LETTERS TO THE EDITOR

ON THE BERNSTEIN-SALZ CONJECTURE IN THE NONLINEAR LEE MODEL

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A conjecture of Bernstein and Salz is investigated in the framework of the nonlinear Lee model; an argument is put forward that this conjecture is true.

In a paper by Bernstein and Salz [1] it was shown that in the Lee model [2] a nice relation between the V particle renormalization constant and the derivate of the bare with respect to the renormalized mass of the V particle holds, namely

$$\frac{\partial m_{0V}}{\partial m_V} = Z_V^{-1}. \tag{1}$$

Recently, Calogero [3] has developed a nonlinear version of the Lee model in which the Hamiltonian is of the form $H + H_0 + f(H_I)$; H_0 and H_I are respectively the free and the interaction parts of the Hamiltonian of the Lee model and $f(x)$ is an arbitrary real function.

In our paper we intend to discuss the BS conjecture in the "Calogero-Lee" model, in the sectors with the quantum numbers $Q_1 = 1$, $Q_2 = 0$ and $Q_1 = 2$, $Q_2 = 1$. It seems to us that this discussion is important, because the Calogero version of the Lee model may serve as a first approximation for the field theories with nonpolynomial Lagrangians.

1

In the sector characterized by $Q_1 = 1$, $Q_2 = 0$ the wave function renormalization constant of the V particle is (see Eqs (4.3), (4.11), Ref. [3])

$$Z_V^{-1} = 1 + \frac{b^2}{c^2} (\bar{\omega} - \Omega)^2 \int \frac{\lambda^2(\omega) d\omega}{(\bar{\omega} - \omega)^2}, \tag{2}$$

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where $\bar{\omega} = m_V - m_N$, $\omega_0 = m_{0V} - m_N + f_e(\Lambda)$, $\Omega = \omega_0 - c^2/b$, $c^2 = f_0(\Lambda)/\Lambda$, $b = f_e(\Lambda)/\Lambda^2$; $f_e(\Lambda)$ and $f_0(\Lambda)$ are respectively the even and the odd parts of the function $f(\Lambda)$ and

$$\Lambda^2 = \int \lambda^2(\omega)d\omega = \int \frac{\lambda_0^2 k f_1^2(k^2)d\omega}{4\pi^2}. \tag{3}$$

From Eq. (4.12) of Ref. [3] we have

$$\delta m_V = m_V - m_{0V} = f_e(\Lambda) - \frac{c^2}{b}\left[1 - \left(1 + b\int \frac{\lambda^2(\omega')d\omega'}{\omega' - m_V + m_N}\right)^{-1}\right] \tag{4}$$

and therefore

$$\frac{\partial m_{0V}}{\partial m_V} = 1 + c^2\left[1 + b\int \frac{\lambda^2(\omega)d\omega}{\omega - m_V + m_N}\right]^{-2} \cdot \int \frac{\lambda^2(\omega')d\omega'}{(\omega' - m_V + m_N)^2}. \tag{5}$$

Substituting (2) into (5) we obtain

$$\frac{\partial m_{0V}}{\partial m_V} = 1 + \frac{c^4(Z_V^{-1} - 1)}{b^2(\bar{\omega} - \Omega)^2\,[1 + bF(\bar{\omega})]^2}, \tag{6}$$

where

$$F(\bar{\omega}) = \int \frac{\lambda^2(\omega')d\omega'}{\omega' - \bar{\omega}}. \tag{7}$$

From (4) and (7) we have

$$bF(\bar{\omega}) = (\omega_0 - \bar{\omega})/(\bar{\omega} - \Omega) \tag{8}$$

and

$$\frac{\partial m_{0V}}{\partial m_V} = 1 + \frac{c^4(Z_V^{-1} - 1)}{b^2(\omega_0 - \Omega)^2}. \tag{9}$$

Because $\omega_0 - \Omega = c^2/b$ we obtain from (9)

$$\frac{\partial m_{0V}}{\partial m_V} = Z_V^{-1} \tag{10}$$

i. e. the BS relation.

2

In this section we discuss the sector characterized by $Q_1 = 2$, $Q_2 = 1$ where two baryons, localized at the fixed positions $\vec{r}_1$ and $\vec{r}_2$ ($r = |\vec{r}_1 - \vec{r}_2|$), are present.

By explicit calculation one finds that two eigenstates are present in this sector: $|NV; r; +\rangle$ — symmetric and $|NV; r; -\rangle$ — antisymmetric under exchange of the coordinates $\vec{r}_1$ and $\vec{r}_2$. The wave function renormalization constants $Z_\pm(r)$ for these two states are

$$Z_\pm^{-1}(r) = 1 + 2\left[\frac{b_\pm(r)\,(\bar{\omega}(r) - \Omega_\pm(r))}{c_\pm(r)}\right]^2 \cdot \int d\vec{k}' \; \frac{\gamma^2(k')\begin{Bmatrix}\cos^2\\ \sin^2\end{Bmatrix}(\tfrac{1}{2}\vec{k}'\cdot\vec{r})}{(\bar{\omega}(r) - \omega')^2}, \tag{11}$$

where $\Omega_{\pm}(r) = \omega_{0\pm}(r) - c_{\pm}^2(r)/b_{\pm}(r)$, $\omega_{0\pm}(r) = 2E + f_e(\Lambda_{\pm}(r))$, $2E = m_{0V} - m_N$, $b_{\pm}(r) = f_e(\Lambda_{\pm}(r))/\Lambda^2{}_{\pm}(r)$, $c_{\pm}(r) = f_0(\Lambda_{\pm}(r))/\Lambda_{\pm}(r)$ and

$$\Lambda_{\pm}^2(r) = \int d\vec{k}'\gamma^2(k')\,[1 \pm \exp(i\vec{k}' \cdot \vec{r})]. \tag{12}$$

In Ref. [3] it is stated that

$$c_{\pm}^2(r)F_{\pm}(\bar{\omega}(r); r) = [\omega_{0\pm}(r) - \bar{\omega}(r)] \cdot [1 + b_{\pm}(r)F_{\pm}(\bar{\omega}(r); r)], \tag{13}$$

i. e.

$$\frac{\partial\omega_{0\pm}(r)}{\partial\bar{\omega}(r)} = 1 + \frac{\partial F_{\pm}(\bar{\omega}(r); r)}{\partial\bar{\omega}(r)}\, b_{\pm}(r)[\bar{\omega}(r) - \Omega_{\pm}(r)]/[1 + b_{\pm}(r)F_{\pm}(\bar{\omega}(r); r)], \tag{14}$$

where

$$F_{\pm}(\bar{\omega}(r); r) = \int d\vec{k}'\gamma^2(k')\,\frac{1 \pm \exp(i\vec{k}' \cdot \vec{r})}{\omega' - \bar{\omega}(r)}. \tag{15}$$

Substituting (11) into (14) and using (13) and (15) we obtain

$$\frac{\partial\omega_{0\pm}(r)}{\partial\bar{\omega}(r)} = Z_{\pm}^{-1}(r) \tag{16}$$

i. e. the BS relation.

Taking into account that in the "Calogero-Lee model", in the sector $Q_1 = 1$, $Q_2 = 0$ there can be two bound states and in the sector $Q_1 = 2$, $Q_2 = 1$ there can be four bound states, we see that the proof of the B S conjecture is not trivial at all.

Incidentally, we have observed that Eqs (43a, b) of Ref. [4] lead also to the BS conjecture.

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