

# ENERGY DEPENDENCE OF THE CROSS-SECTION IN THE DECK TYPE MODELS

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The energy dependence of the cross-section for the diffractive production in the Deck type models is studied. For the reaction  $pp \rightarrow pn\pi^+$  the cross-section decreases with the energy and its dependence on incident momentum can be reasonably represented as  $a + bp^{-\alpha}$ .

## 1. Introduction

The aim of this paper is to find the energy dependence of the cross-section in the Deck type models [1-5]. The calculations were made for the reaction  $pp \rightarrow pn\pi^+$  neglecting the double scattering of the  $n\pi^+$  system on the proton. The values of the cross-section for this diffractive process and the distributions  $d\sigma/dm_{\pi n}$  of the invariant mass of the  $n\pi^+$  system were calculated for the laboratory energies 8, 10, 15, 20, 25 and 30 GeV. Similar

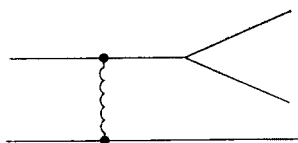


Fig. 1. Graphical representation of the diffractive process  $pp \rightarrow pn\pi^+$ . The fluctuation  $p \rightarrow n\pi^+$  after the elastic scattering

calculations for the energy 500 GeV were made to compare the results with the previous calculations [2] in the infinite energy limit.

The diffractive process  $pp \rightarrow pn\pi^+$  can be described as follows: the incoming proton dissociates diffractively into the  $n\pi^+$  system, the constituents of which interact elastically with the target. The elastic scattering may appear before or after the fluctuation  $p \rightarrow n\pi^+$  (see Fig. 1 and Fig. 2).

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If one assumes that the elastic scattering is correctly described by the Glauber model [6], the elastic scattering of the composite  $n\pi^+$  system is the sum of two single scattering contributions and the double scattering contribution, see Fig. 3.

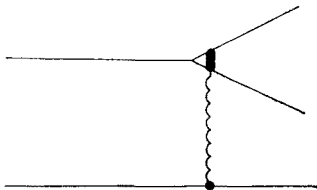


Fig. 2. Graphical representation of the diffractive process  $pp \rightarrow pn\pi^+$ . The fluctuation  $p \rightarrow n\pi^+$  before the elastic scattering

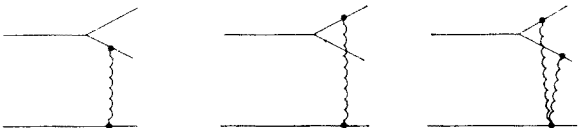


Fig. 3. Graphical representation of the elastic scattering of the composite  $n\pi^+$  system using the Glauber model. The elastic scattering of the  $n\pi^+$  system is the sum of two single elastic scattering contributions and the double elastic scattering contribution

To construct the amplitude for the diffractive process  $pp \rightarrow pn\pi^+$  the method of the generalized Feynman diagrams may be applied.

This amplitude is shown in Fig. 4, where the individual diagrams are understood as the generalized Feynman diagrams. That is to say, the upper parts of the diagrams are calculated according to the Feynman rules, and the lower parts (elastic scattering) are taken from the experimental data. The similar amplitude was earlier considered by Pumplin [7], but without the diagram a of Fig. 4.

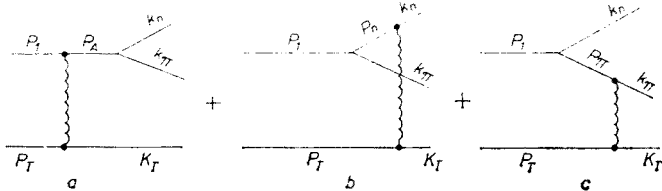


Fig. 4. Three diffractive graphs describing the process  $pp \rightarrow pn\pi^+$ . The kinematics is defined in the laboratory frame

2. The amplitude and the cross-section for the diffractive process  $pp \rightarrow pn\pi^+$

The elastic scattering can be conveniently described by the profile function  $\gamma(t)$ , related to the elastic cross-section by the formula

$$\frac{d\sigma}{dt} = \frac{1}{4\pi} |\gamma(t)|^2. \tag{2.1}$$

The profile function  $\gamma(t)$  was chosen as follows

$$\gamma(t) = \frac{\sigma_{\text{tot}}}{2} \exp\left(\frac{A}{2} t\right) \delta_{\mu\mu'} \delta_{\nu\nu'}, \quad (2.2)$$

because the shape of the diffractive peak can be well approximated by an exponential. Here  $\sigma_{\text{tot}}$  is the total cross-section.  $A$  is the slope of the forward elastic peak and  $\mu, \nu, \mu', \nu'$  are initial and final helicities. Both  $\sigma_{\text{tot}}$  and  $A$  are taken from the experimental data. The amplitude  $A_{\mu\mu'}$  is the sum of the  $A_{\mu\mu'}^a, A_{\mu\mu'}^b, A_{\mu\mu'}^c$  amplitudes, where  $A_{\mu\mu'}^a, A_{\mu\mu'}^b, A_{\mu\mu'}^c$  are constructed from the suitable diagrams of Fig. 4, namely

$$A_{\mu\mu'} = A_{\mu\mu'}^a + A_{\mu\mu'}^b + A_{\mu\mu'}^c. \quad (2.3)$$

If one takes the pseudo-vector coupling in the vertex of the fluctuation  $p \rightarrow n\pi^+$  and uses the normalization from Ref. [9], the expressions for  $A_{\mu\mu'}^a, A_{\mu\mu'}^b, A_{\mu\mu'}^c$  may be written as follows: let

$$N = i \frac{g\sqrt{2}}{2m} \sqrt{\frac{m}{E(p_1)}} \sqrt{\frac{m}{E(k_n)}} \frac{1}{\sqrt{2E(k_\pi)}} \cdot \frac{1}{v^{5/2}} (2\pi)^4 \delta^4(P_i - P_f); \quad (2.4)$$

then

$$A_{\mu\mu'}^a = N \sigma_{\text{tot}}^{pp} \exp\left(\frac{A_p}{2} t\right) \frac{E(p_A)}{p_A^2 - m^2} \bar{u}_\mu(k_n) \gamma_5 k_\pi^\alpha \gamma_\alpha u_\mu(p_A) \bar{u}_\mu(p_A) u_\mu(p_1) F^a F_\perp^a, \quad (2.5)$$

$$A_{\mu\mu'}^b = N \sigma_{\text{tot}}^{np} \exp\left(\frac{A_n}{2} t\right) \frac{E(p_n)}{p_n^2 - m^2} \bar{u}_\mu(k_n) u_\mu(p_n) \bar{u}_\mu(p_n) \gamma_5 k_\pi^\alpha \gamma_\alpha u(p_1) F^b F_\perp^b, \quad (2.6)$$

$$A_{\mu\mu'}^c = N \sigma_{\text{tot}}^{\pi^+ p} \exp\left(\frac{A_\pi}{2} t\right) \frac{E(p_\pi)}{p_\pi^2 - m_\pi^2} \bar{u}_\mu(k_n) \gamma_5 p_\mu^\alpha \gamma_\alpha u_\mu(p_1) F^c F_\perp^c, \quad (2.7)$$

where  $\frac{g^2}{4\pi} = 14$ ,  $m$  and  $m_\pi$  are the nucleon and pion masses,  $\gamma_5$  and  $\gamma_\alpha$  are the Dirac matrices, and  $u_\mu$  and  $\bar{u}_\mu$  are the Dirac spinors.  $F$  and  $F_\perp$  are the phenomenological formfactors:

$$F = \exp[-\gamma(m^{*2} - p^2)], \quad (2.8)$$

$$F_\perp = \exp\left(-\frac{\lambda^2}{2} \vec{p}_\perp^2\right), \quad (2.9)$$

where  $p$  is the four-momentum of the intermediate particle,  $m^*$  is the mass of the suitable real particle,  $\vec{p}_\perp$  is the transverse momentum in the laboratory frame, and  $\gamma$  and  $\lambda^2$  are constants.  $F$  is the off mass-shell formfactor, and  $F_\perp$  refers to the well known experimental fact that the transverse momenta of the produced particles are limited. The exponential forms of the formfactors were taken because the calculations are then reasonably simple. The meaning of the other symbols follows from Fig. 4.

The cross-section for the diffractive process  $pp \rightarrow pn\pi^+$  may be obtained as follows

$$d\sigma = \frac{|M|^2 dN}{V \cdot T |j_{\text{inc}}|}, \quad (2.10)$$

where

$$|M|^2 = \frac{1}{2} \sum_{\mu\mu'} |A_{\mu\mu'}|^2. \quad (2.11)$$

The operator  $\frac{1}{2} \sum_{\mu\mu'} |A_{\mu\mu'}|^2$  signifies averaging over the spins and summing over the final states.

Writing

$$M = \frac{1}{V^{5/2}} (2\pi)^4 \delta^4(P_i - k_n - k_\pi - k_T) T \quad (2.12)$$

one obtains

$$d\sigma = \frac{mE(k_T) |T|^2 k d\Delta^2 d\Omega dm_{\pi n}}{4(2\pi)^4 (\vec{p}_1)^2}, \quad (2.13)$$

where  $E(k_T)$  is the energy of the target in the laboratory frame,  $k$  is the momentum and  $d\Omega = -d \cos \theta d\varphi$  is the solid angle element of the final nucleon, both in the rest frame of the diffractively produced  $n\pi^+$  system;  $\Delta^2 = -t$  is the four-momentum transfer, and  $\vec{p}_1$  is the momentum of the incoming proton in the laboratory frame.

### 3. Results

Calculations were made for the diffractive process  $pp \rightarrow pn\pi^+$  at laboratory energies 8, 10, 15, 20, 25 and 30 GeV. The values of the parameters  $\gamma$  and  $\lambda^2$  were chosen to reproduce the experimental data for the invariant mass distribution. Calculations were made for  $\gamma = 2$  and  $\lambda^2 = 3$ . Integrations in the formula (2.13) were made in the following limits  $\Delta^2$  from 0 to 0.46 GeV<sup>2</sup>,  $m_{\pi n}$  from 1.1 to 1.94 GeV.

The energy dependence of the cross-section for the diffractive process  $pp \rightarrow pn\pi^+$  is shown in Fig. 5 (solid curve). This cross-section decreases with the energy and seems to tend to a constant value. One of the reasons of the cross-section decrease is the decrease of  $\sigma_{\text{tot}}$  and the increase of the forward elastic slopes with the energy. To find the other reasons of the cross-section decrease, the calculations were repeated with constant  $\sigma_{\text{tot}}$  and the elastic slopes  $A$ ; the calculations have been performed for  $\sigma_{\text{tot}}^{pp} = \sigma_{\text{tot}}^{np} = 36$  mb,  $\sigma_{\text{tot}}^{\pi^+p} = 24$  mb,  $A_p = A_n = 10$ ,  $A_\pi = 8$ .

The energy dependence of the cross-section in this case is shown in Fig. 5 (broken curve). In the interval 8–20 GeV the cross-section decreases with the energy, but from 30 GeV it stays constant at the value about 0.9 mb. For the laboratory energy 500 GeV the value of this cross-section is also about 0.9 mb. The previous calculations in the infinite energy limit also give the value of about 0.9 mb for this cross-section. Fig. 6 shows the invariant mass distributions of the diffractively produced  $n\pi^+$  system for different ener-

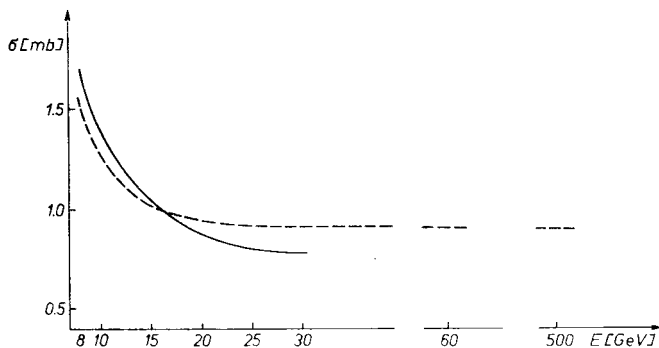


Fig. 5. The energy dependence of the cross-section for the diffractive process  $pp \rightarrow pn\pi^+$  (solid curve). The energy dependence of this cross-section at fixed  $\sigma_{\text{tot}}$  and the elastic slopes (broken curve)

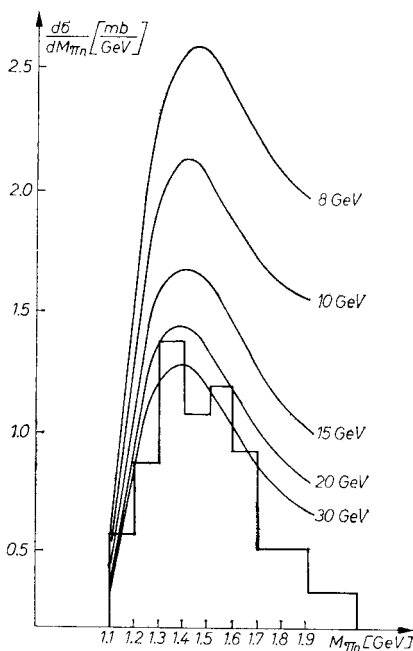


Fig. 6. The invariant mass distributions of the  $n\pi^+$  system for the laboratory energies 8, 10, 15, 20 and 30 GeV. The histogram shows the experimental  $n\pi^+$  mass distribution from the reaction  $pp \rightarrow pn\pi^+$  at 29 GeV. Its normalization is arbitrary

gies. For each energy such a distribution has the maximum for the invariant mass  $m_{n\pi}$  equal to 1.4–1.5 GeV. The maximum goes down with the energy, but its position does not change. In Fig. 6 the experimental mass distribution of the  $n\pi^+$  system for laboratory energy equal to 29 GeV is also shown.

#### 4. Conclusions

In this paper the energy dependence of the cross-section for the diffractive production in the Deck type models was studied. The calculations for the reaction  $pp \rightarrow pn\pi^+$  show that this cross-section decreases with the energy. The decrease of the cross-section is weaker at higher energies, and the cross-section seems to approach a constant. The energy dependence of the cross-section for the diffractive process  $pp \rightarrow pn\pi^+$  may be reasonably described by the formula

$$\sigma = 0.72 + \frac{63.32}{p_1^2} [\text{mb}], \quad (4.1)$$

where  $p_1$  is the laboratory momentum of the incoming proton.

Looking at the shapes of the curves shown in Fig. 5 one can see that one of the reasons for such a behaviour of the cross-section is the decrease of  $\sigma_{\text{tot}}$  and the increase of the elastic slopes with the energy. However, if one takes constant  $\sigma_{\text{tot}}$  and the elastic slopes, the cross-section still decreases. The energy dependence of the cross-section in this case may be reasonably described by the formula

$$\sigma = 0.85 + \frac{42.72}{p_1^2} [\text{mb}]. \quad (4.2)$$

In Fig. 6 the experimental invariant mass distribution of the  $n\pi^+$  system for the laboratory energy 29 GeV is shown. For the higher masses its agreement with the computed mass distribution at 30 GeV incident energy may be improved by taking into account the diagram with the double scattering.

It is to be stressed that the calculations were made at finite energy and that in particular the longitudinal four-momentum transfer was not neglected (in contrast to Ref. [2]).

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