv \frac{a_2^{\text{stat}}}{A^{\text{stat}}} = \frac{M_1^2 - P_1^2 + 3M_1A_2 - \frac{3}{4}A_2^2 - V_2^2}{2M_1^2 + P_1^2 + \frac{3}{2}A_2^2 + V_2^2}. \tag{4}$$

This coefficient is shown in Fig. 2 for the transition to the level at $E_1^* = 0.097$ MeV. For the transitions to the other two higher levels ($E_2^* = 2.37$ MeV and $E_3^* = 4.12$ MeV) a_2 does not reveal any significant dependence on C_P.

4.2. The circular polarization of nuclear deexcitation gamma rays

For the pure electromagnetic transition to ^{19}O (g.s.) the term $B_{3\eta}$ vanishes and so we have from (3) for the circular polarization

$$P_\gamma(\varphi) = -\frac{3}{2\sqrt{5}} \frac{a_1 \boldsymbol{k}\cdot\boldsymbol{q}}{1+0.1a_2P_2(\boldsymbol{k}\cdot\boldsymbol{q})}$$

$$\cos\varphi = \boldsymbol{k}\cdot\boldsymbol{q}. \tag{5}$$

We are interested in the case of the statistically populated hf levels thus we have

$$a_1 = \sqrt{5}\,\frac{M_1^2 + \frac{3}{5}M_1A_2 + \frac{9}{20}A_2^2 + \frac{2}{5}P_1V_2}{2M_1^2 + P_1^2 + \frac{3}{2}A_2^2 + V_2^2}. \tag{6}$$

As it is apparent from Figs 3 and 4 the circular polarization of gamma rays shows a strong dependence on the nuclear model. The situation in unfortunately almost the same as in

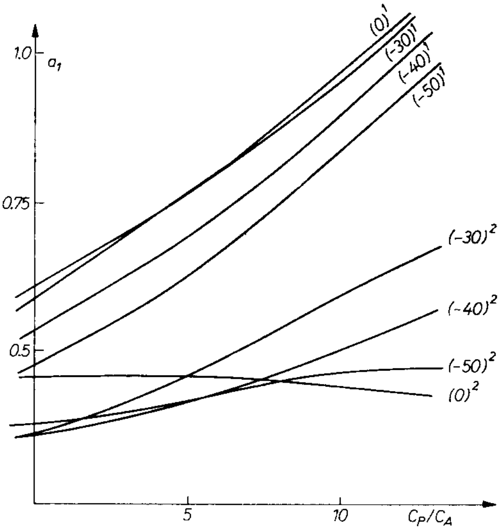

Fig. 3. The circular polarization of nuclear deexcitation gamma rays $P_\gamma(\varphi)$ (with a factor $-2\sqrt{5/3}$) at the angle $\varphi = \arccos 1/\sqrt{3}$. The upper indices ([1], [2]) denote the transitions $\xrightarrow{\mu}$ ^{19}O($3/2^+$, $E_1^* = 0.097$ MeV) and $\xrightarrow{\mu}$ ^{19}O ($3/2^+$. $E_2^* = 2.37$ MeV), respectively

the case of gamma-neutrino directional correlation (Fig. 2) where the second forbidden terms destroy the sensibility of a_2 on C_P.

We may conclude that only the total experiment, *i.e.* the independent determination of all four amplitudes may give valuable information. For this aim it is completely sufficient

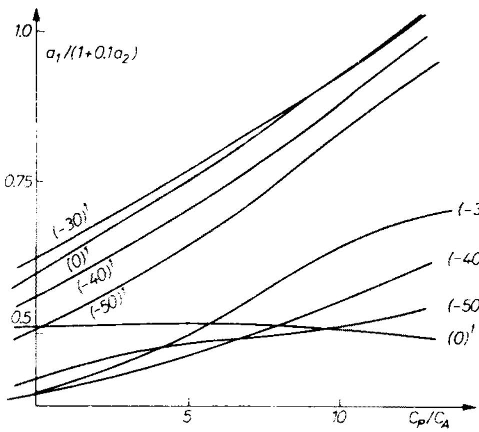

Fig. 4. The circular polarization $P_\gamma(\varphi)$ (with the factor $-2\sqrt{5}/3$) at the angle $\varphi = \arccos 1$; see caption to Fig. 3

to measure the time dependence of the capture rate together with circular polarization $P_\gamma(\varphi)$. The most interesting point is that the amplitude P in the transition to the ^{19}O ($3/2^+$, $E^* = 0.097$ MeV) is independent of nuclear parameter V_0 in the considered region. Moreover the amplitude P is the largest one, being of most interest in our consideration, because by definition only this amplitude contains the induced pseudoscalar C_P.

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