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UNIFIED SPACE-TIME FOR INTERACTION OF ELEMENTARY PARTICLES

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In this paper the eight-dimensional Space-Time is utilized for establishing the Elementary Particles Theory free of the ultraviolet infinities. The main results are as follows. A procedure for evaluating the mean values with respect to the directly unobservable parameters is introduced. Although it is formally equivalent to the nonlocality introduced by [1], the theory is still microcausal and the S-matrix is unitary on the mass shell. In addition, particles are classified in multiplets; the same internal space may be used for both types of particles, leptons and hadrons. It is shown that the Yukawa interaction is deducible from the local gauge transformations in unified Space-Time. The interaction Lagrangians of strong interactions and of leptonic interactions are established.

For a long time the possibility of connecting the Space-Time of the Relativity Theory with the so-called internal space, such as isospace, was considered by many authors. Although no possibility was totally successful, the unified space theory gained some important progress for strong interaction theory. In our opinion the theory proposed by Rayski [2] is of interest.

The main objections for using the multidimensional theory are that it contains the parameters having no physical meaning and encounters divergencies stronger than that of orthodox theory. Therefore in order to remove these divergencies various theories, such as the nonlocal and essentially nonlinear theories [3–8], were proposed.

Fundamental assumptions are given in Section 1, the concept of internal space for weak and strong interactions is suggested.

In Section 2, the spinor equation of motion is studied and then the classification of elementary particles is realized for strongly interacting particles.

The Yang-Mills field corresponding to local gauge transformations is considered in Section 3. It is shown that for free particles unified Space-Time is divided into two parts: external space being Minkowskian Space-Time and internal space. The eight--covariance is required in presence of interactions.

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In Section 4, interaction of leptons and their symmetry is considered. There seems to exist a certain symmetry between four hadrons p, n, Ξ^0, Ξ^- and four leptons e, ν_e, ν_μ, μ instead of the Kiev symmetry.

In Section 5 the quantum field theory without ultraviolet infinities in the framework of unified Space-Time is derived, the S-matrix of this theory is covariant, microcausal and unitary on the mass shell.

1. Metric of unified space-time

In this Section some fundamental assumptions of theory are proposed to establish the metric of unified Space-Time. Firstly, let us discuss physical meaning of "abstract space" used in Particle Theory. As is well known, on the one hand, the spaces such as isospace, are consided conventionally as abstract spaces having no physical sense, *i.e.* they are not physical spaces. On the other hand, the quantities connected with them, such as isospin and hypercharge are of interest in view of experimental verifications. The conservation laws of these quantities for strong interactions were confirmed by actual experimental data.

In our opinion, it is possible that the space of this type has a certain physical meaning. However, it plays the role only in the intermediate states, since it is not directly observable.

Now, the new terminology is introduced and defined. Unified Space-Time is called a differentiable manifold, whose submanifolds are Minkowskian Space-Time and internal space, such as isospace. In addition, we can suppose that its topology is Euclidean. Then this manifold may be described by writting down its metric as follows:

$$ds^2 = \sum_{\mu=0}^{n} g_{\mu\mu} dx^{\mu^2}; \tag{1.1}$$

here $g_{\mu\mu} = \pm 1$, ds^2 will be defined by means of the following principles.

I. It is well known that causality is one of the most fundamental principles of physics. Therefore, for our physical theory this principle is firstly formulated as follows.

The causality principle of conventional sense is still true for physical motions in unified Space-Time manifold.

As is known, the causality is solidly connected with dynamics of motions. The second principle which concerns dynamic property of theory is the assumption that

II. Unified Space-Time manifold is a dynamic one in the sense that all equations of motion written in it are of hyperbolic type.

III. It is known that conservation laws concerning the conserved quantities, such as the electric charge and total angular momentum, are of importance for studying physical systems. They are directly related to the motion. Present elementary particle theory showed that for strong interactions, beside the usual conservation laws concerning the symmetry of Minkowskian Space-Time, there are yet the conservation laws for quantities concerning the symmetry of internal space, such as isospin and hypercharge. For weak interactions it seems that there exist similar conservation laws [19, 20]. Therefore, it is quite possible to assume that the following quantum quantities are conserved in connection with the symmetry property of unified Space-Time manifold: total angular momenta, isospin (or similar quantities), and hypercharge (or similar quantities). In the group theory this means that the rank of the motion group of unified Space-Time manifold must be three.

In the language of group theory, the third principle is formulated as follows:

The rank of the motion group of unified Space-Time manifold equals three.

IV. In order to guarantee that the orthodox theory is a special case of our theory, we are in need of a correspondence principle which can be the following.

There is the correspondence principle for going over to the ordinary theory in the case when the internal degrees of freedom are suppressed.

It will be shown below that the four above mentioned principles are sufficient to build our theory. The main purpose now is to determine the metric (1.1) in detail. The principles I and II assert that the coordinates characterizing our unified Space-Time manifold consist of one time coordinate and $(n-1)$ space coordinates, *i.e.* one has

$$ds^2 = dx^{o^2} - \sum_{i=1}^{n} dx^{i^2}.$$

On the other hand, following Ref. [21] the number of dimensions of space must be odd to ensure causality, *i.e.*

$$n = 2k+1$$

and therefore

$$ds^2 = dx^{o^2} - \sum_{i=1}^{2k+1} dx^{i^2}.$$

The principle III tells us that the rank of the motion group of unified space defined by

$$d\sigma^2 = \sum_{i=1}^{2k+1} dx^{i^2}$$

has to be equal to three. The 3-rank groups are clearly $O(6)$ and $O(7)$. Thus it is clear that

$$2k+1 = 7$$

and our metric is finally written as follows

$$ds^2 = dx^{o^2} - \sum_{i=1}^{7} dx^{i^2}. \qquad (1.2)$$

Hence, our unified Space-Time manifold is a pseudoeuclidean space of eight dimensions. Its motion group is $O(1, 7)$. It is easily seen that the principles I, II, III allow us to determine uniquely this metric. The momentum space corresponding to unified Space-Time has, of course, eight dimensions.

Now the following convention is introduced.

Rich letters, such as $\boldsymbol{a}$ and $\boldsymbol{b}$, correspond to 8-vectors.

The indices of summation with respect to all eight components 0, 1, 2, 3, 4, 5, 6, 7 are represented by Greek letters.
The indices of summation with respect to seven space components 1, 2, 3, 4, 5, 6, 7 — by Latin letters.
Vectors with components 1, 2, 3 are denoted by $\vec{a}$.
Vectors with components 4, 5, 6, 7 are denoted by $\bar{b}$.
The differential operator

$$\sum_{\sigma=0}^{7} g^{\sigma\sigma} \frac{\partial^2}{\partial x^{\sigma 2}}$$

is denoted by $\Box_x^8$.

2. *Equation of motion*

In this Section the equation of motion in unified Space-Time is considered.
The linear wave equation has the form

$$(\Box^8 - M^2)\Phi(x) = 0, \tag{2.1}$$

where M is a certain real number characterizing quantum states in unified Space-Time. It is called the iso-mass of particle. The eight-momentum of a certain "free" particle fulfils the relation

$$p^2 = p^{02} - p^{i2} = M^2$$

and the quantity given by

$$m^2 = p^{02} - \vec{p}^2 = \bar{p}^2 + M^2 \tag{2.2}$$

is identified with the physical mass of particle. $\vec{p}$ is momentum in ordinary 3-space and $\bar{p}$ is called iso-momentum, it is a directly unobservable quantity.

For massless particles, such as photon and neutrino, from $m = 0$ we deduce $\bar{p} = 0$ and $M = 0$. And for small-mass particles, such as electron and muon, from $m \ll 1$ we deduce $\bar{p}^2 \ll 1$ and $M^2 \ll 1$. Thus for leptons it is possible to characterize the internal space by means of the following restriction on iso-momentum

$$\bar{p}^2 \ll 1.$$

In case we choose the lepton mass, for instance the muon mass, to be mass unit, then the above inequality is rewritten

$$\bar{p}^2 \ll 1. \tag{2.3}$$

It is easily seen that for strongly interacting particles, that is mesons and barions, we have

$$\bar{p}^2 > 1. \tag{2.4}$$

The above two inequalities (2.3) and (2.4) lead to the important conclusion. As it is known, the isomomentum space is, of course, dual to internal space, therefore (2.3) and (2.4) impose certainly the restrictions on the geometric aproperties of internal space. Hence, the same internal space may be utilized to describe the multiplet states of both types, leptons and strongly interacting particles. To every case there corresponds one defined geometric property of this space.

We assume, in addition, that in the case when there is no interaction, unified Space--Time is branched off into two subspaces: Minkowskian Space-Time and internal space. And then the eightdimensional covariance may be violated. Inversely, in the case when there is interaction, the equation of motion has to be covariant under the transformations of coordinates in unified Space-Time, that is eightdimensional covariance is required.

Thus our unified Space-Time can be utilized universally for all elementary particles.

By linearization of equation (2.1) the spinor equation is obtained as follows

$$\left(\Gamma^{\mu}\frac{\partial}{\partial x^{\mu}}+M\right)\psi = 0; \tag{2.5}$$

here Γ^{μ} verify the well known relations

$$\Gamma^{\mu}\Gamma^{\nu}+\Gamma^{\nu}\Gamma^{\mu} = 2g^{\mu\nu}.$$

It is easily seen that the algebra of the Γ-matrix consists of 256 linearly independent elements, thereby the Γ^{μ} may be represented under the form of 16×16-matrices. For convenience, they may be represented as follows

$$\begin{aligned} \Gamma^{A} &= \gamma^{A}\times\tau_3\times\tau_3 \qquad (A = 0, 1, 2, 3)\\ \Gamma^{4} &= I_4\times\tau_2\times\tau_3 \qquad \Gamma^{5} = I_4\times\tau_1\times\tau_3\\ \Gamma^{6} &= I_4\times I_2\times\tau_1 \qquad \Gamma^{7} = I_4\times I_2\times\tau_2 \end{aligned}$$

here γ^{A} are the Dirac matrices, (τ_1, τ_2, τ_3) — the Pauli matrices and I_{α} are the unitary matrices of rank α.

Thus, ψ is a column matrix with 16 elements. The spinor $\bar{\psi}$ conjugate to ψ is defined by

$$\bar{\psi} = \psi^{+}\Gamma^{0}.$$

It is easily seen that the combinations $\bar{\psi}\Gamma^{\mu}\Gamma^{\nu}\ldots\psi$ are tensors in unified Space-Time.

The equation (2.1) is deduced simply from the following Lagrangian

$$\mathscr{L} = \frac{i}{2}\bar{\psi}\left(\Gamma^{\mu}\frac{\vec{\partial}}{\partial x^{\mu}} - \frac{\overleftarrow{\partial}}{\partial x^{\mu}}\Gamma^{\mu}\right)\psi - M\bar{\psi}\psi$$

from which we obtain easily the energy-momentum tensor

$$T^{\mu\nu} = \frac{i}{2}g^{\nu\nu}\left(\bar{\psi}\Gamma^{\mu}\frac{\partial\psi}{\partial x^{\nu}} - \frac{\partial\bar{\psi}}{\partial x^{\nu}}\Gamma^{\mu}\psi\right),$$

the eight-vector of current

$$J^{\mu} = \bar{\psi}\Gamma^{\mu}\psi$$

and the spin tensor

$$S^{\mu,\nu\varrho} = \tfrac{1}{4}\bar{\psi}\{\Gamma^{\mu}\overset{\nu\varrho}{\sum} + \overset{\nu\varrho}{\sum}\Gamma^{\mu}\}\psi,$$

where

$$\overset{\nu\varrho}{\sum} = \frac{1}{2i}(\Gamma^{\nu}\Gamma^{\varrho} - \Gamma^{\varrho}\Gamma^{\nu}).$$

The generators of transformations are expressed in terms of space integrals of the zeroth components of the spin tensor in the usual way

$$M^{\nu\varrho} = \int \delta^{0,\nu\varrho}d^7x = \tfrac{1}{2}\int \psi^{+}\overset{\nu\varrho}{\sum}\psi d^7x.$$

For $\nu, \varrho \leqslant 3$, $M^{\nu\varrho}$ are generators of ordinary Lorentz transformations and for $\nu, \varrho \geqslant 4$, $M^{\nu\varrho}$ are the generators of transformations of coordinates of internal space. In order to classify elementary particles, let us consider the group of coordinate transformations of internal space. It is easily seen that the $O(4)$-group is isomorphic locally to $O(3)\times O(3)$-group. Therefore all particles may be classified following the $O(3)\times O(3)$-symmetry given by [22]. Here we have the isospin operator $\vec{\tau}$ and "hyperchargespin" operator $\vec{\xi}$. Then the Gell-Mann-Nishijima formula is of the following form

$$Q = \tau_3 + \xi_3 .$$

The known particles are classified as follows

spinor (p, n, Ξ^0, Ξ^-)	spinor $(K^+, K^0, K^-, \bar{K}^0)$
vector $(\Sigma^+, \Sigma^0, \Sigma^-)$	vector $(\pi^+\ \pi^0\ \pi^-)$
scalar Λ	scalar η

and so on. As is suggested by [22], this classification has some advantages.

Now, consider the continuous and discrete transformations. At first it is easily seen that under the orthogonal transformations of coordinates of unified Space-Time

$$x^{\sigma} \to {}'x^{\sigma} = a^{\sigma}_{\varrho}x^{\varrho}$$

the spinor ψ transforms as follows

$$\psi \to {}'\psi = S\psi,$$

$$\bar{\psi} \to {}'\bar{\psi} = \bar{\psi}S^{-1},$$

in which

$$S^{-1}\Gamma^{\sigma}S = a^{\sigma}_{\varrho}\Gamma^{\varrho}.$$

Among discrete transformations of coordinates only the following one is considered

$$P_E : t \to t', \quad \vec{x} \to -\vec{x}, \quad \bar{x} \to \bar{x}$$

since it will be utilized below.

Under P_E the combinations $\bar{\psi}\Gamma_E\psi$, $\bar{\psi}\Gamma^\sigma\Gamma_E\psi$ transform as follows

$$\bar{\psi}\Gamma_E\psi \to -\bar{\psi}\Gamma_E\psi$$

$$\bar{\psi}\Gamma^\sigma\Gamma_E\psi \to \bar{\psi}\Gamma^\sigma\Gamma_E\psi',$$

and so on. Here $\Gamma_E = \Gamma^{\varrho-1}\Gamma^2\Gamma^3$.

Apart from the transformations of coordinates it is interesting to consider the gauge transformation having the form

$$\psi \to e^{i\alpha}\psi$$

which, of course, leads to the charge conservation law.

3. *Yukawa interaction*

In particular, let us now study the local gauge transformation, in which α is a certain function of coordinates,

$$\alpha = \alpha(\boldsymbol{x}).$$

Then in the spirit of the Yang-Mills theory [23], a compensation field appears

$$\partial_\mu\psi \to (\partial_\mu - i\varepsilon\varphi_\mu)\psi,$$

in which Φ_μ transforms according to the following law

$$\Phi_\mu \to \Phi_\mu + \frac{i}{\varepsilon}\frac{\partial\alpha}{\partial x^\mu}. \tag{3.1}$$

It is clear that Φ_μ is an eight-vector in unified Space-Time. If we introduce the tensor

$$F_{\mu\nu} = \partial_\nu\Phi_\mu - \partial_\mu\Phi_\nu$$

them the Lagrangian of the interacting fields in the following

$$\mathscr{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar{\psi}\Gamma^\mu(\partial_\mu - i\varepsilon\Phi_\mu)\psi - M\bar{\psi}\psi.$$

Hence we obtain the equations

$$\frac{\partial}{\partial x^\sigma}F_{\varrho\sigma} - i\varepsilon\bar{\psi}\Gamma_\varrho\psi = 0,$$

$$\Gamma^\mu(\partial_\mu - i\varepsilon\Phi_\mu)\psi + M\psi = 0.$$

In order to interpret the physical meaning of the Φ-potential, let us notice, on the one hand, that the transformation law (3.1) of the Φ-potential leads to the zero isomass. And, fortunately, it is easily seen that the Φ-field may have nonvanishing physical mass. This feature is different from the Yang-Mills theory in Minkowskian Space-Time. On the other hand, one needs to consider separately the local gauge transformations, whose phases α are functions of the Minkowski coordinates and the internal coordinates respectively.

1) $$\alpha = \alpha(x^0, \vec{x});$$

we obtain then

$$[\Gamma^0(\partial_0 - i\varepsilon\Phi_0) + \vec{\Gamma}(\vec{\partial} - i\varepsilon\vec{\Phi}) + \vec{\Gamma}\vec{\partial} + M]\psi = 0.$$

Using the correspondence principle and the equality (2.2), one can put

$$\vec{\Gamma}\vec{\partial} + M \to m, \ \psi \to \psi(x^0, \vec{x}).$$

Then the above equation becomes

$$(\sum_{A=0}^{3} \Gamma^A(\partial_A - i\varepsilon\Phi_A) + m)\psi = 0$$

which is the Dirac equation in the case when there is interaction between the spinor field and the vector field. The components are invariant under the transformations of coordinates of internal space. Thus $(\Phi_0, \Phi_1, \Phi_2, \Phi_3)$ can be identified with the ω-meson.

2) $$\alpha = \alpha(\vec{x}).$$

Then we obtain

$$[\sum_{A=0}^{3} \Gamma^A\partial_A + \vec{\Gamma}(\vec{\partial} - i\varepsilon\vec{\Phi}) + M]\psi = 0.$$

Using the correspondence principle and (2.2), we have

$$\vec{\Gamma}\vec{\partial} + M \to m, \ \psi \to \psi(x^0, \vec{x})$$

and the above equation takes the form

$$(\sum_{A=0}^{3} \Gamma^A\partial_A - i\varepsilon\vec{\Gamma}\vec{\Phi} + m)\psi = 0.$$

This is the Dirac equation with interaction of the Yukawa type. It is clear that $\vec{\Phi}$ are scalar under the transformations in Minkowskian Space-Time. Therefore $(\Phi_4, \Phi_5, \Phi_6, \Phi_7)$, can be identified to $(\pi^+, \pi^0, \pi^-, \eta)$. Hence, one important result is obtained in the framework of unified Space-Time theory, since the Yukawa interaction cannot be found by using the Yang-mills theory in Minkowskian Space-Time.

4. Lepton interaction

In this section the unified Space-Time is utilized for studying leptons. In this case $\vec{\tau}$ is called leptonic isospin and $\vec{\xi}$-leptoncharge-spin. Assume that τ_3, ξ_3 and electric charge Q of leptons are connected by the relation

$$Q = \tau_3 + \xi_3.$$

Then the four leptons e, ν_e, ν_μ, μ form one quartet as follows

$$L = \begin{pmatrix} e^- \\ \nu_e \\ \nu_\mu \\ \mu^+ \end{pmatrix}.$$

It is easily seen that electron leptonic charge L_e, muon leptonic charge L_μ and ξ_3 are related with each other as follows

$$L_e - L_\mu = 2\xi_3.$$

However, instead of L_e and L_μ, it is convenient to make use only of one type of leptonic charge for all the four leptons. The eigenvalues of ξ_3 are called the leptonic charges of leptons; they are, of course, conserved quantum numbers.

Now consider the weak interaction of leptons. The leptonic current interaction theory of Gell-Mann and Feynman gives us the interaction Lagrangian

$$L_W = \frac{G}{\sqrt{2}} J_\mu J^{\mu +},$$

where the leptonic current consisting of two parts, a vector and an axial one, has the form

$$\bar{J}_\mu = \bar{L}\Gamma_\mu(1-\Gamma_E)L$$

which contains the neutral currents of the following forms

$$\bar{e}o_\sigma e, \quad \bar{\mu}o_\sigma\mu, \quad \bar{\nu}_e o_\sigma \nu_e, \ \ldots,$$

here

$$o_\sigma = \bar{\gamma}_\mu(1 \pm \gamma_5), \quad \bar{\gamma}_A = \gamma_A; \quad \bar{\gamma}_\mu = I_4, \quad \mu \geqslant 4.$$

For leptons it is very interesting to consider some discrete coordinate transformations. Under the time inversion T,

$$T: x^0 \to -x^0, \vec{x} \to \vec{x}, \vec{x} \to \vec{x}$$

the components of L-spinor transforms as follows

$$e \to \gamma^1\gamma^2\gamma^3 e, \quad \nu_e \to \gamma^1\gamma^2\gamma^3\nu_e, \quad \nu_\mu \to \gamma^1\gamma^2\gamma^3\nu_\mu, \quad \mu \to \gamma^1\gamma^2\gamma^3\mu$$

similarly as in orthodox theory.

Under the 3-space reflection

$$P_E : x^0 \to x^0, \vec{x} \to -\vec{x}, \vec{x} \to \vec{x}$$

the transformation law of the L-spinor is the following

$$e \to \gamma^0 e, \quad \nu_e \to \gamma^0\nu_e, \quad \nu_\mu \to \gamma^0\nu_\mu, \quad \mu \to \gamma^0\mu$$

also similarly as in orthodox theory.

Under the reflection of internal space

$$P_I : x^0 \to x^0, \vec{x} \to \vec{x}, \vec{x} \to -\vec{x}$$

the L-spinor transforms as follows

$$e \to e, \quad \nu_e \to -\nu_e, \quad \nu_\mu \to -\nu_\mu, \quad \mu \to \mu.$$

Finally, it is very interesting to notice that in our theory, instead of Kiev symmetry[1] there is a new symmetry of leptons and four barions

$$e^- \leftrightarrow p^+, \quad \nu_e \leftrightarrow n, \quad \nu_\mu \leftrightarrow \Xi^0, \quad \mu^+ \leftrightarrow \Xi^-.$$

5. *S-matrix formalism in unified Space-Time*

Now the quantized field theory in eight-dimensional unified Space-Time is considered. Let us restrict ourselves in studying only scalar and spinor fields. Their Lagrangians are respectively as follows

$$\mathscr{L}(\varphi) = \tfrac{1}{2} \sum_\mu g^{\mu\mu} \left(\frac{\partial \varphi}{\partial x^\mu}\right)^2 - \frac{M_1^2}{2} \varphi^2(x),$$

$$\mathscr{L}(\psi) = \frac{i}{2} \sum_\mu \bar{\psi} \left(\Gamma^\mu \frac{\vec{\partial}}{\partial x^\mu} - \frac{\overleftarrow{\partial}}{\partial x^\mu} \Gamma^\mu\right) \psi - M_2 \bar{\psi} \psi.$$

The operators $\varphi(\boldsymbol{x})$ and $\Psi(\boldsymbol{x})$ are divided into negative-frequency and positive-frequency parts, $\varphi^\pm(\boldsymbol{x})$ and $\Psi^\pm(\boldsymbol{x})$. It is easily seen that in order to obtain always positive energies, the quantization of the scalar field must be subject to Bose-Einstein statistics and the quantization of spinor field must be subject to Fermi-Dirac one, so that $\varphi^+(\boldsymbol{x})$ and $\varphi^-(\boldsymbol{x})$ satisfy the following commutation relation

$$[\varphi^+(\boldsymbol{x}), \varphi^-(\boldsymbol{y})]_- = \frac{1}{i} D^+(\boldsymbol{x}-\boldsymbol{y}) \tag{5.1a}$$

and $\Psi^+(\boldsymbol{x})$, $\bar{\Psi}^-(\boldsymbol{y})$ satisfy the following anticommutation relation

$$[\psi^+(\boldsymbol{x}), \bar{\psi}^-(\boldsymbol{y})]_+ = \frac{1}{i} S^-(\boldsymbol{x}-\boldsymbol{y}), \tag{5.2a}$$

where

$$D^+(\boldsymbol{x}) = -D^-(-\boldsymbol{x}) = \frac{1}{(\pi 2)^7 i} \int e^{ikx} \delta(\boldsymbol{k}^2 - M^2) \theta(k^0) d\boldsymbol{k},$$

$$S^\pm(\boldsymbol{x}) = \left(i \sum_\mu \Gamma^\mu \frac{\partial}{\partial x^\mu} + M\right) D^\pm(\boldsymbol{x}).$$

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[1] As is known, this symmetry was proposed at Kiev Conference on High-Energy Physics 1959.

The Green "causal" functions of scalar and spinor fields are obtained as follows

$$\underbracket{\varphi(x)\varphi(y)} = D^c(x-y), \tag{5.1b}$$

$$\underbracket{\psi(x)\bar{\psi}(y)} = S^c(x-y), \tag{5.2b}$$

where

$$D^c(x) = \frac{1}{(2\pi)^8}\int \frac{e^{ipx}}{p^2-M^2-i\varepsilon}\,dp, \tag{5.3}$$

$$S^c(x) = \left(i\sum_{\mu}\Gamma^{\mu}\frac{\partial}{\partial x^{\mu}}+M\right)D^c(x). \tag{5.4}$$

The vacuum states of free fields are defined as states in which there is no particle. In other woıds, the energy and three-momentum of these states equal zero, *i.e.*

$$\varphi^{-}(k)\Phi_0 = 0$$

and

$$\varphi^{-}(p)\Phi_0 = 0,$$

$$E_0 = 0\ \vec{P}_0 = 0.$$

From the relatiom

$$m_0^2 = E_0^2-\vec{p}_0^{\,2} = \vec{p}_0^{\,2}+M_0^2$$

one deduces

$$m_0 = 0,\ \vec{p}_0 = 0,\ M_0 = 0.$$

This implies that eight-momenta of vacuum states equal zero.

It is clear that the quantum relations (5.1) and (5.2) formally coincide with those of the orthodox theory. However, here their physical meaning is still unclear. It will be shown below, that if we take into account the interaction, *e. g.* the scalar-spinor interaction, the physical sense of (5.1) and (5.2) becomes clarified. To do this, let us now remind the definition of *S*-matrix, starting from which the *S*-matrix formalism of our theory is established satisfactorily. As it is known the *S*-matrix is an operator mapping the in-states to the out-states

$$S\Phi_{\text{in}} = \Phi_{\text{out}},$$

where Φ_{in} and Φ_{out} are asymptotic states, in which the particles are free. As usual, the *S*-matrix satisfies, of course, the fundamental physical requirements, some of which are the following

1. Unitarity condition

$$SS^{+} = 1.$$

2. Microcausal condition of Bogolubov [24]

$$\frac{\delta}{\delta\varphi(x)}\left(\frac{\delta S}{\delta\varphi(y)}S^{+}\right) = 0 \quad \text{for} \quad x \leqslant y.$$

Then the matrix elements are given by

$$A_{i\to f} = \langle \Phi_{\text{out}}|S|\Phi_{\text{in}}\rangle. \tag{5.5}$$

Unfortunately, it is easily seen that in the case when the perturbation theory is utilized, we would encounter the ultraviolet infinities to be stronger than in the orthodox theory, for instance, the self-energy graph of scalar field gives us the divergence of 6 degrees whereas in orthodox theory the divergence is only quadratic. This fact is clearly connected with the introduction of the directly unobservables $\vec{p}$. However, this difficulty can be suppressed immediately by the following procedure.

As it was mentioned above, in the states Φ_{in} and Φ_{out} one has

$$p^2 - M^2 = 0$$

or

$$p^{02} - \vec{p}^2 = \vec{p}^2 + M^2 = m^2$$

i. e. the well-known relation of Einstein is satisfied. Therefore it is possible to consider Φ_{in} and Φ_{out} as observable states for in reality they are in full coincidence with the corresponding states of orthodox theory. And for intermediate states $|n\rangle$ the relation $p^2 - M^2 = 0$, of course, is not fulfilled. $|n\rangle$ and $\vec{p}$ are not directly observable, thereby it is necessary to evaluate the mean values with respect to $\vec{p}$ belonging to $|n\rangle$. Taking into account the eight-covariance, we need to evaluate the mean values with respect to the eight-momenta p^σ belonging to $|n\rangle$ with the aid of an appropriate weight function, that is

$$M \sim \int F(p) V(p)\, dp$$

where $F(p)$ is the matrix element corresponding to the contribution of all intermediate states. Of course, the weight function $V(p)$ has to be chosen so that

1) the theory would be free of ultraviolet infinities,
2) the S-matrix would be unitary on the mass shell,
3) the microcausality in Minkowskian Space-Time would be valid.

We will demonstrate that with the aid of $V(p)$ in the form given by Ref. [1], that is

$$V_\lambda(x-y) = V_\lambda(\Box_x^8)\delta^8(x-y)$$

where

$$V_\lambda(\Box_x^8) = \int_{\varrho^2<\lambda^2} d^8\varrho\, a(\varrho^2) \exp(i\varrho_0\partial_0 + \varrho_a\partial_a),$$

$$V_\lambda(\Box_x^8) = \int_{\varrho^2<\lambda^2} d^8\varrho\, a(\varrho^2) \exp(\varrho_0\partial_0 + i\varrho_a\partial_a),$$

all the above mentioned conditions are fulfilled. To prove this, let us consider for illustration the example of a scalar field. The Lagrangian of this field can be written in the form

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I,$$

where $\mathcal{L}_0$ is the free field Lagrangian and $\mathcal{L}_I$ describes the self-interaction of this field. It is of the form, *e. g.* $g\Phi^4$. Then formally the S-matrix may be written as follows

$$S = T\exp\{-ig\int \mathcal{L}_I dx\}.$$

It is easily seen, however, that this S-matrix leads to strong infinities for large momenta. Hence, it is necessary to introduce the averaging procedure. The latter is formally similar to the nonlocality given by Efimov. We assume that there is no contribution of the Φ field to the interaction Lagrangian, exept the field $\varphi(x)$ defined by

$$\varphi(x) = \int dy V(x-y)\Phi(y) = V(\Box_x^8)\Phi(x),$$

$$V(x-y) = V(\Box_x^8)\delta^8(x-y),$$

then we have

$$D^c(x-y) = \underbrace{\varphi(x)\varphi(y)} = V(\Box_x^8)V(\Box_y^8)\Phi(x)\Phi(y)$$

$$= \int e^{ip(x-y)} \frac{(V[p^2])^2}{p^2-M^2}\, dp$$

that is the propagator is replaced as follows

$$\frac{1}{p^2-M^2} \to \frac{[V(p^2)]^2}{p_2^2-M^2};$$

the commutator is given by

$$[\Phi(x), \Phi(y)]_- = \int dx'dy' V(x-x')\, V(y-y')\, \Delta(x'-y')$$

$$= [V(M^2)]^2\, \Delta(x-y).$$

In order to prove the validity of microcausality, let us first study briefly the distributions given by

$$V_\lambda(x-y) = V_\lambda(\Box_x^8)\delta^8(x-y).$$

The space of fundamental functions D is such a space that the functionals

$$\langle V, f\rangle = \int dy V(x-y)f(y) = V(\Box_x^8)f(x)$$

are defined uniquely for each $f(x) \in D$. As is raised by Ref. [1] the topology of this space is the following:

1. Each $f(x)$ is the value on the real axis of a certain entire function $f(z)$ increasing slower than $|z|^p$ for $z \to \infty$.

2. An arbitrary sequence of functions $\{f_n(x) | f_n \in D\}$ of D converges to zero in a certain region G, if and only if all the functions of this sequence converge uniformly to zero in this region.

Let us now study the local property of the distributions V. Let $\{f_\nu(x, y)\}$ be the sequence of fuctions belonging to D so that the limit function

$$f(x, y) = \lim_{\nu\to 0} f_\nu(x, y)$$

does not belong to D and equals zero for all $x \neq y$. Otherwise, $f_\nu(x, y)$ is normalized by

$$\int d^8 x f_\nu(x, y) = 1.$$

That sequence $\{f_\nu(x, y)\}$ gives clearly a δ-representation in the D space. Thus for every $f_\nu(x-y)$ such that $\lim_{\nu \to 0} f_\nu(x-y) = 0$ for all $x \neq y$ the function

$$g_\nu(x) = V(\Box_x^8) f_\nu(x)$$

has the property

$$g(x) = \lim_{\nu \to 0} g_\nu(x) = 0$$

outside the region $x^0 = 0, \vec{x}^{22} + \vec{x} < \lambda^2$. These result are the generalization of the results obtained in [1].

Now, the terms of the perturbation theory series are considered. Formally we have the S-matrix in the form

$$S = T \exp\{-ig \int \mathscr{L}_I dx\}$$

The matrix element of a certain graph in n-order approximation in configuration space is of the form

$$F(x_1, x_2, \ldots, x_n) = \prod_{i=j} D\,(x_i - y_j).$$

The Fourier transforms of $F(x_1, \ldots, x_n)$ in momentum space is given as follows

$$F(p_1, \ldots, p_n) = \int \cdots \int \prod_i dl_i \prod_j \frac{[V(k_j^2)]^2}{k_j^2 - M^2 + i\varepsilon},$$

where k_j are the eight-momenta corresponding to the interior lines of graph and l—the eight-momenta of the integration.

We can see immediately, that the above integral converges and the weight function plays the role of a form factor.

The unitarity of S-matrix is proved in a way similar to that of Ref. [1].

Finally let us consider the microcausality of S-matrix. As it has been proved in Ref. [1], with the aid of the Green function which is given as follows

$$D^c(x-y) = \int dx dy V(x-x') V(y-y') \Delta^c(x'-y'),$$

the S-matrix satisfies the relation

$$\frac{\delta}{\delta \Phi(x)} \left(\frac{\delta S}{\delta \Phi(y)} S^{-1} \right) = 0$$

outside the regions G and G_l, where

$$G : x^0 \geqslant y^0,\ (x-y)^2 > 0,$$

$$G_l : -l^2 \leqslant (x-y)^2 \leqslant l^2.$$

This implies that the S-matrix fulfils the "macrocausal" condition in unified Space-Time, and that leads to the microcausality of the S-matrix in Minkowskian Space-Time. Indeed, the regions G and G_l can be rewritten as follows

$$G: x^0 \geqslant y^0, (x^0-y^0)^2-(\vec{x}-\vec{y})^2 > (\vec{x}-\vec{y})^2,$$
$$G_l: -l^2+(\vec{x}-\vec{y})^2 \leqslant (x^0-y^0)^2-(\vec{x}-\vec{y})^2 \leqslant l^2+(\vec{x}-\vec{y})^2.$$

Because $(\vec{x}-\vec{y})^2$ is an arbitrary number, then $-l^2+(\vec{x}-\vec{y})^2$ can be arbitrarily small. Then in reality both regions G and G_l projected in Minkowskian Space-Time can be characterized by the following inequality

$$x \geqslant y.$$

Therefore the above mentioned relation for the S-matrix takes the form

$$\frac{\delta}{\delta\Phi(x)}\left(\frac{\delta S}{\delta\Phi(y)} S^{-1}\right) = 0 \quad \text{for} \quad x \leqslant y.$$

This is the microcausality formulated by Bogolubov [24].

Summarizing one can state that the use of the averaging procedure which is closely connected with the unobservable parameters makes the theory free of ultraviolet infinities and the S-matrix microcausal in Minkowskian Space-Time and unitary on the mass shell.

The results obtained may be generalized for studying the interactions of two arbitrary fields, for instance, the interaction of Yukawa type

$$\mathscr{L}_I = g\bar{\psi}\Gamma\psi\Phi$$

in which the propagators of spinor and scalar fields are given by

$$S^c(p) = \frac{V(p^2)}{\tilde{p}-M^2},$$

$$D^c(k) = \frac{W(k^2)}{k^2-M_1^2},$$

where V and W fulfil the conditions mentioned above. Thus it is possible to formulate the Feynman rules for an arbitrary graph in momentum space as follows

a) factor $\dfrac{V(p)^2}{\hat{p}-M_2+i\varepsilon}$ for every interior spinor line with momentum p;

b) factor $\dfrac{W(k^2)}{k^2-M_1^2+i\varepsilon}$ for every interior line of scalar fields with momentum k;

c) factor $g\Gamma$ for every vertex,

d) factor $(2\pi)^{-7/2}\dfrac{1}{2\sqrt{\omega_K}}$ for every exterior scalar line with the energy $\omega_K = (\vec{k}^2+\mu^2)^{1/2}$, here, μ is the physical mass of the scalar particle;

e) factor $(2\pi)^{-7/2}\overline{w}^s(p)$ for every exterior spinor line with the momentum p and spinor index s, leaving the graph;

f) factor $(2\pi)^{-7/2}w^s(p)$ for every exterior spinor line with momentum p and spinor index s, entering the graph;

g) factor $(2\pi)^8\delta^8(p-p'\pm k)$ for every vertex corresponding to energy-momentum conservation;

h) factor (-1) for every closed spinor loop;

i) finally it is necessary to integrate over all interior momenta.

It is easily seen that using the above mentioned Feynman rules the matrix element of an arbitrary graph contains only observable quantities and therefore it may be compared directly with the experiment.

6. *Conclusions*

Let us discuss the results obtained. The present paper was an attempt to construct an elementary particle theory free of ultraviolet infinities in confluence with the symmetry theory. The main results are the following.

1. The metric of unified Space-Time is defined on the basis of several fundamental assumptions starting from the natural requirements, such as causality and the conservation laws.

2. It is shown that the same unified Space-Time is useful for hadrons and leptons. The particles are classified according to multiplets if $SU(2)\times SU(2)$-symmetry and the interaction Lagrangians are established. In particular, the Yang-Mills fields, corresponding to the local gauge transformations in unified Space-Time, may possess nonvanishing physical mass and lead to Yukawa interaction.

3. Instead of Kiev symmetry, a new symmetry between four barions and four leptons is proposed.

4. In order to suppress the unobservable parameters connected with the coordinates of the internal space, a procedure for evaluating the mean values is introduced. Formally this procedure is similar to the nonlocality given by the Ref. [1] in Minkowski Space-Time. However, with the aid of adequate weight functions the theory is not only free of ultraviolet infinities, but its S-matrix is microcausal in Minkowskian Space-Time and unitary on the mass shell.

The unobservable parameters play only the role in the intermediate states, the final results contain only observable quantities. The bove obtained results prove clearly that the unified Space-Time theory has several advantages in comparison with the elementary particle theory based on Minkowskian Space-Time.

Here we have built the theory in which the symmetry theory and the Yang-Mills field theory are combined with the microcausal theory free of ultraviolet infinities.

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