TWO-STEP DEUTERON STRIPPING ON SPHERICAL NUCLEI

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The main features of the two-step effects, connected with one-phonon virtual excitations, are analyzed for allowed neutron transfers in the $^{52}Cr(d, p)^{53}Cr$ reaction. To this end the obvious expression for the cross-sections obtained in the double adiabatic perturbation approach is calculated with the help of an ordinary DWBA-method. A mixture of the two-step effects to the usual one-step cross-section is found to be about 10 per cent, but with increasing incident energy from 10 to 20 MeV the effect increases approximately by a factor of two times.

The multi-step nucleon transfer process is connected with preliminary and posterier excitations of the low-lying collective states in the entrance and exit channels. In some details multi-step effects have been investigated in deuteron stripping reactions in the case of deformed nuclei only (see *e. g.* Refs [1–3]). They were shown to play an important role and often can be of the same order as the direct one-step transitions going without any intermediate excitations. Note that in this case it was natural to use an appropriate method of accounting for strongly coupled channels.

In the case of spherical nuclei such a consideration has not been yet done, with the exception of only two attempts of indirect transfer calculations which were performed in Refs [4,5] and based on a specific "core-excitation stripping model". In this paper we treat another more general method, the so-called generalized distorted wave Born approximation (GDWBA), which was confirmed very well beforehand by the calculations of the multi-step stripping reactions on deformed nuclei. To this end we use the result of the previous work [6] where, in the framework of the mentioned GDWBA method with the corresponding generalized distorted waves calculated according to the so-called double adiabatic perturbation theory, an expression for the multistep cros-ssection has been obtained. The main advantage of using this perturbation approach is the possibility of giving the multi-step cross-section in a very simple and handy form, so all calculations may be carried out with the help of the well-known standard DWBA-method (see *e. g.*

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Ref. [7]. This perturbation approach seems to be a good one because of the experimental fact that inelastic transition probabilities to phonon states in spherical nuclei are small compared with those to rotational states in deformed nuclei. Thus, the corresponding multi--step transfer amplitudes, which are proportional to these probabilities, should also be in the same relations. Therefore, one may hope that the corresponding multi-step stripping calculations on spherical nuclei can be done successfully with the help of the perturbation treatment. Naturally, in practice all cases have to be controlled: does this approach really hold.

We limit ourselves only to the consideration of two-step stripping reactions $A(d, p)B$ on even nuclei, when intermediate transitions *via* the one-phonon states only are taken into account. In this case the corresponding formulae take the following obvious form [6]:

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{2J_B+1}{2J_A+1} \sum_{Lm} \left| c \sum_{l} \hat{B}_l^L \beta_l^{Lm}(\theta) \right|^2, \tag{1}$$

where c is the usual numerical constant, and

$$\beta_l^{Lm} = D_0 \sum_{l_p l_d} (-1)^{l_p} \frac{\hat{l}_d^2 \hat{l}_p}{4\pi \hat{L}^2} (l_p l_d m 0|Lm)\,(l_p l_d 00|L0)\, Y^*_{l_p m}(\hat{k}) \int r^2 dr \varphi_{l_p} R_l \varphi_{l_d} \tag{2}$$

is the modified (when $L \neq l$) one-step stripping amplitude, which can be easily calculated by the ordinary DWBA-method. In the case when the transferred momentum L coincides with a captured neutron orbital momentum l, the β_l^{lm} amplitude is exactly the same as the DWBA-code. The spectroscopic part of the cross-section is included in a weight-step factor:

$$\hat{B}_l^L = \sum_{\alpha I j} \gamma_{Ijl}^{\alpha B} (-1)^{I+j-J_B} \hat{j}\,\hat{l}(lI00|L0) W(jJ_B lL; Is_n) \hat{\mathscr{A}}_I(\alpha A), \tag{3}$$

where

$$\gamma_{Ijl}^{\alpha B} = \sum_{\nu M} (IjM\nu|J_B M_B) \langle J_B M_B B | a^+_{jl\nu\alpha} | IM\alpha \rangle \tag{4}$$

are the generalized spectroscopic amplitudes, which denote the quasiparticle ($I = 0$), quasiparticle + phonon ($I = 2,3$) and other higher components in the whole odd nucleus wave function $|J_B M_B B\rangle$. Here, the $|IM\alpha\rangle$ is a wave function of the initial even nucleus in the ground ($I = 0$) or excited states ($I \neq 0$). It is obvious that the quasiparticle component $\gamma_{0 J_B L}^{AB}$ determines the usual spectroscopic factor $S_L = |\gamma_L|^2$. Then, the operator

$$\hat{\mathscr{A}}_I = \delta_{I0}\delta_{\alpha A} + a_I \left(R_p \frac{\partial}{\partial R_p} + R_d \frac{\partial}{\partial R_d} \right) \tag{5}$$

adjusts a contribution of each step into the stripping amplitude. The coefficient a_I is directly proportional to a reduced matrix element of the phonon state excitation

$$a_I = \frac{\hat{I}\sqrt{4\pi}}{3AR_d^I} \langle I || \mathscr{M}_I || 0 \rangle, \tag{6}$$

where

$$\hat{\mathscr{M}}_{IM} = \sum_{i=1}^{A} r_i^I Y_{IM}(\hat{r}_i) \qquad (7)$$

is a mass transition I-multipolarity operator. In principle, the magnitude of a_I can be extracted independently from the corresponding experimental inelastic cross-sections. Also, it can be calculated in the framework of an appropriate nuclear model or expressed through an electric transition probability by means of introducing a dimensionless effective charge e_{ef} and effective nucleon mass q_{ef} parameters ($q_{ef}/e_{ef} \approx A/Z$):

$$a_I = \left(\frac{q_{ef}}{e_{ef}}\right)\frac{1}{e}\,\frac{\hat{I}\sqrt{4\pi}}{3AR_d^I}\,B^{\frac{1}{2}}_{I\to 0}(EI). \qquad (8)$$

Note that the well known perturbation result of Ref. [2] for the two-step cross-section on deformed nuclei follows from our equations, putting instead of $B(EI)$ the classical rotational model formula, so that a_I becomes equal to $\beta/\sqrt{4\pi}$, where β is the static deformation parameter.

With the help of the above given expressions numerical applications were carried out for the stripping reaction $^{52}Cr(d, p)^{53}Cr$ leading to the following states $3/2^-$ (g. s.), $1/2^-$ (0.567 MeV), $5/2^-$ (1.01 MeV), $3/2^-$ (2.32 MeV). The main purpose of this investigation

TABLE I

The expansion core-excitation γ-coefficients [8] of the odd nucleus ^{53}Cr wave function and the relative contributions of two-step effects in the stripping reaction $^{52}Cr(d, p)\,^{53}Cr$ at $E_d = 10$ MeV

$J_B\ L\ (E$ MeV)	$\gamma_{0J_BL} = \sqrt{S_L}$	γ_{2jl}			max $\frac{d\sigma - d\sigma_{\text{one-step}}}{d\sigma}$
	$j = J_B\ l = L$	$j = {}^1/_2\ l = 1$	$j = {}^3/_2\ l = 1$	$j = {}^5/_2\ l = 3$	
$3/2^-$ 1 (g.s.)	0.832	—	−0.444	−0.077	0.7
$1/2^-$ 1 (0.567)	0.447	—	0.834	−0.212	11.3 (21.4)* (38)**
$5/2^-$ 3 (1.01)	0.185	−0.381	0.433	−0.13	14.8
$3/2^-$ 1 (2.32)	0.507	−0.151	0.799	−0.068	1.8

* The two-step contribution at $E_d = 20$ MeV.
** The two-step contribution at $E_d = 30$ MeV.

was to find typical features of two-step corrections to one-step transitions on these states. Calculations were based on the ordinary DWBA code [7] which was modified slightly due to the necessity of taking into account the small change ($L \neq l$) in the amplitude (2). The mixture coefficients γ were those given by the core-excitation model in Ref. [8] (see Table). The set of optical parameters is the same as in Ref. [9]. The coefficient a_2 was chosen equal to 0.07, corresponding to a known experimental $B(E2)$ probability.

Now consider the obtained results.

1. First of all, one can see from the Table that the contributions of two-step effects are rather small and amount approximately to 10% of the one-step cross-section. Thus,

this result confirms the previous qualitative estimates [6] and also quantitative calculations in the framework or the "core-excitation stripping model" [5].

2. On the other hand, one can see the fast increase of the relative two-step effect contributions (about one or two orders) with increasing nuclear state excitation energy (from g. s. $(3/2^-)_1$ to 2.32 MeV $(3/2^-)_2$). The reason for this is the more complicated structure of high-lying nuclear states and, in particular, a more important role of the higher nuclear component admixtures.

3. However, we can conclude that the smallness of two-step effects in an absolute value ensures that they really may be treated in the framework of the perturbation approach. The other point is that in practice the presented formalism is rather convenient since it uses the ordinary DWBA code only.

4. Fig. 1 is a typical picture which demonstrates the influence of two-step effects on an angular distribution in a (d, p) stripping reaction. On the right-hand side of Fig. 1 the

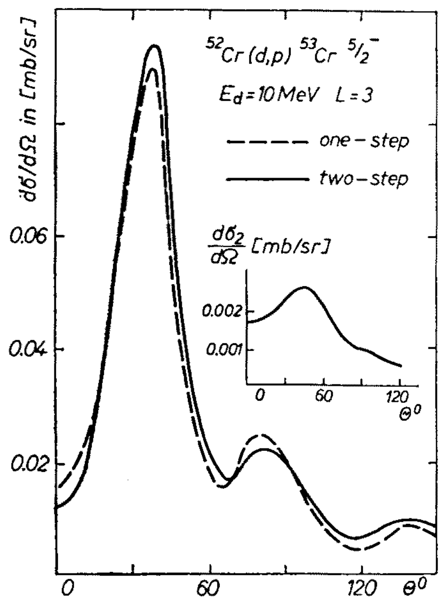

Fig. 1. Differential cross-sections of one- and two-step transferring in (d, p)-stripping at $E_d = 10$ MeV. On the right the pure two-step cross-section is shown when the one-step amplitude is equal zero

pure effect of the square two-step amplitude only is shown. As is easily seen, a two-step contribution is peaked at the angles near the principal maximum of a one-step differential cross-section and results in a smoother aggregate curve.

5. Fig. 2 shows the magnitude of two-step effects at a doubled deuteron energy of $E_d = 20$ MeV. We see that now the two-step contribution is about 10% of the one-step cross-section, *i. e.* 2 times larger than at $E_d = 10$ MeV. Of course, this fact is due to the increased role of virtual one-phonon state excitations.

6. Next, Fig. 3 gives a comparison between an "exact $(L \neq l)$" and an "approximate

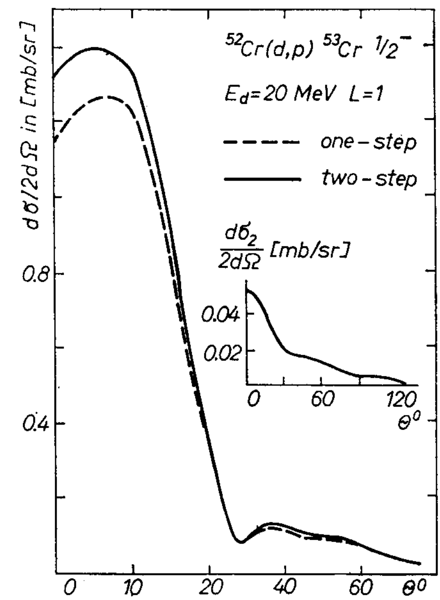

Fig. 2. Differential cross-section (divided by 2) of one- and two-step nucleon transferring at E_d = 20 MeV. On the right is the pure two-step effect

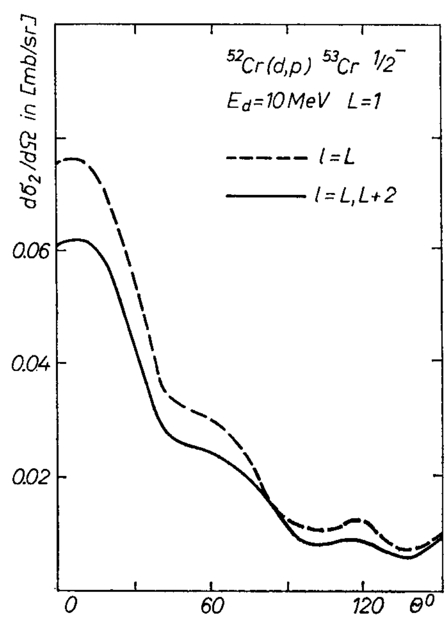

Fig. 3. A comparison of the two-step effects, calculated by means of an "exact" (all l) and an "approximate" ($l = L$) methods (see text)

($L = l$)" way of computing the two-step amplitude (2). The aim of this comparison is to investigate whether we can always use the ordinary DBWA code which calculates only the β_l^{lm} amplitude ($l = L$), and not use its modification for the general case ($l \neq L$), which needs more computing time. We see that the way of putting $L = l$ in Eq. (2) in all cases results in about a 20% change in two-step contributions. Therefore, one can conclude that for the purpose of a qualitative estimation it is actually possible to calculate two--step stripping without increasing the computational difficulties. In this case, the only necessity is to find the derivatives of the standard amplitudes β_l^{lm} by means of calculating them practically only at five points of R_p and R_d with a shift from one to the other by about 0.1 fermi. The parameters R_p and R_d are those of optical potentials in entrance and exit channels.

7. Finally, we note that here we have analyzed the two-step effects in allowed, generally strong, transfer transitions only. They occur with large probabilities, which is due to comparatively large magnitudes of quasiparticle components in the odd nucleus wave functions. However, it is obvious that the two-step effects must play a decisive role in the case of forbidden transitions, generally one or two orders weaker. The latter are observed experimentally rather often, but remain without any identification. An analysis of them in the framework of a multi-step stripping mechanism is of great theoretical interest because of the possibility of determining the nature of higher components in the nuclear wave functions.

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