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CLUSTERING EFFECTS IN HIGH ENERGY MULTIPARTICLE PRODUCTION

By F. Hayot and A. Morel

Department of Theoretical Physics, CEN, Saclay*

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The effects of fireball production on correlations in inclusive spectra are reviewed. Both integrated and differential correlations are discussed.

1. *Introduction*

Clustering will be the word we shall use all along to summarize the tendency which particles have, when they are produced at very high energy, not to be independent of each other, but rather stick together in various ways. A typical example of clustering is resonance production: if π production goes through ϱ meson production, say, then the fact that one π is seen implies that another π has been produced together with the one seen. Note that in this case, the two π's which emerge from the same ϱ have similar momenta at the scale where we look at the energy of the secondaries. Such an effect leads obviously to positive contributions to correlations: pairs of π's are produced in the same region of phase space (short range correlations). Thus we see that in general clustering will affect both integrated correlations (f_k coefficients for example), and, say, rapidity correlations. Now, one knows that other mechanisms are able to yield correlations. The simplest one is obvious: charge (or baryon number, or strangeness) conservation implies pair production. But conservation principles do not tell us that conservation of quantum numbers has to be verified locally (in the phase space). Correlations of this kind may be *a priori* as well short range correlations, as long range correlations. We shall keep the word cluster for objects which decay into relatively low energy secondaries in their cm system. A typical example is given by the fragmentation process: by definition, the fragments of particle *a* are those secondaries which are slow in its rest frame. The fragmentation pattern of the projectile (or target) is then a cluster. On the contrary, we know that the whole "compound nucleus" which is formed by the two incident particles when they hit each other is not a cluster: the mass $\sqrt{s}$ of the object is too large as compared to the relatively low

* Address: Dph/T — CEN — Saclay BP No 2, 91 Gif-sur-Yvette, France.

($\sim$ log s) average number of decay products. A way to share a large energy into a small number n of secondaries is to group them into f clusters containing each p particles of small relative momenta, most of the energy being then spoiled in giving high momenta to at least a part of the clusters. $p = 1$ is the trivial case.

Now, why to look for clusters? The first motivation is very general: since we know for sure that particles are not produced independently, we can try to simplify the description of their correlations by assigning them to be correlated inside clusters, which in turn will be considered in a first approximation as independently produced. A second motivation is based on experiment: two-body short range rapidity correlations are actually well observed at ISR energies [1, 2], showing that very often particles like to be produced at low relative momenta. Leaving aside correlations in the phase space, it is also known that correlations exist in the multiplicity distributions between various kind of particles, π^- and π^0's for example [3], a property which may well originate in a primary production of resonances [4]. We thus expect that at least two-particle clusters, of the resonance type for example, do exist. Is clustering a more general feature of high energy multiproduction is the question we would like to discuss, if not answer. There are basically two difficulties in trying to solve this problem. The first one is that clustering, if it exists, is certainly not the only mechanism which yields correlations. In particular, the existence of long range correlations is experimentally proved [5]. The second one is that as soon as clusters are independently produced, two particles which are neighbouring in rapidity may perfectly belong to two different clusters. In other words, we cannot infer from the assumption of clustering that many events will present in a rapidity plot two or three well separated sets of two or three particles each. So clustering will not be seen by just a glance at individual events [6].

We shall first discuss the implication of the clustering hypothesis on the multiplicity distributions, and in particular on the energy dependence of the Mueller integrated correlation coefficients (Section 2). Then we shall examine the question of the one and two particle rapidity distributions, and study the shape of the corresponding short range correlations. This will be done in Section 3 of these notes.

2. *Clustering and multiplicity distributions*

The framework of the present discussion is essentially that of a kind of multiperipheral model (MPM), where the objects produced are clusters. In such a model at very high energy baryon exchange is expected to be highly dominated by meson and Pomeron exchanges, so that the clusters associated with the projectile and target in pp collisions are very likely to be different from those produced along the multiperipheral chain. They will be called fragmentation clusters (FC), have more or less the proton quantum numbers, and generally contain a fast proton (leading particle). We shall concentrate on the other ones, which we consider as the true multiperipheral clusters (MC).

We shall now continue as if the MC production was the main production process, or equivalently as if we were able to separate this process from the others. In this picture, we expect the MC to be responsible for the increase of the multiplicity when the incident energy is increased. In the spirit of a conventional multiperipheral model, we now assume that we produce f independent MC, according to a Poisson distribution with average value M:

$$P(f) = e^{-M} \frac{M^f}{f!}. \tag{2.1}$$

For simplicity, we assume further that all MC are clusters of the same nature, each of them decaying into p_i particles according to a multiplicity distribution $f(p_i)$ which we do not specify for the moment. The processes of cluster production and decay are independent of each other. The total number of particles produced by f MC containing $p_1, \ldots, p_f$ particles is of course

$$m_f = p_1 + p_2 + \ldots + p_f \tag{2.2}$$

and the probability for finding m particles through this process is

$$P(m) = \sum_{f, p_1, \ldots, p_f} P(f) f(p_1) \ldots f(p_f) \delta(m - p_1 - \ldots - p_f). \tag{2.3}$$

The general properties of such a distribution have already been discussed in the Le Bellac's lectures, together with its generalization to the case when one identifies different kinds of particles. In particular, with Eq. (2.1), we find for the Mueller's correlation coefficients [7]

$$f_q = M \langle p(p-1) \ldots (p-q+1) \rangle_f, \tag{2.4}$$

where the symbol $\langle \ \rangle_f$ means averaging with the distribution $f(p)$. Let us make some comments on this formula.

(*i*) All f_q's are linear in M, as a reflection of the assumed Poissonian distribution of the clusters.

(*ii*) One has, of course,

$$\langle m \rangle = M \langle p \rangle_f. \tag{2.5}$$

(*iii*) If clusters never contain more than P particles ($f(p > P) = 0$), then

$$f_q = 0 \text{ if } q > P. \tag{2.6}$$

(*iv*) Since we are in the framework of a multiperipheral model, we expect $\langle m \rangle$ to increase more or less as $\log s$ with energy. Now, it is interesting to discuss how this will be actually realized, namely what are the energy dependences of M and $\langle p \rangle_f$ in Eq. (2.5). We shall discuss only two particular extreme cases:

(a) Conventional MPM model: M increases logarithmically with s, whereas $\langle p \rangle_f$ is constant: the nature and composition of the clusters are energy independent. Eq. (2.4) becomes

$$f_q = \alpha_q \langle m \rangle \tag{2.7}$$

with

$$\alpha_q = \frac{\langle p(p-1) \ldots (p-q+1)\rangle_f}{\langle p\rangle_f}$$

so that all f_q's are linear functions of the multiplicity. Of course the remark (*iii*) above applies.

(b) Modified MPM model: instead of applying really the MPM model to clusters one may also consider that when the energy is increased, the dynamics prefer to create heavier clusters rather than make the MC chain longer. The extreme case is of course $M = \text{const.}$, $\langle p\rangle_f \sim \log s$. The energy dependence of the f_q's now depends on the particular form chosen for the distribution $f(p)$[1].

In order to illustrate the kind of multiplicity distribution which can be obtained in this case, and its relation to KNO scaling, let us quote here the model of type (b) proposed by Lévy [9]. He assumes that $f(p)$ is a distribution which is sharply peaked around $p = \langle p\rangle$. Then, replacing $m_f = p_1 + \ldots + p_f$ by $m_f = \langle p\rangle f$, one obtains

$$\text{Prob.}\,(m) \propto P\left(f = \frac{m}{\langle p\rangle}\right), \tag{2.8}$$

which, owing to Eq. (2.5) yields

$$\text{Prob.}\,(m) \propto P\left(f = M\frac{m}{\langle p\rangle}\right). \tag{2.9}$$

This formula is valid whatever the cluster distribution P is, and under the assumption that M is constant (case b), yields exact KNO scaling. By the way, when normalizing Prob. (m) to $\sum_m$ Prob. $(m) = 1$ one obtains

$$\text{Prob.}\,(m) = \frac{\sigma_m}{\sigma_{\text{in}}} = \frac{2M}{\langle m\rangle} P\left(M\frac{m}{\langle m\rangle}\right) \tag{2.10}$$

so that the KNO function is

$$\psi\left(\frac{m}{\langle m\rangle}\right) = 2MP\left(M\frac{m}{\langle m\rangle}\right). \tag{2.11}$$

Specializing P to be a Poisson distribution with average value M, we obtain an explicit form for ψ which depends only on M. A numerical value for M can be easily deduced from the observed result that, above 50 GeV/c, $\langle n_c\rangle^2/D_c^2 \simeq 4$, and it is easy to convince oneself that under the assumption made, $M = \langle n_c\rangle^2/D_c^2$. So, the final Lévy's result for the KNO function is:

$$\psi(x) = \frac{8e^{-4}4^{4x}}{\Gamma(4x+1)}. \tag{2.12}$$

The agreement with experiment is nearly perfect.

[1] Cases (a) and (b) can be considered as extreme cases of the continuous sets of models which have been considered in the thermodynamical framework [8].

The preceding discussion shows that clustering effects may be present, and if they are, lead to well defined predictions on multiplicity distributions and integrated correlation coefficients f_q. Now, we have to take some care when we compare to data and try to deduce something about the very existence of clusters of multiperipheral origin. Even if it is true they populate the central region, the existence of diffractive dissociation events, which are exclusive of central production, certainly gives rise to long range correlations. When integrated over the whole phase space, these LR correlations give rise to non linear terms in the expression of the f_q coefficients [10, 11, 12]. It is thus necessary, if one wants to investigate further the cluster problem, to go into more details, and especially to look at distributions in the phase space, according to our definition of clustering.

3. *Clustering and momentum distributions*

3.1. Small transverse momentum behaviour

We shall mainly discuss here the one and two-particle distributions in rapidity $y = \frac{1}{2} \log [(E+p_L)/(E-p_L)]$ or in the cosmic variable $\eta = -\log \operatorname{tg} \theta/2$, after averaging over all transverse momenta. Let us make before a simple remark on a possible clustering effect on the $p_\perp$ distribution at small transverse momenta (turn over effect). This remark has already been done in the hypothesis of ϱ meson production [13], but it is in fact more general. In order to simplify the calculations, we consider that a cluster has been emitted along the longitudinal axis, and decays isotropically in its rest frame. Let $f(k)$ be the momentum distribution of a particle of mass μ inside the cluster. The 1-particle decay distribution is

$$dI(k, \Omega) = f(k) d \cos \theta d\Phi \frac{k^2 dk}{k_0} \tag{3.1}$$

and the corresponding $p_\perp$ distribution

$$\frac{d\sigma}{dp_\perp^2} \propto \int f(k) \delta(p_\perp^2 - k^2 \sin^2 \theta) d \cos \theta \frac{k^2 dk}{k_0},$$

which, after integration upon cos θ takes on the form:

$$\frac{d\sigma}{dp_\perp^2} \propto \int_{p_\perp}^{\infty} f(k) \frac{k dk}{k_0 \sqrt{k^2 - p_\perp^2}}. \tag{3.2}$$

Now we assume $f(k)$ to be peaked around some value K of k (for a ϱ resonance, decaying into two π's, $f(k) \propto \delta(k^2 - \frac{1}{4} m_\varrho^2 + \mu^2)$). For $p_\perp \ll K$, we can estimate $d\sigma/dp_\perp^2$ by

$$\frac{d\sigma}{dp_\perp^2} \sim \int_{K}^{\infty} f(k) \frac{k dk}{k_0 \sqrt{k^2 - p_\perp^2}} \tag{3.3}$$

which is an increasing function of $p_\perp$. For large $p_\perp$ values, the behaviour of the $p_\perp$ distribution will be essentially governed by $f(p_\perp)$. Thus in general, we expect clustering to give a turn over effect, or at least a flatter $p_\perp$ distribution at small values.

3.2. One particle rapidity distribution

We now turn to the rapidity distributions, and consider that we have averaged over all transverse momenta [14]. We want to obtain a quantitative estimate of clustering effects in 1 and 2-particle rapidity distributions, and for doing that we choose a simple MPM for cluster production, and again a simple isotropic decay distribution in the cluster rest frame [15]. Furthermore, we do not distinguish the cluster rest frame from the one where its rapidity y is zero. Let z be the rapidity of a secondary in this frame, and k the length of its 3-momentum. It is easy to find [16] for the z distribution

$$p(z) = \begin{cases} \dfrac{k}{2k_0 \operatorname{ch}^2 z} & \text{for} \quad |z| < \log \dfrac{k+k_0}{\mu}, \\ 0 & \text{for} \quad |z| > \log \dfrac{k+k_0}{\mu}. \end{cases} \tag{3.4}$$

For clusters decaying into π's, it is certainly correct in general to set $k/k_0 = 1$. If it is not so (maybe for heavier decay products, φ production of K's, production of pairs baryon-antibaryon), an approximate formula would be:

$$p(z) = \begin{cases} \left\langle \dfrac{1}{v} \right\rangle \dfrac{1}{2 \operatorname{ch}^2 z} & \text{for} \quad |z| < \operatorname{arcth}(1/\langle 1/v \rangle), \\ 0 \quad \text{otherwise}, \end{cases} \tag{3.5}$$

with $\langle 1/v \rangle$ being the average inverse velocity of secondaries in the cluster. In a more refined version of the model, one should take into account the transverse momentum of the cluster, but the model is too crude in its present form to go further into details. Now we are going to calculate the rapidity distribution $P(y)$ in the total cm system. Let $M(Y)$ be the cluster rapidity distribution; we have

$$P(y) = \int dY\, M(Y)\, p(y-Y). \tag{3.6}$$

In the spirit of a MPM model, we take simply for M a plateau of width Δ, and find (with $\langle 1/v \rangle = 1$)

$$P(y) = \frac{1}{2\Delta}\left(\operatorname{th}\left(\frac{\Delta}{2} + y\right) + \operatorname{th}\left(\frac{\Delta}{2} - y\right)\right). \tag{3.7}$$

The result is essentially a plateau with a little smaller width as Δ, supplemented by smooth edges yielded by those clusters which decay at the extremities of their allowed range. Eq. (3.7) is certainly well adapted to fit the one-particle data in the central region, but of course this cannot be considered as an evidence for clustering. Note that the only free parameter is Δ. Remembering that the things we describe are only what we have defined as multiperipheral clusters (excluding the two fragmentation clusters and diffraction dissociation), the best way to estimate Δ is to look at the experimental rapidity distribution for high multiplicities. We expect these events to be essentially produced by MC. Using the Pisa-Stony Brook data [1], we obtain the reasonable value $\Delta \simeq 3.5$ at $\sqrt{s} = 30$ GeV. The contributicn of the process under consideration to the inclusive cross-section normalized to the non diffractive total inelastic cross-section $\sigma_{\rm nd}$ is

$$\frac{1}{\sigma_{\rm nd}} \frac{d\sigma^{\rm mc}}{dy} = \langle p \rangle M P(y). \tag{3.8}$$

Here, $\sigma_{\rm nd}$ also contains the contribution of possible non diffractive fragmentation clusters. The energy behaviour depends on $\langle m \rangle = \langle p \rangle M$, the average number of the multiperipheral particles, and Δ. Scaling corresponds to $\langle m \rangle / \Delta =$ const. The height of complete inclusive cross-section at $y = 0$, where most probably only the MC clusters contribute, is given by

$$\frac{1}{\sigma_{\rm in}} \frac{d\sigma}{dy}(y = 0) = \frac{\sigma_{\rm nd}}{\sigma_{\rm in}} \frac{\langle m \rangle}{\Delta} \operatorname{th}\left(\frac{\Delta}{2}\right). \tag{3.9}$$

With $\Delta \simeq 3.5$, an estimation $\langle m_c \rangle$ can be obtained from the experimental value of $\frac{1}{\sigma_{\rm in}} \frac{d\sigma}{dy}$ $(y = 0)$, which is about 1.8–2.

We have

$$\frac{\sigma_{\rm nd}}{\sigma_{\rm in}} \langle m_c \rangle \sim 6 \text{ to } 7 \text{ at } \sqrt{s} = 30 \text{ GeV}, \tag{3.10}$$

and we are left with 3 to 4 particles in average from the fragmentation and diffractive dissociation processes.

3.3. Two particle rapidity distribution

Let us now consider the two-particle rapidity distribution. Once we know that two particles are produced, either they belong to two different clusters and their distribution is

$$V(y_1, y_2) = P(y_1) P(y_2) \tag{3.11}$$

or they belong to the same cluster. In this case, their distribution is

$$I(y_1, y_2) = \frac{1}{4\Delta} \int_{-\Delta/2}^{\Delta/2} dY \frac{1}{\operatorname{ch}^2 (y_1 - Y)} \frac{1}{\operatorname{ch}^2 (y_2 - Y)} \tag{3.12}$$

which is no more the product of individual distributions. Let us now construct the two-body cross-section, normalized again to $\sigma_{\rm nd}$, the non-diffractive cross-section. It is just a matter of simple counting to find

$$\frac{1}{\sigma_{\rm nd}}\frac{d\sigma^{\rm mc}}{dy_1dy_2} = M^2\langle p\rangle^2P(y_1)P(y_2)+M\langle p(p-1)\rangle I(y_1, y_2) \tag{3.13}$$

for identical particles, and

$$\frac{1}{\sigma_{\rm nd}}\frac{d\sigma^{\rm mc}}{dy_1dy_2} = M^2\langle p_1\rangle \langle p_2\rangle P(y_1)P(y_2)+M\langle p_1p_2\rangle I(y_1, y_2) \tag{3.14}$$

for particles of type 1 and 2. M is as before the average number of clusters, and the symbol $\langle\ \rangle$ always means averaging over the multiplicity distribution of the individual clusters.

Let us study the shape of the function $I(y_1, y_2)$, which contains all short range correlations [17]. From (3.12) we have

$$I(y_1, y_2) = \frac{1}{4\Delta}\int_{-\frac{\Delta}{2}-y_1}^{\frac{\Delta}{2}-y_1} du\,\frac{1}{\mathrm{ch}^2\,u}\,\frac{1}{\mathrm{ch}^2\,(u-(y_2-y_1))} \tag{3.15}$$

which shows that for $|y_1|$ small enough as compared to $\Delta/2$, one obtains a function of $|y_1-y_2|$ only, exponentially decreasing with $|y_1-y_2|$. The function is no more symmetric in y_1-y_2 as soon as y_1 approaches the edges of the distribution, $y_1 \simeq \pm\Delta/2$, and it is smaller at $y_1 = y_2 = \Delta/2$ than at $y_1 = y_2 = 0$, because one now integrates upon only half of the preceding interval in u. Thus, I represents a short range correlated part of the two-body cross-section, which falls off exponentially in $|y_1-y_2|$, and is symmetric in (y_1-y_2) for small $|y_1|$.

Let us now turn to the question of the size and composition of the clusters. For doing that, we first compute the contribution of I to the two-body correlation function

$$R(y_1, y_2) = \sigma_{\rm in}\frac{d^2\sigma/dy_1dy_2}{\dfrac{d\sigma}{dy_1}\,\dfrac{d\sigma}{dy_2}}. \tag{3.16}$$

This contribution is, for two charged particles,

$$E^{\rm cc}(y_1, y_2) = \frac{\sigma_{\rm in}}{\sigma_{\rm nd}}\,\frac{\langle p_c(p_c-1)\rangle}{M\langle p_c\rangle^2}\,\frac{I(y_1, y_2)}{P(y_1)P(y_2)} \tag{3.17}$$

and for one charged particle and one π^0:

$$E^{\rm c0}(y_1, y_2) = \frac{\sigma_{\rm in}}{\sigma_{\rm nd}}\,\frac{\langle p_cp_0\rangle}{M\langle p_c\rangle\langle p_0\rangle}\,\frac{I(y_1, y_2)}{P(y_1)P(y_2)}. \tag{3.18}$$

Let us first concentrate at the point $y_1 = y_2 = 0$. It is easy to find

$$\frac{\sigma_{\rm in}}{M\sigma_{\rm nd}}\,\frac{I(0,0)}{(P(0))^2} = \frac{\sigma_{\rm in}\Delta}{\sigma_{\rm nd}M}\,\frac{1-\mathrm{th}^2\,(\Delta/2)/3}{2\,\mathrm{th}\,\Delta/2} \tag{3.19}$$

or using Eq. (3.9), and th $\Delta/2 \sim 1$

$$\frac{\sigma_{\text{in}}}{M\sigma_{\text{nd}}}\frac{I(0,0)}{(P(0))^2} = \frac{\langle p_c\rangle}{3\dfrac{1}{\sigma_{\text{in}}}\dfrac{d\sigma^c}{dy}(y=0)}. \tag{3.20}$$

Owing to equations (3.16) and (3.20), and recalling that at $y_1 \simeq y_2 \simeq 0$ only the MC clusters contribute, we obtain for the R functions:

$$R^{cc}(0,0) = \frac{\sigma_{\text{in}}-\sigma_{\text{nd}}}{\sigma_{\text{nd}}} + \frac{\langle p_c(p_c-1)\rangle}{3\langle p_c\rangle}\frac{\sigma_{\text{in}}}{\dfrac{d\sigma^c}{dy}(y=0)},$$

$$R^{c0}(0,0) = \frac{\sigma_{\text{in}}-\sigma_{\text{nd}}}{\sigma_{\text{nd}}} + \frac{\langle p_c p_0\rangle}{3\langle p_0\rangle}\frac{\sigma_{\text{in}}}{\dfrac{d\sigma^c}{dy}(y=0)}. \tag{3.21}$$

Let us evaluate from Eq. (3.21) the characteristics of the cluster. We take for definiteness $R^{cc}(0,0) = 0.7$ [1], $R^{c0}(0,0) = 0.6$ [2], $\sigma_{\text{in}} = 32$ mb, $\sigma_{\text{nd}} = 26$ and $\dfrac{1}{\sigma_{\text{in}}}\dfrac{d\sigma^c}{dy}(0) = 1.8$. With these numbers we get

$$\frac{\langle p_c(p_c-1)\rangle}{\langle p_c\rangle} \simeq 2.5, \qquad \frac{\langle p_c p_0\rangle}{\langle p_0\rangle} \simeq 2. \tag{3.22}$$

Let us try to interpret these numbers. The first remark is that they are large as compared to clusters which would be single resonances like ϱ, ω, ... *etc.* For example, $\langle p_c(p_c-1)\rangle/ /\langle p_c\rangle = 1$ for ϱ_0 or ω production, and σ production. Now, we have to remember that our calculations hold for independent clusters, whereas production of ϱ, ω, ... *etc.* would require exchanges of π, ϱ, ω, ... *etc.*, which are known to lead to short range correlations. So, if these ϱ or ω resonances are produced by uncorrelated clusters, they should emerge from Pomeron exchange and thus be produced by pairs at least. Thus a simple idea is to consider pairs of resonances, coupled to an isospin zero state [7]. We obtain in the simple case of a pair of ϱ's coupled to $I = 0$:

$$\frac{\langle p_c(p_c-1)\rangle}{\langle p_c\rangle} = 2, \qquad \frac{\langle p_c p_0\rangle}{\langle p_0\rangle} = 2. \tag{3.23}$$

Clearly a 2ϱ model gives the right order of magnitude. We do not claim, of course, that data prove in any way that pairs of ϱ's are independently produced. We just want to point out that if the cluster model has any sense, we do have rather large objects. Certainly, uncorrelated single resonances cannot work. We infer from this discussion that very probably clusters exist which contain more than $1\pi^-$, or in other words that we expect sizable short range correlations between negative tracks.

Independently of the example of a 2ϱ cluster model, from Eq. (3.22) and the reasonable assumption that charge is conserved in a cluster we obtain

$$\frac{\langle p_-(p_- -1)\rangle}{\langle p_-\rangle} = \frac{1}{2}\left[\frac{\langle p_c(p_c-1)\rangle}{\langle p_c\rangle} - 1\right] \simeq 0.75. \tag{3.24}$$

So, with the same input experimental quantities as for the charged particle correlations, we get at the same energies ($\sqrt{s} = 30$ GeV)

$$R^{--}(0, 0) \simeq 0.23+0.25 \simeq 0.5. \tag{3.25}$$

The first term is the same long range contribution as before. In this case, both contributions are of the same order of magnitude.

Let us end up with a few comments about energy dependence of two-body correlations. The fact that at $y_1 = -2.$ [1] or -2.5 [2], $R(y_1, y_1)$ is smaller than $R(0, 0)$ has to be attributed to the presence there of fragmentation or diffraction events which do not contain short range correlations. Now, if the energy is increased, the same value of y_1 will fall ultimately in the central plateau so that $R(y_1, y_1)$ will tend to $R(0, 0)$. On the other hand Eq. (3.21) implies the constancy of $R(0, 0)$ if $d\sigma/dy(0)$ is constant (scaling). So we expect all short range correlations at y_1 and $y_2 \neq 0$, to tend towards a constant value at larger energy. An indication that this actually happens can be found in Ref. [2].

As a conclusion, let us say that equations (3.21) provide a simple tool for getting an insight into possible independent clusters production in the central region. The isotropy hypothesis allows simple calculations, but is not really fundamental. Other distributions in the cluster rest frame could be looked at. We consider that isotropy probably yields a lower bound to the correlation length. Indeed there is more chance that clusters remember the longitudinal direction. Secondaries would then have larger momenta along this direction than along the transverse one, as it is in a multiperipheral model where exchange of Regge poles with intercept 0.5 leads to a correlation length equal to 2, rather than 1 [15]. The structure of Eqs (3.21) would not be modified, and the relative strengths of the two-body short range correlations at $y = 0$ for various configurations not affected. Obviously the question of the existence of short range correlations between negative particles is very important in this aspect, and good data about them will be welcome.

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