QUANTUM ELECTRODYNAMICS WITHOUT ULTRAVIOLET INFINITIES

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Quantum Electrodynamics without ultraviolet infinities is constructed satisfactorily. The cosmic field, discovered earlier is utilized as the regularized field of the theory. The mechanism of violation of the scale invariance principle is outlined for the first time. In the second order of the perturbation theory the corrections for mass and charge of electron are obtained as follows

$$\frac{\delta m}{m} = \frac{3\alpha}{4\pi} \ln\left(\frac{1}{lm}\right)^2 + O((lm)^2 \ln (lm)^2),$$

$$\frac{\delta e}{e} = -\frac{\alpha}{6\pi} \ln\left(\frac{1}{lm}\right)^2 + O((lm)^2 \ln (lm)^2).$$

The procedure for securing the gauge invariance of the theory is established. The problem of the equivalence theorem is discussed. Finally the system of equations for propagators of electron and photon is obtained and the physical meaning of the two renormalization constants Z and Z_1 is discussed.

1

As it is known, quantum electrodynamics encountered the ultraviolet infinities some seventy years ago. Weisskopf [3], using positron theory, had found the infinity related with the electromagnetic mass of electron to be as follows:

$$\frac{\delta m}{m} \approx \frac{6\alpha}{4\pi} \lim_{R\to\infty} \log \frac{1}{Rm} + \text{finite terms}.$$

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In all higher orders, as was suggested by Weisskopf, $\delta m/m$ may diverge as fast as $\alpha_n\left(\log\frac{1}{Rm}\right)^n$. This fact leads to the existence of a "critical" length

$$R_{\text{crit}} = \frac{1}{m}\exp\left(-\frac{1}{\alpha}\right) \approx 10^{-56}/m.$$

In the modern formalism of quantum electrodynamics, the above result obtained by Weisskopf was confirmed by Dyson [4] who in addition proved that a further infinity exists related with the so-called self-charge of electron. The pioneering paper of Bethe [5] led to the so-called renormalization method by which this important value in modern quantum field theory was obtained. The renormalization in quantum electrodynamics seems to satisfy us with its results which are in good agreement with experiment as for instance, the Lamb displacement of levels $2S_{\frac{1}{2}}$ and $2P_{\frac{1}{2}}$ of hydrogen atom and the anomalous magnetic moment of the electron. However, it is clear that the physical foundations as well as the mathematical basis of renormalization are yet obscure.

In order to suppress the ultraviolet infinities, various theories were developed. The nonlocal relativistic quantum field theory is of considerable importance [6–20]. Recently, the theory of field interactions with the nonpolynomial Lagrangian, to provide a theory free of infinities, was indicated by Efimov and Fradkin [21, 22]. The technique for this theory had been developed by Volkov and other authors [23–25]. The papers of Salam, Isham and Strathdee [26, 27] gave, probably, a very important contribution to this effort. With the aid of the gravitational field, as a regularized field, quantum electrodynamics is described by the following non-polynomial Lagrangian

$$\mathscr{L}_{em} = ie_0\frac{1}{\sqrt{-g}}\bar{\psi}\gamma_\mu\psi A^\mu$$

or, in vierbein (or tetrad) formalism,

$$\mathscr{L}_{em} = ie_0\frac{L^{\mu a}}{\det L}\bar{\psi}\gamma_a\psi A_\mu,$$

here $L^{\mu a}$ are the vierbein components expressed by means of the formula

$$g_{\mu\nu} = L_{\mu a}L_{\nu b}\eta^{ab}$$

$$\eta_{ab} = (1, -1, -1, -1).$$

The self-energy graphs of the second order for electron and photon are free of infinities. In general, Volkov [27] has shown that by means of gravitational propagator defined by Salam [27] all the infinities encountered in quantum electrodynamics are suppressed.

However, various difficulties, some of which were suggested by the authors themselves, arise as follows:

1. The equivalence theorem asserts that on the mass shell matrix elements are identical, irrespective of which interpolating field $g_{\mu\nu}$ or $g^{\varrho\sigma}$, we start from, provided these fields

possess the same asymptotic states and also belong to the same locally-commutative equivalence class. The same remark applies to coordinate transformations considered in general relativistics by means of which one can incorporate the factor $(\det g^{-\frac{1}{4}})$ into the definition of matter field, that is

$$\psi' = (\det g)^{-\frac{1}{4}}\psi.$$

The earlier difficulties of the theory appear again. After [28], in this case it is convenient to make use of the Lagrangian given by [29]

$$\mathscr{L}(\psi, A, h) = \left\{\frac{i}{2}\, r^{\mu\nu}(\alpha)\, [\bar{\psi}\gamma_\mu(\partial_\nu - \Gamma_\nu - ieA_\nu)\psi - \right.$$

$$\left. - \bar{\psi}(\overleftarrow{\partial}_\nu + \Gamma_\nu + ieA_\nu)\psi] - m\bar{\psi}\psi \right\} (\det g)^{-\frac{1}{4}}$$

where $r^{\mu\nu}(x)$ is defined by means of the series expansion

$$r^{\mu\nu}(x) = \sqrt{\eta^{\mu\nu} + Kh^{\mu\nu}(x)} = \eta^{\mu\nu} + \frac{K}{2}\, h^{\mu\nu}(x) - \frac{K^2}{8}\, h^{\mu\alpha}h^{\alpha\nu} + \ldots$$

$h^{\mu\nu}(x)$ are quantized field operators, commutators of which are defined as follows

$$\underbrace{h^{\alpha\beta}(x)h^{\gamma\delta}(y)} = -\frac{i}{2}\,[\eta^{\alpha\gamma}\eta^{\beta\delta} + \eta^{\alpha\delta}\eta^{\beta\gamma} - \eta^{\alpha\beta}\eta^{\gamma\delta}]\Delta^c(x-y).$$

The determinant of the metric tensor can be eliminated by the equivalence transformations, but it is clear that the non-polynomiality of this Lagrangian is conserved. However, it is unfortunate that the superpropagator consists of two operators $\gamma^{\alpha\beta}(x)$ and $\gamma^{\gamma\delta}(y)$, which may only regularize the logarithmic infinity. Otherwise, as is seen above, different formulations of gravity theory lead to different possibilities of removing ultraviolet infinities.

2. In our opinion, the gauge invariance of the theory in any order of perturbation theory is again unsolvable. It is probable that the theory will be considered to be correct if and only if the above difficulties are solved. However, it is possible that they cannot be suppressed in any manner if the gravitational field is still utilized as a regulator.

Before passing to the study of quantum electrodynamics by means of the cosmic field, it is very interesting to note that some twenty years ago Pauli and Villars [30] have mentioned that the above-discussed critical length R_{crit} would be the length where the theory may be expected to need some fundamental modifications—either a "realistic" one (through inclusion of other forces the influence of which may remove the infinities) or a purely mathematical modification. In our opinion, this is a proof of an insight!

Basing on our papers [1, 2] it will be shown that both modifications raised by Pauli are accounted for in our theory. The other "force" is expressed by the cosmic field and the mathematical modification is expressed by a non-polynomial Lagrangian for electron and photon interacting with the cosmic field. Therefore, quantum electrodynamics will be constructed satisfactorily.

In paragraph 2 the symmetry group of the theory is considered. It is shown that the mechanism of violation of the scale invariance principle is connected with the existence of cosmic field.

The corrections of self-mass and self-charge of electron are calculated in paragraph 3. The obtained values are of the order of those obtained by Salam [27].

In paragraph 4 the problems of gauge invariance and of equivalence theorem are concerned and discussed. The procedure for securing the gauge invariance is established.

The general equations for propagators of electron and photon are obtained in paragraph 5. Paragraph 6 is concerned with other important problems of quantum electrodynamics.

2

It is shown that the cosmic field is characterized by the non-Galilean structure of space--time geometry [1, 2]

$$ds^2 = e^{-2l\chi(x)}(dt^2 - d\vec{r}^2) \tag{2.1}$$

here $\chi(x)$ is the cosmic field operator and l is new constant which has the dimension of length in the system of units where $h = c = 1$.

The geometry given by (2.1) is conformly Euclidean, thus according to Taub [31] it possesses a maximum group of transformations on itself, as follows

$$x'^{\sigma} = a^{\sigma} + \frac{\alpha^{\sigma}_{\varrho}x^{\varrho} + \alpha^{\sigma}x^2}{1 + \alpha^{0}_{\varrho}x^{\varrho} + \alpha^{0}x^2}, \tag{2.2}$$

in which

$$\alpha^{\sigma} = \alpha^{0}_{\varrho}\alpha^{\sigma}_{\varrho},$$

$$\alpha^{0} = \alpha^{0}_{\varrho}\alpha^{0}_{\varrho},$$

$$\alpha^{\sigma}_{\mu}\alpha^{\sigma}_{\nu} = \alpha^{2}\delta_{\mu\nu}.$$

This 15-parameter group is one of the possible, physically interesting generalizations of the Poincaré group. It includes the following groups:

1. Space-time translations

$$'x^{\sigma} = x^{\sigma} + a^{\sigma}$$

2. Homogenous Lorentz transformations

$$'x^{\sigma} = \Lambda^{\sigma}_{\varrho}x^{\varrho}$$

3. Special conformal transformations

$$'x^{\mu} = \frac{x^{\mu} + \beta^{\mu}x^2}{1 + 2\beta x + \beta^2 x^2}$$

4. Scale transformations

$$'x^\mu = \alpha x^\mu, \qquad \alpha > 0.$$

This 15-parameter group is locally isomorphic to the SU(2,2)-group containing SU(2,1)-group as a subgroup. The finite-dimension representations of SU(2,1)-group are in coincidence with those of the SU(3)-group. Therefore, SU(3)-symmetry is obtained in our theory.

On the other hand, all physical equations that are written in this geometry are covariant under the group of transformations (2.2). Following the Noether theorem, to these transformations there correspond local currents, for instance, to the special conformal and scale transformations correspond the local currents expressed in terms of the momenta-energy tensor $\theta^{\alpha\beta}$ as follows

$$C^{\mu\nu} = \theta^{\mu\alpha}(2x^\nu x_\alpha - g^\nu_\alpha x^2) \tag{2.3}$$

$$S^\mu = \theta^{\mu\alpha} x_\alpha. \tag{2.4}$$

The generators of transformations are expressed in the usual way in terms of space integrals of zero components of the currents

$$C^\mu = \int (2\theta^{\mu\alpha} x^0 x_\alpha - \theta^{\mu 0} x^2) d\vec{x}$$

$$S = \int \theta^{0\alpha} x_\alpha d\vec{x}.$$

The conservation laws have the following forms

$$C^{\mu\nu}_{;\nu} = 0,$$

$$S^\mu_{;\mu} = 0,$$

or

$$\partial_\nu C^{\mu\nu} = \Gamma^\mu_{\nu\sigma} C^{\nu\sigma} + \Gamma^\nu_{\nu\sigma} C^{\mu\sigma} \tag{2.5}$$

$$\partial_\nu S^\nu = \Gamma^\nu_{\nu\sigma} S^\sigma \tag{2.6}$$

here

$$\Gamma^\varrho_{\mu\nu} = l(\delta^\varrho_\mu \partial_\nu \chi + \delta^\varrho_\nu \partial_\mu \chi - \mathring{g}_{\mu\nu} \mathring{g}^{\varrho\sigma} \partial_\sigma \chi)$$

from where it is easily seen that, in general, there are no usual conservation laws. By (2.6) the mechanism of violation of the scale invariance principle is clarified. As suggested in [32] it would be very interesting to understand the mechanism of violation of the scale invariance principle and to develop a method of calculation of the corrections to this approximation. The problem of a possible spontaneous violation of this symmetry is being extensively discussed in literature in connection with violation of chiral symmetry [33–36]. Basing on the assumption that scale invariance is guaranteed for some inelastic lepton-hadron interactions, some interesting results were obtained [32]. For the first time this assumption was suggested by N. N. Bogolubov in connection with the so-called automodel solutions of classical hydrodynamics. The study of the mechanism of violation of scale invariance suggested in this paper will be continued in the next paper.

3

It is shown in [1, 2] that in the case when cosmic field is taken into account as the regularized field, all the ultraviolet infinities encountered in quantum field theory are absent. In this paragraph the corrections of self-mass and self-charge of electron are obtained in the second order of perturbation theory.

Let us now study the electromagnetic interaction in the space defined by (2.1). It is easily seen that the equations of quantum electrodynamics are built simply as follows.

Let γ^μ be the Dirac matrix which of course veryfies the following relations

$$\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu = 2\mathring{g}_{\mu\nu}e^{-2l\chi(x)}$$

from where we obtain

$$\gamma_\mu = e^{-l\chi(x)}\mathring{\gamma}_\mu$$

and the Lagrangian of quantum electrodynamics is obtained

$$\mathscr{L}_{\text{tot}} = \sqrt{-|g|}\left\{\frac{i}{2}\,\bar{\psi}\gamma_\mu(\partial_\mu-G_\mu-ieA_\mu)\psi-\right.$$

$$\left.-\frac{i}{2}\,\bar{\psi}(\overleftarrow{\partial}_\mu+G_\mu+ieA_\mu)\psi-m\bar{\psi}\psi\right\}. \tag{3.1}$$

For our purpose we shall consider only the interaction Lagrangian

$$\mathscr{L}_{em} = ie_0 : \bar{\psi}\hat{A}\psi e^{-4l\chi}: \tag{3.2}$$

The S-matrix corresponding to (3.2) is given by

$$S = T\exp\{ie_0\textstyle\int : \bar{\psi}\hat{A}\psi e^{-4l\chi} : d^4x\}. \tag{3.3}$$

The interaction Lagrangian (3.2) depends non-polynomially on the cosmic field operator. Volkov [28] has shown that an interaction of this type is microcausal and free of all the infinities encountered in quantum field theory.

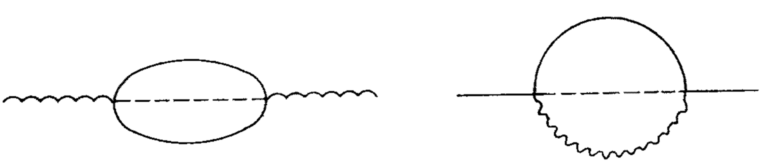


Fig. 1

Now, let us evaluate the self-energy graphs of electron and photon, given by Fig. 1. In order to do this, let us at first consider the superpropagator of the cosmon. We have

$$G(x) = \exp\{l^2D_0^c(x)\} = \sum_{n=0}^{\infty}\frac{1}{n!}\,[l^2D_0^c(x)]^n$$

which can be represented as follows

$$G(x) = \frac{1}{2i}\int\limits_{c} \frac{e^{i\pi z}}{\sin \pi z \Gamma(1+z)} [l^2 D_0^c(x)]^z dz$$

or

$$G(x) = \frac{i}{2}\int\limits_{c} \Gamma(-z) e^{i\pi z}[l^2 D_0^c(x)]^z dz$$

here the contour is shown by Fig. 2.

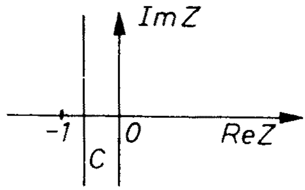


Fig. 2

For this representation of G it is very convenient to compute the matrix elements in configuration space [23, 24]. The computation presented below is similar to that of Salam *et al.* [27]. The matrix elements corresponding to Fig. 1 are obtained easily, as follows:

$$\Sigma(p) = \int\limits_{c} dz \Gamma(-z) e^{i\pi z} \Sigma(p, z) \tag{3.4a}$$

$$\Pi_{\mu\nu}(k) = \int\limits_{c} dz \Gamma(-z) e^{i\pi z} \Pi_{\mu\nu}(k, z) \tag{3.4b}$$

here

$$\Sigma\,(p, z) = e_0^2 \int d^4x e^{ipx} \mathring{\gamma}_\mu S(x) \mathring{\gamma}_\nu D_{\mu\nu}^c(x)\,[l^2 D_0^c(x)]^z \tag{3.5a}$$

$$\frac{1}{i}\,\Pi_{\mu\nu}(k, z) = e_0^2 \int d^4x e^{ikx} Sp(\mathring{\gamma}_\mu S(x) \mathring{\gamma}_\nu S(-x))\,[l^2 D_0^c(x)]^z. \tag{3.5b}$$

Into (3.5) one can substitute for propagators as follows:

$$S^c(x) = (i\gamma^\mu \partial_\mu + m)\,\frac{m K_1(m\sqrt{-x^2})}{4\pi^2\sqrt{-x^2}}$$

$$D_{\mu\nu}^c(x) = \frac{1}{4\pi^2 x^2}\,\mathring{g}_{\mu\nu}, \qquad D_0^c(x) = -\,\frac{1}{4\pi^2 x^2}$$

and write $\Sigma(p, z), \Pi_{\mu\nu}(k, z)$ in the following form

$$\Sigma(p, z) = \hat{p} A(p, z) + m B(p, z) \tag{3.6a}$$

$$\Pi_{\mu\nu}(k, z) = (k_\mu k_\nu - \mathring{g}_{\mu\nu} k^2) C(k, z) + \mathring{g}_{\mu\nu} D(k, z). \tag{3.6b}$$

Let us now compute firstly the self-mass of electron $\Sigma(p)$. For $A(p, z)$ and $B(p, z)$ we obtain the expressions

$$A(p, z) = -\frac{2e^2}{l^2 p^2}\int d^4x e^{ipx}[l^2 D_0^c(x)]^{z+1}(ip^\nu \partial_\nu)\Delta(x)$$

$$B(p, z) = -\frac{4e^2}{l^2}\int d^4x e^{ipx}[l^2 D_0^c(x)]^{z+1}\Delta(x).$$

The integrals are taken over the Euclidean region in x-space and defined initially for $p^2 < 0$. By analytic continuation these integrals are obtained for $p^2 > 0$.

By using the polar coordinates

$$dx = 4\pi r^2 \sin\theta\, dr\, d\theta$$

$$px = -\sqrt{-p^2}r\cos\theta$$

$$\sqrt{-x^2} = r$$

and the standard relations [31]

$$\int_0^{+\infty} du u^{-2z-1} K_1(u) J_1\left[u\left(-\frac{p^2}{m^2}\right)^{\frac{1}{2}}\right] =$$

$$= \left(-\frac{p^2}{m^2}\right)^{\frac{1}{2}} 4^{-z-1}\Gamma(1-z)\Gamma(-z)\,{}_2F_1\left(1-z, -z; 2; \frac{p^2}{m^2}\right)$$

$$\int_0^{+\infty} du u^{-2z-1} K_2(u) J_2\left[u\left(-\frac{p^2}{m^2}\right)^{\frac{1}{2}}\right] =$$

$$= -\frac{p^2}{2m^2} 4^{-z-1}\Gamma(2-z)\Gamma(-z)\,{}_2F_1\left(2-z, -z; 3; \frac{p^2}{m^2}\right)$$

we shall arrive at the following expressions for $A(p, z)$ and $B(p, z)$

$$A(p, z) = -\frac{\alpha}{4\pi}\left(\frac{lm}{4\pi}\right)^{2z}\Gamma(-z)\Gamma(2-z)\,{}_2F_1\left(2-z, -z; 3; \frac{p^2}{m^2}\right),$$

$$B(p, z) = \frac{\alpha}{\pi}\left(\frac{lm}{4\pi}\right)^{2z}\Gamma(-z)\Gamma(1-z)\,{}_2F_1\left(1-z, -z; 2; \frac{p^2}{m^2}\right).$$

Then the expressions for $\Sigma(p)$ takes the form,

$$\Sigma(p) = \frac{\alpha}{\pi}\int_c dz(\Gamma(-z))^2\left(\frac{lm}{4\pi}\right)^{2z} \times$$

$$\times\left\{\frac{\hat{p}}{4}\Gamma(2-z)\,{}_2F_1\left(2-z, -z; 3; \frac{p^2}{m^2}\right) + m\Gamma(1-z)\,{}_2F_1\left(1-z, -z; 2; \frac{p^2}{m^2}\right)\right\}.$$

We obtain for the mass correction

$$\delta m = \Sigma(p)|_{\hat{p}=m}$$

or

$$\frac{\delta m}{m} = \frac{\alpha}{\pi}\int_c dz(\Gamma(-z))^2\left(\frac{lm}{4\pi}\right)^{2z}\times$$

$$\times\left\{\tfrac{1}{4}\Gamma(2-z)_2F_1\left(2-z,\ -z;\ 3;\ \frac{p^2}{m^2}\right)+\Gamma(1-z)_2F_1\left(1-z,\ -z;\ 2;\ \frac{p^2}{m^2}\right)\right\}_{\hat{p}=m}.$$

In order to compute this integral we remember that the leading singularity of the integrand is the dipole at $z = 0$. The next singularity which occurs at $z = 1$ is a dipole in A and a tripole in B. The remaining singularities which occur at $z = 2, 3, \ldots$ are all tripoles.

The contribution of the leading singularity takes the form

$$\frac{\delta m}{m} = \frac{\alpha}{\pi}\frac{\partial}{\partial z}\left\{\frac{1}{[\Gamma(1+z)]^2}\left(\frac{lm}{4\pi}\right)^{2z}\times\right.$$

$$\left.\times\left[\tfrac{1}{4}\Gamma(2-z)_2F_1\left(2-z,\ -z;\ 3;\ \frac{p^2}{m^2}\right)+\Gamma(1-z)_2F_1\left(1-z,\ -z;\ 2;\ \frac{p^2}{m^2}\right)\right]\right\}_{\substack{\hat{p}=m\\ z=0}}$$

or

$$\frac{\delta m}{m} = \frac{3\alpha}{4\pi}\ln\left(\frac{1}{lm}\right)^2 + O((lm)^2\ln(lm)^2)$$

which depends on l in the form an effective cutoff

$$R_{\text{Cutoff}} = l^{-1}.$$

The contributions of the tripoles at $z = 1, 2, \ldots$ to the self-mass of electron will give the terms like $l^{2n}(\ln l)^{2n}$, $n \gg 1$, and they can be of course neglected.

Now the computation of self-charge $\Pi_{\mu\nu}(k)$ is dealt with. For transversal and longitudial parts $C(k, z)$ nad $D(k, z)$ we obtain the following expressions

$$C(k^2, z) = -\frac{\alpha}{6}\Pi^{3/2}\left(\frac{lm}{2\pi}\right)^{2z+2}\frac{1}{z}\frac{[\Gamma(2-z)]^2\Gamma(4-z)}{\Gamma(\frac{5}{2}-z)}\times$$

$$\times\,{}_3F_2\left(4-z, 2-z,\ -z;\ 4, \frac{5}{2}-z;\ \frac{k^2}{4m^2}\right)$$

$$D(k^2, z) = -3\alpha\Pi^{3/2}\left(\frac{lm}{2\pi}\right)^{2z+2}\frac{1}{z+1}\frac{[\Gamma(1-z)]^2\Gamma(2-z)}{\Gamma(\frac{3}{2}-z)}\times$$

$$\times\,{}_3F_2\left(2-z, 1-z,\ -1-z;\ 3; \frac{3}{2}-z;\ \frac{k^2}{4m^2}\right).$$

$C(k)$ and $D(k)$ are computed in analogy with $A(p)$ and $B(p)$. The results are as follows

$$C(k^2) = \frac{1}{6}\alpha\Pi^{3/2}\frac{\partial}{\partial z}\left\{\left(\frac{lm}{2\pi}\right)^{2z+2}\frac{1}{z}\frac{[\Gamma(2-z)]^2\Gamma(4-z)}{\Gamma(\frac{5}{2}-z)}\times\right.$$

$$\left.\times\,{}_3F_2\left(4-z,2-z,-z;4,\frac{5}{2}-z;\frac{k^2}{4m^2}\right)\right\}_{z=0}$$

or

$$C(k^2) = -\frac{\alpha}{3\pi}\left\{-2\ln\frac{2\pi}{lm}+O((lm)^2\ln(lm)^2)\right\}$$

and

$$D(k^2) = -\frac{3\alpha}{2}\frac{m^2}{\pi}\,{}_3F_2\left(2,1,-1;3;\frac{3}{2};\frac{k^2}{4m^2}\right)\neq 0!$$

The transversal part $C(k^2)$ gives the charge renormalization of electron

$$\frac{e_c^2}{e_0^2} = z = 1-\frac{2\alpha}{3\pi}\ln\frac{2\pi}{lm}+O((lm)^2\ln(lm)^2).$$

The nonvanishing value of $D(k^2)$ shows that our technical procedure is incorrect, since for quantum electrodynamics gauge invariance must be guaranteed for any order of perturbation theory. This problem will now be discussed.

4

In this paragraph the gauge invariance as well as the equivalence theorem in quantum electrodynamics with nonpolynomial Lagrangian will be discussed in detail. As it is shown, the main difficulties of quantum electrodynamics without ultraviolet infinities, constructed by Salam *et al.* are gauge invariance and the equivalence theorem. Volkov [28] discusses the formalism assumed by Budini and Calucci [38] and finds it unsatisfactory for the reason that in the case when consideration is restricted to the class of graphs having a defined number of superpropagators, there are of course some graphs which give divergent matrix elements. Also the formalism assumed already by Feynman [39] places us in a dilemma. In order to apply the Feynman procedure we need to attach photon lines in all possible ways to the electron loop including gravitons, and there are of course some vertices which are not connected by superpropagators. In the case when these vertices are connected by superpropators of graviton, the Feynman procedure for securing gauge invariance would now demand that we include more complicated configurations.

We shall now prove that the Budini-Calucci procedure applied to our theory reduces in reality tothe approximate gauge invariance for a certain order of perturbation theory series. In effect, we consider independently in each order of the classes of graphs to have a defined number of superpropagators of cosmon, the sum of all these classes, of course, satisfying the gauge invariance condition of the theory. The main feature of our theory is as follows. We are always able to construct graphs, whose matrix elements are free of divergences. Indeed, the cosmic field operator participates in terms of the S-matrix in the form of an exponential function, $\exp\{-l\chi(x)\}$ and has the following important property

$$\exp\{a+b\} = \exp\{a\}\exp\{b\}.$$

Therefore, for an arbitrary graph, say for a graph of n vertices, $n > 1$, it is possible to "cut" this operator into several "smaller" parts so that the matrix elements of the graph are convergent (in an extreme case, all the vertices of graph are connected by superpropagators). As an illustrative example, let us consider the graph of 3 vertices, we have of course

$$S_3(x, y, z) = -e_0^3 T\{N(\bar{\psi}(x)\hat{A}(x)\psi(x))N(\bar{\psi}(y)\hat{A}(y)\psi(y)) \times$$
$$\times N(\bar{\psi}(z)\hat{A}(z)\psi(z))N(e^{-4l\chi(x)})N(e^{-4l\chi(y)})N(e^{-4l\chi(z)})\}.$$

The cosmic field operator can be "cut" as follows

$$\exp\{-4l\chi(x)\} = \exp\{-2l\chi(x)\}\exp\{-2l\chi(y)\}.$$

Then the 3-vertice graph given by Fig. 3 corresponds to the matrix element

$$\Gamma_3^\nu(x, y, z) = ie_0^2 \sum_\varrho \mathring{g}^{\varrho\varrho}\mathring{\gamma}^\varrho S^c(x-y)G_{\frac{1}{2}}(x-y)\mathring{\gamma}^\nu S^c(y-z) \times$$

$$\times G_{\frac{1}{2}}(y-z)\mathring{\gamma}^\varrho D_0^c(x-z)G_{\frac{1}{2}}(x-z)$$

here

$$G_{\frac{1}{2}}(x-y) = \underbrace{\exp\{-2l\chi(x)\}\exp}\{-2l\chi(y)\}.$$

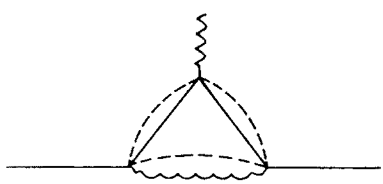


Fig. 3

All the vertices of this graph are connected by cosmon superpropagators and therefore its matrix elements are convergent. It is easily seen that the above-mentioned procedure can be applied to all other graphs. In addition, in our theory the electron-cosmon loops also appeared in connection with the following part of Lagrangian

$$\frac{i}{2} e^{-2l\chi(x)}\bar{\psi}\gamma_\mu \overleftrightarrow{\partial}_\mu \psi.$$

However, these loops give convergent matrix elements. Hence, the gauge invariance is guaranteed approximately in every order of perturbation theory.

Next, the equivalence theorem is concerned. It is different from the gravitational field, the cosmic field is not a metric one and the formalism of the theory is unique. The geometry defined by (2.1) only allows us to have coordinate transformations (2.2) and therefore, $(\det g)^{-\frac{1}{4}}$ cannot be eliminated. Finally the cosmic field operator is always present under the form $e^{-2l\chi}$, for which there is no sense of speaking about covariant and contravariant components. In resuming, the difficulty concerning the equivalence theorem raised in Salam's theory, is absent in our theory. Therefore, it is possible to say that we are now ready to construct quantum electrodynamics.

5

In this paragraph general equations for propagators and vertex function are established.

In the Heisenberg picture propagators of electron and photon are defined as follows.

$$G^e_{\alpha\beta}(x-y) = \langle 0|T(\check{\psi}_\alpha(x)\check{\bar{\psi}}_\beta(y))|0\rangle$$

$$G^\gamma_{\mu\nu}(x-y) = \langle 0|T(\check{A}_\mu(x)\check{A}_\nu(y))|0\rangle$$

here $\check{O}$ denotes an operator in the Heisenberg picture. It is easily seen that for $G^e_{\alpha\beta}$ and $G^\gamma_{\mu\nu}$ we obtain also the following spectral representations in momentum space

$$G^e(p) = -z_1\left\{\frac{1}{\hat{p}-m_c-i0} + \int_{m_c}^{+\infty} \frac{\varrho(M)dM}{\hat{p}-M-i0} + \int_{m_c}^{+\infty} \frac{\varrho(-M)dM}{\hat{p}+M+i0}\right\}$$

$$G^\gamma(k) = \frac{z}{i}\left\{\frac{1}{k^2-i0} + \int_0^{+\infty} \frac{\varrho(M^2)dM^2}{k^2-M^2-i0}\right\}$$

which are identical to those of "ordinary" quantum electrodynamics [40].

The self-energy graphs of electron and photon are defined respectively to be the graphs with two electron lines and two external lines of photon. To these graphs correspond the matrix elements denoted by $\Sigma(p)$ and $\Pi(k)$. Then we obtain the following equations for G^e and G^γ:

$$G^e(p) = S^c(p)+S^c(p)\Sigma(p)S^c(p) \tag{5.1a}$$

$$G^\gamma(k) = D^c(k)+D^c(k)\Pi(k)D^c(k) \tag{5.1b}$$

from the relation

$$\langle 0|T(\check{A}\check{B})|0\rangle = \langle 0|T(ABS)|0\rangle$$

here A and B are given in the interaction picture. It is convenient to introduce the mass operator $M(p)$ and the polarization operator $P(k)$ to be defined as follows

$$G^e(p)^{-1} = S^c(p)^{-1}+iM(p) \tag{5.2a}$$

$$G^\gamma(k)^{-1} = D^c(k)^{-1}+iP(k). \tag{5.2b}$$

From which we obtain the equations for the Green functions

$$(\hat{p}-m-M)G^{e}(p) = -1,$$

$$i(k^2+P(k))G^{\gamma}(k) = 1.$$

Since our theory as a whole is gauge invariant, it is necessary for $P(0)$ to vanish

$$P(0) = 0$$

and therefore

$$Z^{-1} = 1+\left(\frac{\partial P}{\partial k^2}\right)_{k^2=0}.$$

The mass correction δm and the constant Z_1 are given as follows

$$\delta m = b(m_c^2)$$

$$Z_1^{-1} = 1+a(m_c^2)$$

here

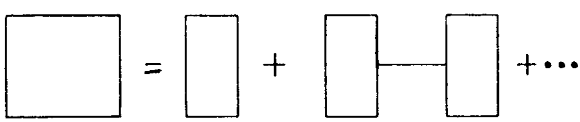
$$M(p) = a(p^2)(\hat{p}-m_c)-b(p^2).$$

The graphs are divided into compact and irreducible graphs. Functions corresponding to the compact self-energy graphs of electron and photon are denoted by Σ^* and Π^* respectively. It is easily seen that

$$\Sigma(p) = \Sigma^*+\Sigma^* S^c \Sigma^*+\ldots =$$

$$= \Sigma^*(1-S^c\Sigma^*)^{-1}$$

which corresponds to the graph equation



Similarly, we have

$$\Pi(k) = \Pi^*(1-D^c\Pi^*)^{-1}.$$

In comparing with (5.2) we reduce

$$\Sigma^*(p) = iM(p),$$

$$\Pi^*(k) = -iP(k).$$

The general vertex function Γ_μ is defined by

$$\Gamma_\mu = \overset{\circ}{\gamma}_\mu+\Lambda_\mu$$

here Λ_μ is the matrix element corresponding to a vertex graph without any external cosmic line. Beside this vertex function it is convenient to introduce another vertex function

$$\bar{\Gamma}_\mu = \mathring{\gamma}_\mu + \bar{\Lambda}_\mu$$

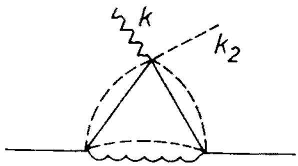
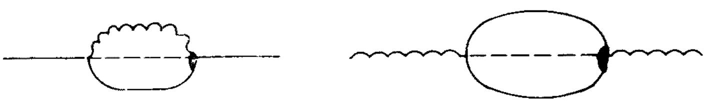


Fig. 4

corresponding to graph with one cosmon external line, here $\bar{\Lambda}_\mu$ is given by graph 4. It is clear that Λ_μ and $\bar{\Lambda}_\mu$ are connected as follows

$$\Lambda_\mu = \bar{\Lambda}_\mu \Big|_{\substack{k_2 \to 0 \\ l \to \alpha l \quad 0<\alpha<1}} .$$

Now with the aid of the above-mentioned function the equation for propagators are found easily by using the skeleton graphs



Self-energy graphs for electron and photon give

$$\Sigma^*(p) = \frac{e_0^2}{(2\pi)^4} \int \mathring{\gamma}_\mu G^e(p-k_1-k_2)\bar{\Gamma}_\nu(p, p-k_1-k_2; k_1) G^\gamma_{\mu\nu}(k_1) G(k_2) dk_1 dk_2$$

$$\Pi^*(k) = \frac{e_0^2}{(2\pi)^4} \frac{1}{3} Sp \int \mathring{\gamma}_\mu G^e(p_1) \bar{\Gamma}_\mu(p_1, p_1-p_2-k; k) G^e(p_1-p_2-k) G(p_2) dp_1 dp_2 .$$

From here one obtains by taking into account (5.1)

$$G^e(p) = S^c(p) + \frac{e_0^2}{(2\pi)^4} S^c(p) \int \mathring{\gamma}_\mu G^e(p-k_1-k_2) \bar{\Gamma}_\nu(p, p-k_1-k_2; k) \times$$
$$\times G^\gamma_{\mu\nu}(k_1) G(k_2) G^e(p) dk_1 dk_2,$$

$$G^\gamma(k) = D^c(k) + \frac{e_0^2}{(2\pi)^4} \frac{1}{3} D^c(k) Sp \int \mathring{\gamma}_\mu G^e(p_1) \bar{\Gamma}_\mu(p_1, p_1-p_2-k; k) \times$$
$$\times G^e(p_1-p_2-k) G(p_2) G^\gamma(k) dp_1 dp_2 .$$

To end we notice that because our theory is gauge invariant the Ward identity is easily derived

$$\Lambda_\mu(p, p; 0) = \frac{\partial \Sigma^*(p)}{\partial p^\mu}$$

the proof of which is similar to that given by [39].

6

In this paragraph we consider the asymptotic structure of propagators and vertex function for large momenta, $|k^2| \gg m^2$. It is known, that vertex function $\Gamma_\mu(p_1, p_2; k)$ is logarithmically infinite as l tends to zero. Therefore it can be expanded in a power series as follows

$$\Gamma(p_1, p_2; k) = \sum_{n=0}^{\infty} a_n \lambda^n$$

here $\lambda = \ln \frac{1}{lm}$ and $a_n = a_n\left(\frac{p_1^2}{m^2}, \frac{p_2^2}{m^2}, \frac{k^2}{m^2}, e^2\right)$. The parameters of this expansion are e^2 and $e^2\lambda$, therefore the expansion is correct if and only if these parameters are very small,

$$e^2 \ll 1 \quad \text{and} \quad e^2\lambda = e^2 \ln \frac{1}{lm} \ll 1.$$

If a_n are expanded in terms of e, $\Gamma(p_1, p_2; k)$ has the form

$$\Gamma(p_1, p_2; k) = \sum_{n,r=0}^{\infty} a_{nr}^0 (e^2)^r (e^2\lambda)^n.$$

Similarly the expanded forms of G_e and G^γ are easily obtained:

$$G^e(p) = s(p) S^c(p)$$

$$G^\gamma(k) = d(k) D^c(k)$$

where

$$s(p) = \sum s_{mn} (e^2)^n \left(e^2 \ln \frac{1}{l^2 |p^2|}\right)^m$$

$$d(k) = \sum d_{mn} (e^2)^n \left(e^2 \ln \frac{1}{l^2 |k^2|}\right)^m$$

in which s_{mn} and d_{mn} are constants for $|k^2| \gg m^2$, $|p^2| \gg m^2$. In our theory the charge e, propagators, vertex function and matrix elements are also renormalized by using two renormalization constants Z and Z_1 to be the functions of l. However, it is necessary to remember that the terminology "renormalization" is used here in a different sense from that of "ordinary" quantum electrodynamics. In our theory renormalization is not connected with the removal of infinity for the theory is free of divergences. The actual sense of this terminology is as follows.

1. With the aid of two constants Z and Z_1 we may establish formally the new quantities

$$G_c^e(p) = Z_1^{-1} G^e(p)$$

$$G_c^\gamma(k) = Z^{-1} G^\gamma(k)$$

$$\Gamma_c = Z_1 \Gamma, \quad \Lambda_1 = Z_1 \Lambda$$

$$e_c^2 = Ze^2$$

$$\psi_c = Z_1^{\frac{1}{2}}\psi, \quad a_c = Z^{\frac{1}{2}}a,$$

as $l \to 0$ they tend to well known quantities of quantum electrodynamics.

2. If G_c^e, G_c^γ and Γ_c are considered to be the functions of momenta and $Ze^2 = e_c^2$, then they are free of l. The dependence on l is concentrated in Z and Z_1. In that sense it is possible to say that two renormalization constants Z and Z_1 characterize the interaction between photon-electron field and the cosmic field. The absence of the latter leads to infinities in the two constants. Thus the physical meaning of two renormalization constants is explained. For any other theory, we can assert that the renormalizability implies, mathematically, that a finite number of constants are present to characterize the interaction between the matter field and cosmic field in order for the fundamental quantities of theory to be expressed in terms of those constants. In other words, the interaction with the cosmic field can be "factorized" by means of a finite number of factors or renormalization constants.

As it is proved by [1, 2] with the aid of cosmic field, as a regularized field, all theories are convergent, thereby it is possible to divide them into two classes:

A. Class of theories, in which there exists a finite number of factors, by means of which the fundamental quantities, for instance, propagators, are "factorized".

B. Class of theories, in which there are no such factors, or in the other words, the number of these factors is infinite.

The theory belonging to the first class is said to be renormalizable and the theory of second class is said to be unrenormalizable. It is clear that as $l \to 0$ the sense of this terminology is the usual one. Finally, it is shown that for large momenta $|k^2| \gg m^2$, the propagators depend only on a variable which is a combination of two variables e^2 and $1/l^2k^2$. The proof of this assertion is similar to that of [40].

7

In this paragraph conclusion and discussion are given. The cosmic field developed in [1, 2] is utilized to construct quantum electrodynamics without ultraviolet infinities. By Salam *et al.* [26, 27] the gravitational field was utilized for this purpose, but their theory encountered many difficulties. On the one hand, it is known that the quantization of gravitational field is yet unlikely and the localization of gravitational energy is not solved decisively in vierbein formalism by Moller [41]. A satisfactory construction of localized energy-momentum complex of gravitational field is realized in [42] by using the imbedding theorem of Friedman [43]. But the physical meaning of this formalism is not clear. In addition, spinor formalism in the general theory of relativity is established non-uniquely. On the other hand, in our opinion, at least for the present stage, the difficulty connected with gauge invariance is unsolvable if we use the gravitational field as the regularized field. Inversely, as it is prove by this paper, with the aid of a cosmic field, quantum electrodynamics without ultraviolet infinities may be constructed satisfactorily. The obtained essential results are the following.

1. It is shown that the theory enables the SU(2,2)-group to be a symmetry group. In the presence of cosmic field the mechanism of the violation of the scale invariance principle is outlined.

2. In the second order of perturbation theory the self-mass and self-charge of electron are calculated.

3. The procedure for securing gauge invariance for each order of perturbation theory series is established. In our theory the difficulty connected with the equivalence theorem is suppressed automatically. These problems are solved successfully for the reason that the cosmic field possesses special features as compared with other fields.

4. The physical meaning of the two renormalization constants is explained. The concepts on renormalizability and unrenormalizability of a certain field theory are introduced.

The obtained results explain in part the problem: Why does "ordinary" quantum electrodynamics give good agreement with experiment by using the renormalization method, although the physical foudation of as well as the mathematical basis of this procedure are so feeble.

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