

# ON THE ANGULAR DISTRIBUTION OF SECONDARY PARTICLES IN $\pi^-p$ INTERACTIONS AT 60 GeV

BY B. FURMAŃSKA, J. GIERULA, R. HOLYŃSKI, S. KRZYWDZIŃSKI

Institute of Nuclear Physics, Cracow\*

AND A. LINSCHIED

Physics Department, Agricultural College, Cracow

(Received September 6, 1972)

In the present paper the asymmetry of the angular distribution of secondary particles in 60 GeV  $\pi^-p$  interactions is investigated. In particular, the influence of the use of the ultrarelativistic transformation on the obtained value of the asymmetry parameters of angular distribution is estimated.

The angular distributions of secondary particles in  $\pi^-p$  interactions with primary energies 16 GeV and 60 GeV are compared.

A simple approximate method of transformation of the angular distribution from the lab-system to the CM-system in the case of unknown masses and energies of produced particles is presented. The method is tested on 16 GeV  $\pi^-p$  and 67 GeV  $p-p$  data.

## 1. Introduction

In a number of papers published by the Alma-Ata, Budapest, Cracow, Dubna, Moscow, Sofia, Tashkent and Ulan-Bator Collaboration [1, 2, 3], an asymmetry of the angular distributions of secondary particles in 60 GeV  $\pi^-p$  interactions registered in emulsion has been reported. This asymmetry is seen as an excess of particles emitted forwards in the pion-proton CM system.

However, in order to estimate the significance of the asymmetry of angular distribution in the discussed experiment, one should take into account some facts which can considerably influence the obtained values of the so-called asymmetry parameters.

As masses and momenta of secondary particles were not measured, the approximate, ultrarelativistic transformation of the angular distribution to the CM-system was applied:

$$\log \tan \frac{\theta_{\text{CM}}}{2} = \log \tan \theta_{\text{L}} + \log \gamma_c \quad (1)$$

which is obtained under the assumption  $\beta_c = \beta_{\text{CM}}$  where  $\theta_{\text{L}}$  — emission angle of particle

---

\* Address: Instytut Fizyki Jądrowej, Zakład V, Radzikowskiego 152, 31-342 Kraków, Poland.

in the lab-system,  $\theta_{\text{CM}}$  — emission angle of particle in the CM-system,  $\beta_c$  — velocity of the CM-system with respect to the lab-system,  $\beta_{\text{CM}}$  — velocity of particle in the CM-system,

$$\gamma_c = \frac{1}{(1 - \beta_c^2)^{1/2}}.$$

There is a risk that the application of the ultrarelativistic transformation may produce an artificial, unphysical asymmetry of the angular distribution. In many papers dealing with proton-proton interactions in emulsion at energies ranging from a few GeV up to 30 GeV (Winzeler *et al.* [4], Meyer *et al.* [5], Marzari *et al.* [6], Barbaro-Galtieri *et al.* [7], Lohrman *et al.* [8]) a small asymmetry of the composite angular distribution of all particles

in  $\log \tan \frac{\theta_{\text{CM}}}{2} = \log \tan \theta_L + \log \gamma_c$  variable was also observed, although the symmetry

in the CM-system in this case is obvious. The same has been found in the recent studies on the 67 GeV  $p$ - $p$  interactions (Alma-Ata, Cracow, Dubna, Leningrad, Moscow, Tashkent, Ulan-Bator Collaboration [9]).

Therefore, in the following chapters, we shall discuss the exact relation between  $\log \tan \theta_L$  and  $\log \tan \frac{\theta_{\text{CM}}}{2}$  variables.

Then, using the experimental material of 16 GeV  $\pi^-$ - $p$  interactions in the two-metre HBC at CERN, gathered by the Aachen-Berlin-Bonn-CERN-Cracow-Warsaw Collaboration [10], we will try to estimate the influence of the application of the ultrarelativistic transformation to the CM-system on the value of the asymmetry parameter. Finally we will present the CMS angular distributions for 60 GeV  $\pi^-$ - $p$  interactions obtained by the introduced approximate transformation procedure, called the folding procedure. This transformation is much closer to the exact one than the ultrarelativistic transformation.

## 2. Influence of the use of the ultrarelativistic transformation on the estimate of the asymmetry of angular distribution

Figures 1 and 2 show the relation between  $\log \tan \theta_L$  and  $\log \tan \frac{\theta_{\text{CM}}}{2}$  for pions (Fig. a) and protons (Fig. b). The curves correspond to different values of transverse momentum. They were calculated for two primary energies: 16 GeV ( $\gamma_c = 3.05$ ) and 60 GeV ( $\gamma_c = 5.72$ ). In each figure the curve corresponding to  $\beta_L = 0.7$  is drawn. This is the conventional limit between the relativistic and slow tracks in the lab-system. (For the details of the calculation, see Appendix.)

From Figs 1 and 2 we can conclude the following:

a) the ultrarelativistic transformation formula (1) is a fairly good approximation for relativistic pions ( $\beta_L > 0.7$ ), but it is completely inadequate for protons. The ultrarelativistic transformation leads to the appearance of an artificial asymmetry of angular distribution which manifests itself as an excess of particles in the forward cone.

b) The correspondence of the exact transformation formula with the ultrarelativistic approximation is the better the smaller is the emission angle  $\theta_L$  in the lab system, though it does not improve with increasing energy in the range 16–60 GeV.

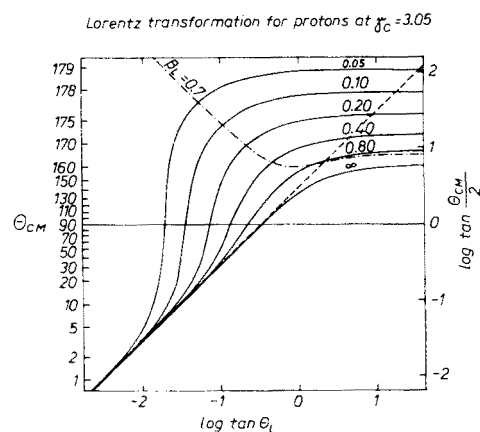
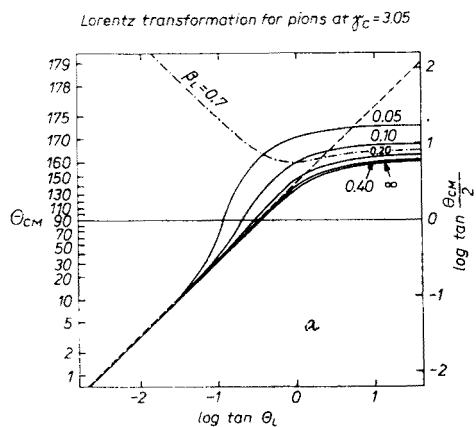


Fig. 1

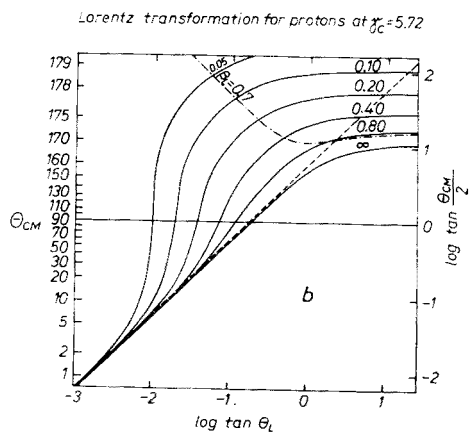
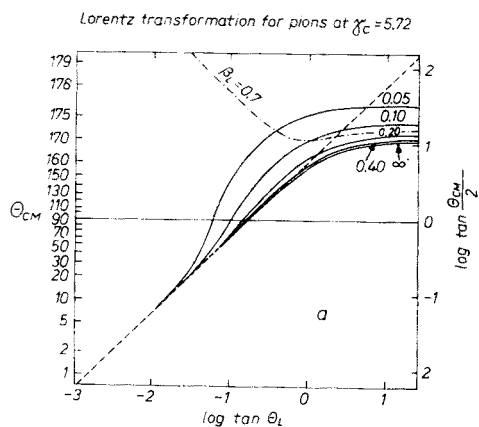


Fig. 2

Fig. 1. Comparison of the ultrarelativistic transformation with the exact one. Full curves represent the exact Lorentz transformation between lab and CMS for different values of transverse momentum. The curves were calculated from the formula (A5) given in the Appendix, for  $\gamma_c = 3.05$  (16 GeV), for pions (Fig. 1a) and protons (Fig. 1b). The dotted line corresponds to the ultrarelativistic transformation formula (1).

The curve  $\beta_L = 0.7$  separates the relativistic particles from the slow ones

Fig. 2. Lorentz transformation for pions and protons at  $\gamma_c = 5.72$  (60 GeV). For details see caption of figure 1

A comparison between the ultrarelativistic transformation and the exact one has been done also on the 16 GeV  $\pi^-p$  HBC data from the Aachen-Berlin-Bonn-CERN-Cracow-Warsaw Collaboration [10]. The sample consisted of more than eighty thousand events with the number of prongs ranging from two to ten. Higher multiplicities were not considered as they contribute a very small percentage (about 0.1%) to the inelastic cross-section.

For each charged multiplicity all reaction channels were taken into account.

For each secondary particle, its charge, mass, momentum and the angle of emission in the lab-system are known here. Thus we can transform the angular distribution to the

CM-system by means of the exact formulae. On the other hand, we may apply the approximate formulae and then estimate the influence of the approximation on the value of the asymmetry parameter.

We denote by  $n_{ch}$  the number of prongs in a star, by  $n_s$  the number of relativistic tracks with ionization less than  $1.4 g_{pl}$ , where  $g_{pl}$  is the plateau ionization, and by  $N_h$  — the number of slow particles with ionization greater than  $1.4 g_{pl}$ . Such a distinction allows us to compare the results with those obtained by emulsion techniques.

The features of ultrarelativistic transformation are illustrated in Fig. 3, on the example of the angular distributions of relativistic pions and protons for 4-prong and 6-prong

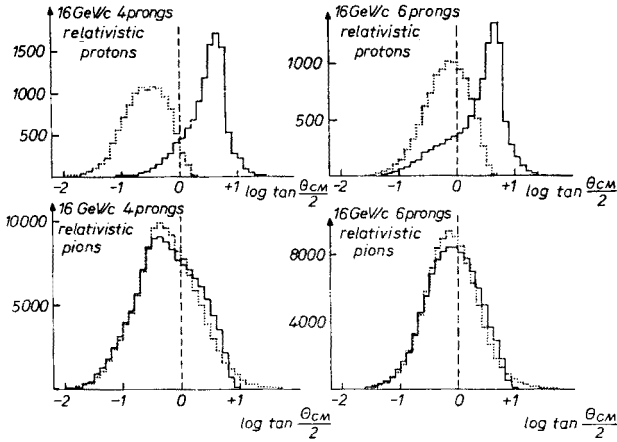


Fig. 3. CMS angular distributions of relativistic pions and protons obtained by the exact transformation (full line histograms) and by the ultrarelativistic transformation (dotted line histograms)

stars in 16 GeV  $\pi^-p$  interactions. The strong influence of protons in producing the artificial forward asymmetry is clearly seen.

In the further analysis of the 16 GeV HBC data and of the 60 GeV emulsion data, we will use as a convenient parameter of asymmetry the median value  $M$  of the  $\log \tan \frac{\theta_{CM}}{2}$  distribution. For a symmetric distribution the median value equals zero. This corresponds to the angle  $\theta_{CM} = 90^\circ$ . The excess of the emitted particles in the forward direction gives a negative value of the asymmetry parameter  $M$ . Very often the median value  $M$  overlaps with the mode of the  $\log \tan \frac{\theta}{2}$  distribution and indicates a system in which there is an exact symmetry of emitted particles. In connection with the quark model it is customary to look for such systems (Elbert, Erwin, Walker [11], Ko, Lander [12]).

In Fig. 4 the comparison is shown of the angular distributions of relativistic secondary particles ( $n_s$ ) in 16 GeV  $\pi^-p$  interactions transformed to the CM-system in the exact way and by means of the ultrarelativistic transformation, respectively. One can see that the

<sup>1</sup> This corresponds to the velocity of the particle in the lab-system  $\beta_L > 0.7$ .

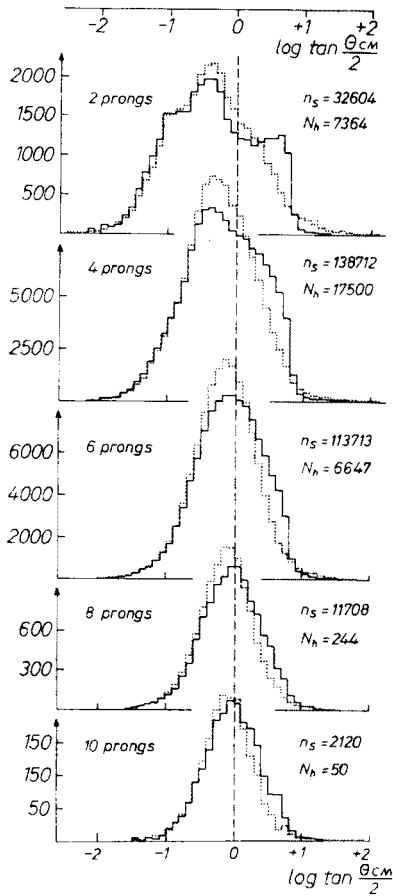


Fig. 4. Comparison of the CMS angular distributions for relativistic particles ( $n_s$ ) at 16 GeV  $\pi^-p$  interactions, obtained by the exact Lorentz transformation from lab to CMS (full line) and by the ultrarelativistic transformation (dotted line). The numbers of slow particles ( $N_h$ ) are also indicated. The vertical broken line indicates  $90^\circ$  in the CMS

TABLE I

Asymmetry parameters  $M$  for  $\pi^-p$  interactions at 16 GeV

$n_{ch}$	Exact transformation used	Ultrarelativistic transformation used
	1	2
2	-0.20	-0.23
4	-0.09	-0.16
6	-0.03	-0.11
8	-0.02	-0.11
10	-0.01	-0.11
All prongs	$-0.058 \pm 0.001$	$-0.136 \pm 0.001$

angular distributions transformed with the help of the ultrarelativistic formula are systematically more forward-asymmetric than those transformed by the exact formula.

It can be seen in Figs 1 and 2 that slow particles in the lab-system ( $N_h$ ) are always emitted backwards in CM-system. Their numbers are given in Fig. 4.

The asymmetry parameters  $M$  for 16 GeV  $\pi^-p$  interactions presented in Table I were calculated always by taking into account all particles, independently of their ionization.

The statistical errors of the asymmetry parameter for each prong number group given in Table I are less than 0.005.

### 3. Asymmetry of angular distributions of secondary particles in 60 GeV $\pi^-p$ interactions

The experimental material discussed here has been gathered by collaboration between several laboratories on the emulsion stack irradiated by 60 GeV negative pions at the Serpukhov accelerator (Alma-Ata, Cracow, Dubna, Moscow, Sofia, Tashkent, Ulan-Bator Collaboration [1, 2, 3]).

From all events found in the emulsion, 469  $\pi^-p$  interactions have been chosen by means of the strong selection criteria [13].

For each event the angles of emission of secondary particles in the lab system,  $\theta_L$ , were measured. The momenta and the particle masses are unknown, thus the transformation of the angular distribution to the CM-system using the exact formula is impossible. On the

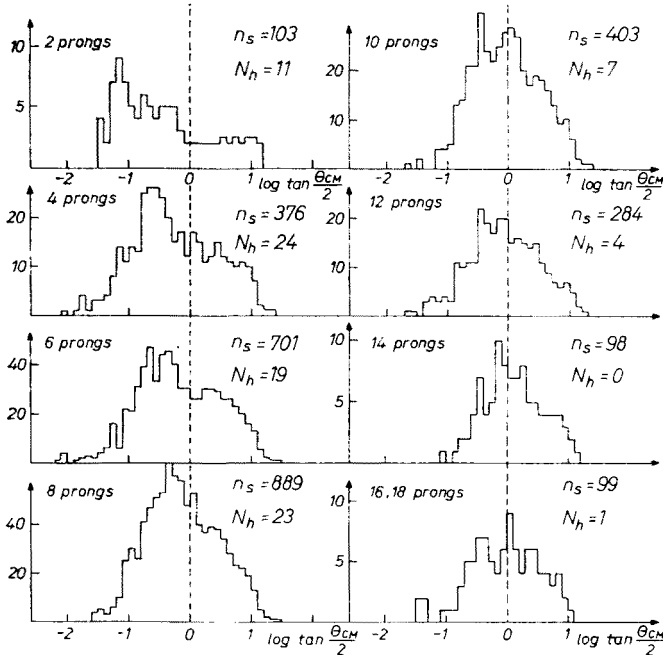


Fig. 5. Angular distributions of relativistic particles ( $n_s$ ) at 60 GeV  $\pi^-p$  interactions transformed to the CMS by the folding procedure described in the Appendix. The numbers of slow particles ( $N_h$ ) are also indicated. The vertical broken lines indicate  $90^\circ$  in the CMS

other hand, as was shown in the previous chapter, the application of the ultrarelativistic transformation to study the asymmetry of the angular distribution is incorrect. Hence we suggest the transformation to the CM-system using the folding procedure described in detail in the Appendix. The correctness of this method has been verified on a sample of 16 GeV  $\pi^-p$  interactions. In any case, we consider that this method approaches the correct transformation much better than does the ultrarelativistic one.

In Fig. 5 the angular distributions are shown of relativistic secondary particles ( $n_s$ ) in  $\pi^-p$  interactions at 60 GeV primary energy, transformed to the CM-system by means of the folding procedure. The numbers of slow particles in the lab-system ( $N_h$ ), emitted backwards in the CM-system, are also indicated. The corresponding asymmetry para-

TABLE II  
Asymmetry parameters  $M$  for  $\pi^-p$  interactions at 60 GeV

$n_{ch}$	Folding procedure transformation used 1	Ultrarelativistic transformation used 2
2	$-0.44 \pm 0.10$	$-0.54 \pm 0.08$
4	$-0.28 \pm 0.05$	$-0.37 \pm 0.04$
6	$-0.22 \pm 0.03$	$-0.30 \pm 0.03$
8	$-0.15 \pm 0.03$	$-0.23 \pm 0.02$
10	$-0.04 \pm 0.03$	$-0.13 \pm 0.03$
12	$-0.09 \pm 0.04$	$-0.17 \pm 0.04$
14	$+0.06 \pm 0.07$	$-0.06 \pm 0.06$
16, 18	$+0.03 \pm 0.07$	$-0.05 \pm 0.07$
All prongs	$-0.142 \pm 0.013$	$-0.237 \pm 0.013$

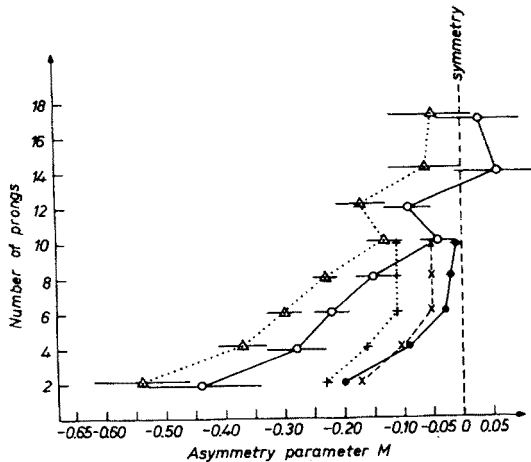


Fig. 6. Median value  $M$  of the CMS angular distribution vs number of prongs for:  $\bullet$  — 16 GeV  $\pi^-p$ , exact transformation;  $\times$  — 16 GeV  $\pi^-p$ , folding procedure transformation;  $+$  ····· 16 GeV  $\pi^-p$ , ultrarelativistic transformation;  $\circ$  — 60 GeV  $\pi^-p$ , folding procedure transformation;  $\Delta$  ····· 60 GeV  $\pi^-p$ , ultrarelativistic transformation. Horizontal bars represent statistical errors

meters  $M$ , calculated for all particles, independently of their ionization, are presented in column 1 of Table II. For comparison, the asymmetry parameters for angular distributions transformed to CMS by the ultrarelativistic transformation are given in column 2.

In Fig. 6 we compare the asymmetry parameters of the angular distributions for 16 GeV  $\pi^-p$  interactions as a function of the multiplicity  $n_{ch}$ . The 60 GeV angular distributions were transformed to the CM-system by the folding procedure; to the 16 GeV interactions the exact transformation was applied. In order to provide a direct comparison, the asymmetry parameters for 16 GeV interactions obtained by the folding procedure are also presented in Fig. 6. One can observe that the values of the asymmetry parameters of these distributions are still slightly overestimated. This shows that the presented values of the asymmetry parameter for 60 GeV interactions should be considered as upper limits.

#### 4. Discussion and conclusions

The analysis of the asymmetry parameters for  $\pi^-p$  interactions (see Fig. 6) leads us to the conclusion that for a given primary energy the asymmetry of the angular distribution decreases with the increase of the number of prongs. On the other hand, for a fixed number of prongs the asymmetry increases with the increase of the primary pion energy.

Recently it was shown by Elbert *et al.* [11] that for 25 GeV  $\pi^-p$  interactions the asymmetry of the angular distribution cannot be explained by the presence of the leading  $\pi^-$ , the distribution of produced pions being also asymmetric.

Elbert *et al.* [11] introduced another measure of the asymmetry, namely the  $R$  parameter:

$$R = \frac{p^* + \beta_s E_p^*}{p^* - \beta_s E_\pi^*} \quad (2)$$

where  $p^*$  is the CM-momentum of primary particle,  $E_p^*$ ,  $E_\pi^*$  are CM-energies of incoming proton and pion respectively,  $\beta_s$  is the velocity of the symmetry system in the CM-system.

$R$  gives the ratio of the momentum of the incoming proton to the momentum of the incoming  $\pi^-$  in the symmetry system. Under the assumption  $\beta_{CM} \approx 1$ ,  $R$  is related to our asymmetry parameter  $M$  via  $\beta_s$ , as follows:

$$M \approx -\frac{1}{2} \log \frac{1 + \beta_s}{1 - \beta_s}. \quad (3)$$

Thus for high energies where  $p^* \approx E_p^* \approx E_\pi^*$ ,

$$M \approx -\frac{1}{2} \log R. \quad (4)$$

Elbert *et al.* [11] have found that for  $\pi^-p$  interactions at 25 GeV,  $R$  equals about 1.5.

Taking the value of the overall asymmetry parameter  $M$  from Table II and using the above equation, we have calculated  $R$  obtaining the value of  $1.9 \pm 0.1$  for 60 GeV  $\pi^-p$  interactions. The error quoted is purely statistical one. The value of  $R$  is probably overestimated. The systematic error introduced by using the approximate method of transformation



of the angular distribution, may be as high as  $-0.2$ . This value was estimated on the basis of the 67 GeV proton-proton emulsion data (see Appendix).

The present analysis is in qualitative agreement with the previous observation of the Alma-Ata, Budapest, Cracow, Dubna, Moscow, Sofia, Tashkent and Ulan-Bator Collaboration [1, 2, 3] that in 60 GeV  $\pi^-p$  interaction pions are emitted predominantly into the forward hemisphere in the  $\pi$ -nucleon CM-system and that this asymmetry is stronger in events of lower multiplicity. It was shown, however, that quantitative conclusions about the values of this asymmetry cannot be drawn directly from the laboratory angular distributions on the basis of the ultrarelativistic transformation. Therefore a considerable care should be exercised when using the present experimental data in speculations concerning the physical models of the production mechanisms.

## APPENDIX

### *I. Approximate methods of transformation of the angular distribution*

In the introduction to the present work we recall the approximate relation of the angle transformation from the lab to the CM-systems:

$$\log \tan \frac{\theta_{\text{CM}}}{2} = \log \tan \theta_{\text{L}} + \log \gamma_c \quad (\text{A1})$$

derived from the exact transformation formula:

$$\tan \theta_{\text{L}} = \frac{\sin \theta_{\text{CM}}}{\gamma_c (\cos \theta_{\text{CM}} + \beta_c / \beta_{\text{CM}})} \quad (\text{A2})$$

under the assumption  $\beta_c = \beta_{\text{CM}}$ .

Equation (A1) is commonly applied in all cases where the identification of secondary particles and the measurements of their momenta cannot be made.

The transformation inverse to (A1) is of the following form:

$$\tan \theta_{\text{CM}} = \frac{\sin \theta_{\text{L}}}{\gamma_c (\cos \theta_{\text{L}} - \beta_c / \beta_{\text{L}})} \quad (\text{A3})$$

Expressing the velocity of the particle  $\beta_{\text{L}}$  in the lab system with the use of transverse momentum  $p_t$

$$\beta_{\text{L}} = \left( \frac{m^2}{p_t^2} \sin^2 \theta_{\text{L}} + 1 \right)^{-\frac{1}{2}} \quad (\text{A4})$$

we obtain from (A3) and (A4)

$$\text{ctg } \theta_{\text{CM}} = \gamma_c \left[ \text{ctg } \theta_{\text{L}} - \beta_c \left( \frac{m^2}{p_t^2} + \text{ctg}^2 \theta_{\text{L}} + 1 \right)^{-\frac{1}{2}} \right]. \quad (\text{A5})$$

This form of the transformation formula is convenient when a comparison with the ultra-relativistic form (A1) is made, because the distribution of its parameter — the transverse momentum — is well known. Besides, the transverse momentum is Lorentz-invariant and to a good approximation its distribution for secondary particles does not depend on the primary energy.

Figs 1 and 2 in the text illustrate the relations between  $\log \tan \theta_L$  and  $\log \tan \frac{\theta_{CM}}{2}$  calculated on the basis of formula (A5). Incidentally, it is worth noting that the  $p_t$  parametrization ensures that  $\theta_{CM}$  is for every  $p_t$  a singlevalued function of  $\theta_L$ ; this is not the case when  $\beta_L$  parametrization (formula (A3)) is used.

Apart from the ultrarelativistic transformation, another approximate method of transformation is sometimes used based on the relation (A5) assuming a constant value of transverse momentum — usually the mean value  $\langle p_t \rangle = 0.4 \text{ GeV}/c$  and a pion mass for each particle. In Figs 1 and 2 it can be seen that for relativistic particles this method practically does not differ from the ultrarelativistic transformation.

Owing to this, the authors suggest a different method of transformation procedure, where the CMS angular distribution is obtained from the lab one by folding in the assumed transverse momentum distribution. Such a method is well justified for a transformation of the composite angular distributions which contain a great number of particles.

## II. Details of the folding procedure transformation of the angular distribution from lab to CM-system

1. The input experimental data were the  $\log \tan \theta_L$  values of relativistic particles in every event.

2. It was assumed that the charged secondary particles consist only of pions and protons. The assumptions concerning the fraction of protons and their laboratory angular distribution were the following ones:

a) particle emitted at the smallest angle is a leading proton with probability 0.7 in case of  $p$ - $p$  interaction or a pion with probability 1 in  $\pi$ - $p$  event;

b) slow particles with  $\beta_L > 0.7$ , occurring in events with  $N_h = 1$  are recoil protons. In events with  $N_h = 0$ , the angular distribution of recoil protons was assumed the same as the composite distribution of all particles in those events, after excluding the smallest angle particles;

c) total number of protons per event in each prong number group is 1.4 and 0.7 in  $p$ - $p$  and  $\pi$ - $p$  interactions respectively. The assumptions 2a, b, c are based on the results obtained at the lower energies at 16 GeV  $\pi$ - $p$  [14] and 30 GeV  $p$ - $p$  [15] interactions.

3. The following transverse momentum distributions were accepted:

$$\frac{dN}{dp_t} = p_t \exp(-p_t/0.175), \quad \langle p_t \rangle = 0.35 \text{ GeV}/c \text{ for pions} \quad (\text{A6})$$

$$\frac{dN}{dp_t} = p_t^2 \exp(-p_t/0.166), \quad \langle p_t \rangle = 0.50 \text{ GeV}/c \text{ for protons.} \quad (\text{A7})$$

4. As the laboratory angular distributions contained only relativistic particles, a cut off at low values of  $p_t$  was introduced. To the criterion  $\beta_L > 0.7$ , accepted for relativistic particles, corresponds the following condition:

$$p_t > p_t^{\min} = 0.98 m \sin \theta_L. \quad (\text{A8})$$

The upper limit of  $p_t$ , also depending on  $\theta_L$ , was taken to be the smaller value of the two ones:

$$p_t < p_t^{\max} = \min(1.5, p_0 \sin \theta_L) \quad (\text{A9})$$

where  $p_0$  is the primary particle momentum.

5. For each value of  $\log \tan \theta_L$  the mass and normalization were assigned according to the assumptions 2a, b, c and then the transverse momentum distribution ((A6) or (A7)) was translated into the corresponding histogram in  $\log \tan \frac{\theta_{CM}}{2}$  variable with the help of the formula (A5).

Fig. 7 shows, as an example, the distributions obtained by an application of the folding procedure transformation to the 3 different values of  $\log \tan \theta_L$  in case of a pion and a proton.

The method outlined above was tested using the 16 GeV  $\pi^-p$  sample [10] and the 67 GeV  $p-p$  emulsion data [9]. In particular, it was tested whether the replacement of the

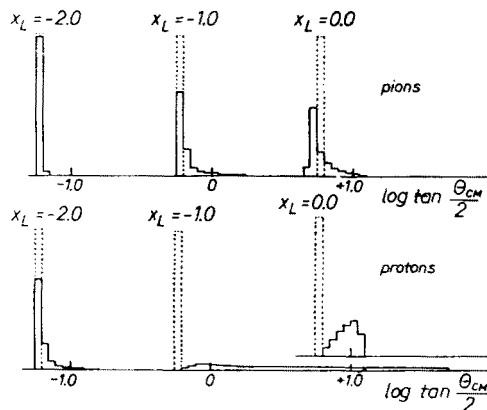


Fig. 7. An illustration of the folding procedure transformation. 3 values of  $X_L (= \log \tan \theta_L)$  are transformed to the CMS by the folding procedure — dashed line histograms, and by the ultrarelativistic formula (A1) — full line histograms, in case of a pion (upper part of the figure) and a proton (lower part of the figure). Primary energy  $E_0 = 67$  GeV

actual transverse momenta by the transverse momentum distribution is a sufficiently good approximation. In Fig. 8 CMS angular distributions for relativistic pions and protons in the 16 GeV  $\pi^-p$  interactions, for 4 and 6 prong stars, are shown. In the transformations, the known laboratory pion and proton angular distributions were used, replacing their

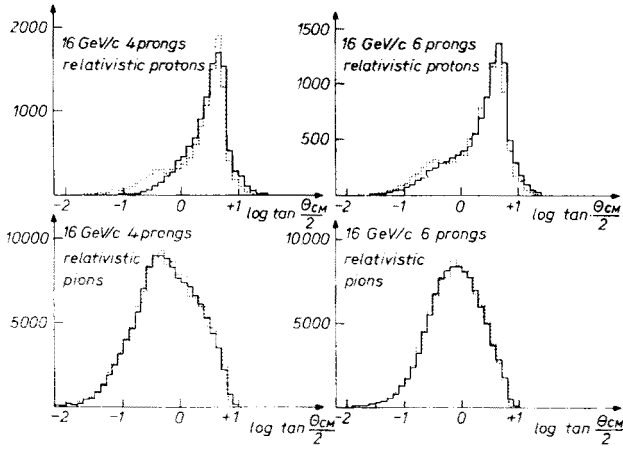


Fig. 8. CMS angular distributions of relativistic pions and protons in 16 GeV  $\pi$ - $p$  interactions. In the transformations to the CMS the formula (A5) was applied, using actual values of transverse momenta (full line histograms) and the transverse momentum distributions (A6) and (A7) (dotted line histograms)

transverse momenta by the distributions (A6) and (A7). One may observe a very good reproduction of the actual CMS angular distributions.

In case of the 67 GeV  $p$ - $p$  interactions the folding procedure transformation should give a symmetric CMS angular distributions. In Fig. 9 the asymmetry parameters obtained

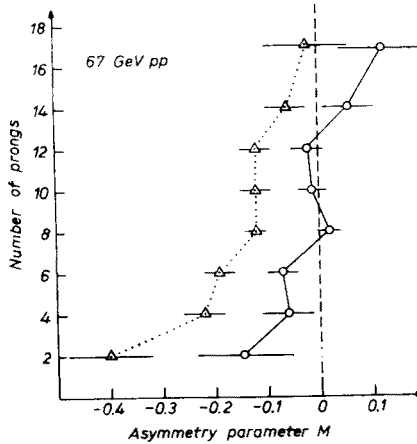


Fig. 9. Median value  $M$  of the CMS angular distribution *vs* number of prongs in 67 GeV  $p$ - $p$  interactions. The CMS distributions were obtained by using:  $\circ$ — folding procedure transformation;  $\Delta$ ..... ultra-relativistic transformation

by applying the folding procedure are compared with those obtained by using the ultra-relativistic formula (A1). One may observe a considerable improvement when applying the folding procedure, however a small forward asymmetry ( $M < 0$ ) in a low multiplicity events and a small backward asymmetry ( $M > 0$ ) in a high multiplicity events is still visible. The overall asymmetry parameter  $M$  (i. e.  $M$  calculated for the composite angular

distribution containing all events) is close to zero and equals  $-0.02 \pm 0.01$ . This value was used in estimating the systematic error of the overall asymmetry parameter in the 60 GeV  $\pi^-p$  interactions.

Fig. 10 shows the composite angular distribution of relativistic particles at 67 GeV  $pp$  interactions.

Our sincere thanks are due to the Aachen-Berlin-Bonn-CERN-Cracow-Warsaw Collaboration, particularly to Dr D.R.O. Morrison for permission to use some unpublished experimental data on 16 GeV  $\pi^-p$  interactions.

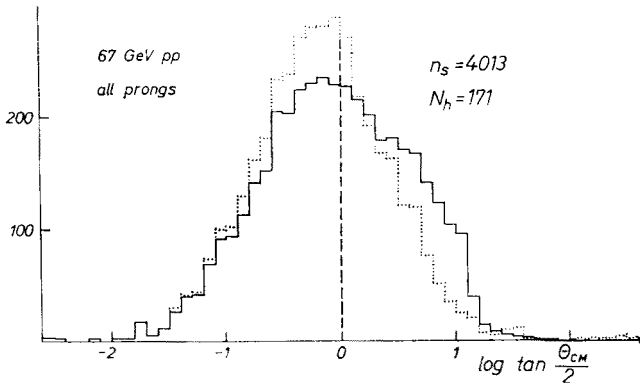


Fig. 10. Comparison of the composite CMS angular distributions for relativistic particles ( $n_s$ ) in 67 GeV  $p$ - $p$  interactions, obtained by the folding procedure transformation (full line) and by the ultrarelativistic transformation (dotted line). The number of slow particles ( $N_h$ ) are also indicated. The vertical broken line indicates  $90^\circ$  in the CMS

We are very indebted to our late colleague Dr O. Czyżewski for his interest in this work and for performing calculations on the CERN computer which enabled us to compare the HBC data with the emulsion experimental results.

It is a pleasure to thank Dr J. Bartke for reading the manuscript and for useful comments.

#### REFERENCES

- [1] Alma-Ata, Cracow, Dubna, Moscow Collaboration, *Proceedings of the Lund International Conference on Elementary Particles*, paper No 97, June 1969.
- [2] Alma-Ata, Budapest, Cracow, Dubna, Moscow, Sofia, Tashkent, Ulan-Bator Collaboration, *Proc. 11th Intern. Conf. on Cosmic Rays*, Budapest 1969; *Acta Phys. Hungar.*, **29**, Suppl. 3, 173 (1970).
- [3] Alma-Ata, Budapest, Cracow, Dubna, Moscow, Sofia, Tashkent, Ulan-Bator Collaboration, *Phys. Letters*, **31B**, 237 (1970).
- [4] H. Winzeler, B. Vlaiber, W. Koch, M. Nikolic, M. Schneeberger, *Nuovo Cimento*, **7**, 8 (1960).
- [5] H. Meyer, M. W. Teucher, E. Lohrmann, *Nuovo Cimento*, **28**, 1399 (1963).
- [6] A. Marzari-Chiesa, R. Rinando, S. Ciurlo, E. Picasso, A. M. Cartacci, *Nuovo Cimento*, **27**, 155 (1963).
- [7] A. Barbaro-Galtieri, G. Baroni, H. Manfredini, C. Castagnoli, C. Lamborizio, I. Ortalli, *Nuovo Cimento*, **20**, 487 (1961).
- [8] E. Lohrmann, K. Teucher, M. Schein, *Phys. Rev.*, **122**, 672 (1961).

- [9] Alma-Ata, Cracow, Dubna, Leningrad, Moscow, Tashkent, Ulan-Bator Collaboration, *Izv. Akad. Nauk SSSR*, in press. See also *Phys. Letters*, **39B**, 282 (1972) and JINR — Dubna preprint (in press).
- [10] Aachen, Berlin, Bonn, CERN, Cracow, Warsaw Collaboration, private communication.
- [11] J. W. Elbert, A. R. Erwin, W. D. Walker, *Phys. Rev.*, **D3**, 2042 (1971).
- [12] W. Ko, R. L. Lander, *Phys. Rev. Letters*, **26**, 1284 (1971).
- [13] Alma-Ata, Cracow, Dubna, Moscow, Tashkent Collaboration, JINR — Dubna preprint (in press), submitted to the 15th Intern. Conf. on High Energy Physics, Batavia 1972.
- [14] B. Furmańska, J. Gierula, R. Hołyński, S. Krzywdziński, A. Linscheid, *INP report* No 761/HP, Cracow 1971.
- [15] E. W. Anderson, E. J. Bleser, G. B. Collins, T. Fujii, J. Menes, F. Turkot, R. A. Carigan, Jr., R. M. Edelstein, N. C. Hien, T. J. Mc Mahon, I. Nadelhaft, *Phys. Rev. Letters*, **19**, 198 (1967).