NUCLEAR INERTIAL MASS PARAMETER B IN THE ADIABATIC APPROXIMATION

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(Received December 4, 1972)

The nuclear inertial mass parameter B is investigated by means of two slightly different approaches, both making use of the adiabatic approximation for the collective motion. The numerical results are given for two regions: the superheavy nuclei $108 \leqslant Z \leqslant 124$, $172 Z \leqslant N \leqslant 188$ and the neutron-rich nuclei $88 \leqslant Z \leqslant 104$, $172 \leqslant N \leqslant 194$.

The influence of the choice of the Nilsson model parameters $\varkappa$ and μ is investigated together with the dependence on the deformation parameters ε and ε_4.

1. Introduction

One of the most interesting topics in the nuclear physics of the last few years is the problem of stability of superheavy elements (SHE) around $Z = 114$ and $N = 184$. In investigating the properties of the nuclei in this new region and in looking for the possible means of producing them one has to consider the spontaneous fission half-lives in the SHE region and — as was pointed out in Ref. [1] — for the nuclei on the r-process path: $88 \leqslant Z \leqslant 104$, $172 \leqslant N \leqslant 194$.

The probability for the penetration of the potential energy barrier $V(\varepsilon)$ in the WKB approximation is given by

$$P = \exp\left\{-2\int\limits_{\varepsilon_1}^{\varepsilon_2}\sqrt{\frac{2B(\varepsilon)}{\hbar^2}}\,[V(\varepsilon)-E]\,d\varepsilon\right\}, \tag{1.1}$$

where E is the initial excitation energy of the nucleus towards fission and $B(\varepsilon)$ is the inertial mass parameter associated with fission.

The estimates of the fission half-lives in the SHE region were made either using the phenomenological B values independent of deformation ($BA^{-5/3} = 0.054\,\hbar^2\,\mathrm{MeV}^{-1}$) or with $B(\varepsilon)$ calculated from the microscopic model (see Refs [2] and [3]). On the r-process path the estimates are given only with B_{phen} [1].

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It is easily seen that the determination of B influences strongly the probability of fission (1.1). The aim of this paper is to compare the results of microscopic calculations of the mass parameter B in the two slightly different approaches.

In Section 2 we describe the microscopic formalism which starts with the two-body Hamiltonian. An approximation is made consisting in replacing this Hamiltonian by a one-body operator and the formula for the mass parameter B_ε is found in the harmonic approximation, *i. e.* to the second order in α, where α is the deviation of the nuclear multipole moment from its extremal (equilibrium or saddle-point) value. One can thus expect that this approximation will be less reliable far from the extremal points. In the investigation of the nuclear fission we would like to cover a wide range of nuclear shapes, taking into account the vicinity of the extremal points as well as the region far from those points.

In Section 3 we derive another microscopic formula for B (we shall denote it by $\tilde{B}_\varepsilon$), which is valid in the whole range of the deformation. This approach starts with the one-body time-dependent Hamiltonian. Nevertheless we are faced with another problem, namely we derive $\tilde{B}_\varepsilon$ in terms of the potential deformation instead of the density deformation.

2. *Derivation of B in the microscopic model*

We follow here the description of the collective motion given in Refs [4] and [5]. One assumes the Hamiltonian of the system in the form

$$H = H_0 - \frac{\kappa}{2} \hat{\alpha} \cdot \hat{\alpha}, \tag{2.1}$$

where $\hat{\alpha}$ is a one-particle operator corresponding to the deviation of the nuclear multipole moment (in our case — the quadrupole moment Q) from its value in the extremal point, *i. e.* equilibrium or the saddle-point deformation. H_0 is the average deformed potential plus the short-range two-body forces.

Instead of (2.1) we consider a one-body operator replacing $\hat{\alpha}\hat{\alpha}$ forces by an average field of the multipolarity considered:

$$H_g = H_0 - \kappa \alpha_V(t) \hat{\alpha}, \tag{2.2}$$

where the average time-dependent multipole moment $\alpha_V(t)$ describing the deformation of the potential is chosen in such a way that the expectation value of $\hat{\alpha}$ in the eigenstate $|\alpha_V\rangle$ of (2.2) is:

$$\langle \alpha_V | \hat{\alpha} | \alpha_V \rangle = \alpha. \tag{2.3}$$

Here $\alpha = \alpha(t)$ is the nuclear multipole moment of the density, not the potential. At the extremal points however (see [4])

$$\alpha_V = \alpha. \tag{2.4}$$

The wave function $|\alpha_V\rangle$ is taken as

$$|\alpha_V\rangle = \sum_j c_j(t) e^{-\frac{i}{\hbar} E_j t} |j\rangle, \tag{2.5}$$

where $|j\rangle$ and E_j are the eigenstates and eigenvalues of the time-independent part of H *i. e.* H_0. Assuming the harmonic time dependence of our field:

$$\alpha_V = \alpha_0 \cos \omega t \tag{2.6}$$

we look for the coefficients $c_j(t)$ of the expansion (2.5). From the Schrödinger equation for the wave function (2.5)

$$i\hbar \frac{\partial}{\partial t} |\alpha_V(t)\rangle = H_g(t)|\alpha_V(t)\rangle \tag{2.7}$$

we get in a straightforward way

$$i\hbar \dot{c}_k e^{-\frac{i}{\hbar}E_k t} = -\kappa\alpha_V \sum_j \langle k|\hat{\alpha}|j\rangle c_j e^{-\frac{i}{\hbar}E_j t}. \tag{2.8}$$

We assume that the ground state $|0\rangle$ is the leading term in the expansion (2.5) by putting

$$c_j(t) = \delta_{j0} + a_j(t) \tag{2.9}$$

so that

$$|\alpha_V\rangle = |0\rangle e^{-\frac{i}{\hbar}E_0 t} + \sum_{j\neq 0} a_j(t)|j\rangle e^{-\frac{i}{\hbar}E_j t} \tag{2.10}$$

and (2.9) gives

$$i\hbar \dot{a}_k e^{-\frac{i}{\hbar}E_k t} = -\kappa\alpha_V \langle k|\hat{\alpha}|0\rangle e^{-\frac{i}{\hbar}E_0 t} \tag{2.11}$$

or

$$a_k = -\frac{\kappa}{i\hbar}\langle k|\hat{\alpha}|0\rangle \int \alpha_V(t) e^{\frac{i}{\hbar}(E_k - E_0)} dt. \tag{2.12}$$

Together with (2.6) this gives

$$a_k = \frac{\kappa}{2}\langle k|\hat{\alpha}|0\rangle\alpha_0 \left[\frac{e^{\frac{i}{\hbar}(E_k - E_0 + \hbar\omega)t}}{E_k - E_0 + \hbar\omega} + \frac{e^{\frac{i}{\hbar}(E_k - E_0 - \hbar\omega)t}}{E_k - E_0 - \hbar\omega}\right]. \tag{2.13}$$

If we express the total energy of the nucleus in terms of α and $\dot{\alpha}$ we obtain the inertial mass parameter B from the term proportional to $\dot{\alpha}^2$ according to the relation:

$$E = E_0 + \tfrac{1}{2} C\alpha^2 + \tfrac{1}{2} B\, \dot{\alpha}^2. \tag{2.14}$$

When we include the terms up to the second order in a_k together with the selfconsistency condition for the expectation value of $\hat{\alpha}$ in the state $|\alpha_V\rangle$ we get:

$$E = \frac{\langle\alpha_V|H|\alpha_V\rangle}{\langle\alpha_V|\alpha_V\rangle} = E_0 + \kappa^2\alpha_0^2(\hbar\omega)^2 \sum_{j\neq 0} \frac{|\langle j|\hat{\alpha}|0\rangle|^2(E_j - E_0)}{[(E_j - E_0)^2 - (\hbar\omega)^2]^2}. \tag{2.15}$$

When α is given by (2.6) we get the relation

$$\dot{\alpha}^2+\alpha^2\omega^2 = \omega^2\alpha^2 \tag{2.16}$$

and

$$B = 2\kappa^2\hbar^2 \sum_{j\neq 0} \frac{|\langle j|\hat{\alpha}|0\rangle|^2(E_j-E_0)}{[(E_j-E_0)^2-(\hbar\omega)^2]^2}\,. \tag{2.17}$$

In the adiabatic approximation we put

$$\hbar\omega \ll E_j-E_0 \tag{2.18}$$

so that

$$B_\alpha = \hbar^2 \frac{2\Sigma_3}{(2\Sigma_1)^2}, \tag{2.19}$$

where

$$\Sigma_i = \sum_{j\neq 0} \frac{|\langle j|\hat{\alpha}|0\rangle|^2}{(E_j-E_0)^i}, \qquad i = 1, 3. \tag{2.20}$$

For the superconducting nucleus we obtain for Σ_i:

$$\Sigma_i = \sum_{\nu,\mu} \frac{|\langle\nu|\hat{\alpha}|\mu\rangle|^2(u_\nu v_\mu+u_\mu v_\nu)^2}{(E_\nu+E_\mu)^i}, \qquad i = 1, 3, \tag{2.21}$$

with

$$E_\nu = \sqrt{(e_\nu-\lambda)^2+\Delta^2}, \tag{2.22}$$

where λ is the Fermi level and Δ the energy gap. The occupation factors for the single-particle level $|\nu\rangle$ fulfill

$$u_\nu^2+v_\nu^2 = 1. \tag{2.23}$$

In our case α describes the quadrupole moment Q so that

$$\langle\nu|\hat{\alpha}|\mu\rangle = q_{\mu\nu} = \langle\nu| \sqrt{\frac{16\pi}{5}}\, r^2 Y_{20}|\mu\rangle \tag{2.24}$$

and

$$B_\varepsilon = B_Q\left(\frac{dQ}{d\varepsilon}\right)^2. \tag{2.25}$$

It has to be stressed that during this derivation we made use of the assumption that we are close to the extremal point of the nuclear potential energy. First, the operator $\hat{\alpha}$ was defined as the deviation of the nuclear moment from its extremal value and the expansion of the total energy in α and $\dot{\alpha}$ was made up to the second order terms in α and $\dot{\alpha}$.

Second, the mass parameter B is related to the nuclear density deformation α, whereas the calculation is made in terms of the potential deformation α_V. We are able to pass from one to another because of the self-consistency conditions (2.3) and (2.4).

3. *Alternative formula for B*

In this section we review the adiabatic approach to the collective motion outlined in Ref. [6].

As before, we are looking for the solutions of the Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \psi(t) = H(t)\psi(t), \tag{3.1}$$

where $H(t)$ varies slowly with time. Now the solutions of the energy eigenvalue equation at each instant of time are assumed to be known:

$$H(t)u_n(t) = E_n(t)u_n(t). \tag{3.2}$$

We assume that the wave function $\psi(t)$ is given by

$$\psi(t) = \sum_n a_n(t) \exp\left[-\frac{i}{\hbar} E_n(t)t\right] u_n(t). \tag{3.3}$$

If we assume that the system is in the ground state $|0\rangle$ at $t = 0$, we can put as before

$$a_n(t) = \delta_{n0} + c_n(t). \tag{3.4}$$

Substitution of (3.3) into (3.1) gives

$$i\hbar \sum_n \left(\dot{a}_n u_n + a_n \frac{\partial u_n}{\partial t} - \frac{i}{\hbar} a_n u_n E_n\right) \exp\left[-\frac{i}{\hbar} \int_0^t E_n(t')dt'\right] =$$

$$= H \sum_n a_n u_n \exp\left[-\frac{i}{\hbar} \int_0^t E_n(t')dt'\right] \tag{3.5}$$

so that

$$\dot{a}_k = -\sum_n a_n \exp\left[\frac{i}{\hbar} \int_0^t (E_k - E_n)dt'\right] \int \bar{u}_k \frac{\partial u_n}{\partial t} d\tau. \tag{3.6}$$

For the last integral in (3.6) we get

$$\int \bar{u}_k \frac{\partial u_n}{\partial t} d\tau = -\frac{\int \bar{u}_k \frac{\partial H}{\partial t} u_n d\tau}{E_k - E_n} = -\frac{\langle k| \frac{\partial H}{\partial t} |n\rangle}{E_k - E_n}. \tag{3.7}$$

In addition we make here the adiabatic approximation assuming that all the quantities a_k, u_k, $E_k - E_n$, $\partial H/\partial t$ are slowly varying with time. This allows us to find

$$c_k(t) = -i\hbar \frac{\langle k| \frac{\partial H}{\partial t} |0\rangle}{(E_k - E_0)^2} e^{\frac{i}{\hbar}(E_k - E_0)t}. \tag{3.8}$$

As before we calculate the total energy of the system

$$E = \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle} = E_0 + h^2 \sum_{k \neq 0} \frac{|\langle k| \frac{\partial H}{\partial t} |0\rangle|_2}{(E_k - E_0)^3}. \tag{3.9}$$

When we put here

$$\frac{\partial H}{\partial t} = \frac{\partial H}{\partial \alpha} \frac{\partial \alpha}{\partial t} = \frac{\partial H}{\partial \alpha} \dot{\alpha}, \tag{3.10}$$

where α describes the deformation of the nucleus, we get

$$E = E_0 + \dot{\alpha}^2 \frac{\hbar^2}{2} 2 \sum_{k \neq 0} \frac{|\langle k| \frac{\partial H}{\partial \alpha} |0\rangle|^2}{(E_k - E_0)^3}; \tag{3.11}$$

hence

$$\tilde{B}_\varepsilon = \hbar^2 2 \sum_{k \neq 0} \frac{|\langle k| \frac{\partial H}{\partial \varepsilon} |0\rangle|^2}{(E_k - E_0)^3} \equiv \hbar^2 2 \tilde{\Sigma}_3. \tag{3.12}$$

In the case of the superconducting nucleus $\tilde{\Sigma}_3$ is given by the formula

$$\tilde{\Sigma}_3 = \sum_{\mu, \nu} \frac{|\langle \nu| \frac{\partial H}{\partial \varepsilon} |\mu\rangle|^2 (u_\nu v_\mu + u_\mu v_\nu)^2}{(E_\nu + E_\mu)^3}. \tag{3.13}$$

In this derivation we did not assume that we are close to the extremal-point deformation. Unfortunately it is evident that in this case we are dealing with the deformation of the potential instead of the deformation of the nuclear density. Moreover, we have no means of relating those deformations between themselves because from the beginning we deal with the average field and we have no self-consistency conditions analogous to (2.3) and (2.4). We can only hope that because of the short-range character of the nuclear forces the shape of the nuclear density and that of the equipotential surfaces are very similar.

4. Description of the calculations

Inertial mass parameter B was calculated according to the formulae (2.19) and (3.12) in the region R: $88 \leqslant Z \leqslant 104$, $172 \leqslant N \leqslant 194$. In the SHE region only the formula (3.12) was used because the B_ε values in this region were recently calculated in [3] with the formula (2.19).

The dependence of B on the quadrupole ε and hexadecapole ε_4 deformations was investigated. To obtain the single-particle energies we used the Nilsson model single-particle Hamiltonian with the deformed oscillator potential:

$$V(\varepsilon, \varepsilon_4) = \frac{\hbar\omega_0(\varepsilon, \varepsilon_4)}{2} \varrho^2 \left[1 - \frac{2}{3}\varepsilon P_2(\cos\theta_t) + 2\varepsilon_4 P_4(\cos\theta_t)\right] - $$

$$-\kappa\hbar\overset{0}{\omega}_0[2l_t s + \mu(l^2 - \langle l^2\rangle_N]. \tag{4.1}$$

Here ξ, η, ζ are the stretched coordinates:

$$\xi^2 = \frac{M\omega_0}{\hbar}\left(1 + \frac{1}{3}\varepsilon\right)x^2, \quad \eta^2 = \frac{M\omega_0}{\hbar}\left(1 + \frac{1}{3}\varepsilon\right)y^2, \quad \zeta^2 = \frac{M\omega_0}{\hbar}\left(1 - \frac{2}{3}\varepsilon\right)z^2,$$

$$\xi^2 + \eta^2 + \zeta^2 = \varrho^2.$$

$P_2(\cos\theta_t)$ and $P_4(\cos\theta_t)$ denote the Legendre polynomials and the subscript t denotes that the quantity is expressed in the stretched coordinates.

In the formula (2.19) we put

$$\langle\nu|\alpha|\mu\rangle = q_{\mu\nu} = \langle\mu|\sqrt{\frac{16\pi}{5}}\, r^2 Y_{20}|\nu\rangle. \tag{4.2}$$

The corresponding formula (3.12) requires the knowledge of the matrix elements of

$$\frac{\partial V(\varepsilon, \varepsilon_4)}{\partial\varepsilon} = \frac{\partial}{\partial\varepsilon}\left\{\frac{\hbar\omega_0(\varepsilon, \varepsilon_4)}{2}\left[\varrho^2 - \frac{2}{3}\varepsilon\varrho^2 P_2(\cos\theta_t) + 2\varepsilon_4\varrho^2 P_4(\cos\theta_t)\right]\right\}. \tag{4.3}$$

We find the derivative of the first term from the constant volume condition

$$[\omega_0(\varepsilon, \varepsilon_4)]^3 = \overset{0}{\omega}{}_0^3 \int_{-1}^{1} \frac{[(1 + \frac{1}{3}\varepsilon)\sqrt{1 - \frac{2}{3}\varepsilon}]^{-1}dx}{[1 - \frac{2}{3}\varepsilon P_2(x) + 2\varepsilon_4 P_4(x)]^{3/2}}. \tag{4.4}$$

The second term when differentiated gives

$$-\frac{2}{3}\left\{\tilde{q}\left(1 + \frac{\varepsilon_4}{\alpha}\right) - \frac{\varepsilon_4}{\alpha}\left[\frac{5}{11}2\varrho^2 P_6(\cos\theta_t) - \left(\frac{5}{11} + \frac{\varepsilon}{3}\right) \times 2\varrho^2 P_4(\cos\theta_t)\right]\right\}, \tag{4.5}$$

where

$$\alpha = (1 + \tfrac{1}{3}\varepsilon)(1 - \tfrac{2}{3}\varepsilon) \tag{4.6}$$

and

$$\tilde{q} = 2\varrho^2 P_2(\cos\theta_t). \tag{4.7}$$

The pairing forces were included in the BCS approximation; the energy gap Δ and Fermi level λ were found from the equations

$$\frac{2}{G} = \sum_{\nu} \frac{1}{E_\nu}, \quad n = \sum_{\nu} 2v_\nu^2 = \sum_{\nu} \left(1 - \frac{e_\nu - \lambda}{E_\nu}\right), \tag{4.8}$$

where

$$E_\nu = \sqrt{(e_\nu - \lambda)^2 + \Delta^2} \tag{4.9}$$

and n is the number of particles (*i. e.* Z or N). The equations (4.8) were solved for the number of levels included in the summation equal to the number of the particles.

The pairing forces strength G was assumed to increase with the increase of the nuclear surface:

$$G(\varepsilon, \varepsilon_4) = G(0, 0)\,\frac{S(\varepsilon, \varepsilon_4)}{S(0, 0)} \tag{4.10}$$

and the initial values in both regions were:

$$\left.\begin{aligned} G_p(0, 0) &= 19.55 \text{ MeV}/A \\ G_n(0, 0) &= 13.40 \text{ MeV}/A \end{aligned}\right\} \text{ in the R region} \tag{4.11}$$

and

$$\left.\begin{aligned} G_p(0, 0) &= 18.6 \text{ MeV}/A \\ G_n(0, 0) &= 13.30 \text{ MeV}/A \end{aligned}\right\} \text{ in the SHE region.} \tag{4.12}$$

In the SHE region the fission barriers and B_ε values were calculated in Ref. [3] with the Nilsson model parameters κ and μ fitted to the levels obtained in the Woods-Saxon potential. We used now those parameters to calculate B_ε with the formula (3.12).

The parameters κ and μ fitted to the experimental levels in the Rare-Earth and Actinide regions can be extrapolated with the mass number A (see Ref. [2]). This extrapolation was made along the beta-stability line. As the region R departs seriously from this line we took into the calculations three different sets of extrapolated κ, μ parameters, corresponding to $A = 264$, $A = 280$ and $A = 294$, in order to explore the sensitivity of the B values on the choice of those parameters.

In the SHE region the calculations were performed for

$$\varepsilon = 0.0(+0.1)0.7 \text{ with } \varepsilon_4 = \varepsilon/10. \tag{4.13}$$

In the R region the range of ε was

$$\varepsilon = 0.1(+0.1)1.0 \tag{4.14}$$

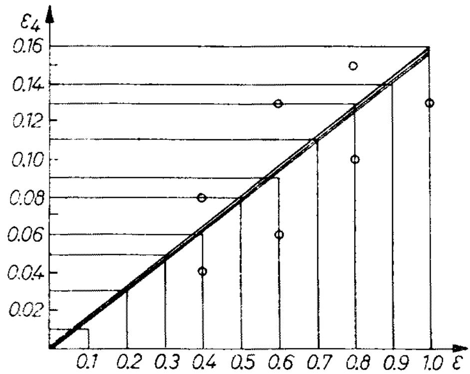


Fig. 1. Deformation parameters ε and ε_4 taken into account in the calculations in the neutron-rich region

and ε_4 was taken as indicated by the solid line on Fig. 1. The additional ε_4 values included in the calculation are marked by the open circles.

5. *Results and discussion*

The numerical results for B_ε in both versions are presented in Table I (R region) nad Table II (SHE region) for different values of the deformation parameter ε. In both cases the first column specifies the atomic number Z, the second — the neutron number N. The third and fourth columns give $B_\varepsilon = \hbar^2(2\Sigma_3)/(2\Sigma_1)^2$ and $\tilde{B}_\varepsilon = \hbar^2\, 2\tilde{\Sigma}_3$ for $\varepsilon = 0.1$; the next columns give the same for $\varepsilon = 0.4$, 0.7 and in the case of R region for $\varepsilon = 1.0$. The deformation dependence of both B_ε and $\tilde{B}_\varepsilon$ can be seen from Fig. 2 (R region) and Figs 3 and 4 (SHE region). The solid lines give $\tilde{B}_\varepsilon$ and the dashed ones represent B_ε. The differences between those two can be as big as 40% B_ε.

As mentioned above, in the case of R region three different sets of κ and μ parameters were used in the calculation. The results for four cases: $^{180}94$, $^{188}94$, $^{180}100$ and $^{188}100$ are presented in Figs 5 and 6 for B_ε and $\tilde{B}_\varepsilon$ respectively. It is immediately seen that B is not very sensitive to the changes of κ and μ: the biggest discrepancy is less than 10% of the lowest B value for a given deformation. In general the discrepancies in $\tilde{B}_\varepsilon$ are larger than those in B_ε.

The lower parts of Figs 3 and 4 give the fission barriers in the SHE region as calculated in Ref. [3]. Table III gives the spontaneous fission life-times calculated as

$$T_{SF} = \frac{\ln 2}{n}\frac{1}{P}, \tag{5.1}$$

where $n = 10^{20.30}$ sec^{-1} in the number of assaults of a nucleus on the fission barrier per unit time and P is given by (1.1). The first two columns specify the nucleus, the third gives T_{SF} calculated with B_ε (taken from [3]) and the fourth contains T_{SF} calculated with B_ε. The fifth column gives the difference

$$\Delta(\log T_{SF}) = \log T_{SF}(\tilde{B}_\varepsilon) - \log T_{SF}(B_\varepsilon). \tag{5.2}$$

TABLE I

B_ε and $\tilde{B}_\varepsilon$ values in $\hbar^2$MeV^{-1} in the neutron-rich region ($\varkappa$ and μ values extrapolated to $A = 280$)

Z	N	ε=0.1		ε=0.4		ε=0.7		ε=1.0	
		B_ε	$\tilde{B}_\varepsilon$	B_ε	$\tilde{B}_\varepsilon$	B_ε	$\tilde{B}_\varepsilon$	B_ε	$\tilde{B}_\varepsilon$
88	172	359.2	299.4	700.4	657.4	1184.9	1084.6	1727.6	1823.2
	174	376.6	305.2	673.4	573.4	1167.1	1000.6	1745.9	1337.3
	176	430.9	349.5	622.8	519.9	1049.9	835.6	1765.0	1829.6
	178	557.9	481.6	719.2	550.9	1035.2	825.1	1786.9	1810.0
	180	566.6	526.0	790.9	606.5	1083.3	916.6	1912.8	1783.3
	182	568.8	553.5	758.5	561.9	1104.8	1000.5	1943.6	1754.9
	184	537.5	528.3	733.2	532.1	1091.7	1052.7	[illegible]	1726.1
	186	495.0	480.6	730.4	541.5	1069.3	1092.6	1920.3	1696.5
	188	470.2	451.1	748.1	588.3	1055.2	1136.7	1962.1	1665.7
	190	467.9	448.3	782.9	666.1	1051.2	1185.3	2006.9	1636.2
	192	457.1	438.7	815.4	749.9	1064.3	1233.7	2056.8	1615.5
	194	425.1	407.7	835.5	818.8	1060.1	1276.2	2114.9	1613.1
94	172	408.1	379.8	712.7	637.2	1240.5	1095.6	1732.9	1691.6
	174	427.0	386.2	661.4	532.8	1210.8	995.3	1742.6	1689.1
	176	477.5	430.0	631.8	464.6	1020.2	762.9	1750.3	1666.9
	178	632.6	607.6	705.0	508.1	1003.9	749.7	1758.9	1630.5
	180	632.8	655.7	810.9	589.1	1078.3	862.5	1770.8	1587.2
	182	637.6	693.8	749.5	521.3	1110.2	956.9	1787.5	1543.0
	184	602.1	659.6	715.2	483.3	1091.9	1006.2	1809.3	1499.2
	186	555.1	598.6	714.1	491.8	1061.6	1040.6	1833.8	1454.3
	188	529.7	563.2	745.0	546.1	1044.4	1083.0	1856.9	1406.7
	190	532.3	566.4	802.8	640.6	1040.5	1132.1	1881.2	1358.8
	192	524.7	558.5	857.2	743.5	1044.7	1181.0	1910.8	1319.9
	194	491.3	522.0	893.8	827.6	1051.5	1225.7	1952.0	1303.0
98	172	440.4	429.6	753.2	727.1	1280.9	1076.8	1809.5	1699.6
	174	460.4	436.9	698.2	605.8	1237.1	963.6	1815.6	1691.2
	176	507.3	476.4	663.8	523.6	958.7	670.9	1818.2	1660.6
	178	690.0	691.6	737.9	573.6	940.9	653.7	1820.8	1614.5
	180	681.7	739.5	851.6	672.7	1043.1	779.1	1826.7	1561.3
	182	688.4	785.6	769.6	578.6	1084.9	876.5	1837.9	1508.0
	184	647.6	742.7	725.8	528.3	1058.8	917.7	1854.3	1455.6
	186	595.1	670.2	712.0	526.7	1017.8	941.9	1872.5	1401.8
	188	567.3	628.9	731.1	575.6	994.9	977.0	1887.5	1343.8
	190	572.6	634.2	780.1	670.0	988.8	1020.1	1901.6	1287.8
	192	566.2	628.6	826.6	771.2	991.5	1062.5	1920.6	1232.5
	194	529.1	586.1	850.3	847.3	996.5	1099.8	1953.6	1205.9
104	172	475.6	454.4	827.6	954.3	1538.9	1141.1	2018.2	1888.3
	174	497.8	460.1	777.1	814.3	1522.0	1024.2	2026.8	1880.9
	176	538.2	489.3	747.5	719.3	1138.8	633.3	2031.0	1847.4
	178	795.4	775.2	832.5	793.1	1155.3	631.2	2035.7	1796.8
	180	767.2	816.6	969.8	942.9	1324.4	800.3	2045.5	1739.8
	182	778.8	876.1	873.4	813.2	1408.6	929.0	2063.3	1684.9
	184	723.6	815.7	836.8	761.5	1406.1	986.6	2088.8	1633.0
	186	655.5	722.1	821.3	761.1	1391.1	1032.6	2116.9	1579.8
	188	619.9	669.7	836.8	821.5	1429.4	1106.5	2140.9	1520.3
	190	630.5	680.3	884.1	939.6	1503.9	1206.6	2162.3	1455.9
	192	625.2	676.8	928.1	1065.5	1620.7	1330.0	2189.3	1400.3
	194	577.0	622.8	950.9	1159.7	1794.4	1487.9	2234.0	1374.7

TABLE II

B_ε and $\tilde{B}_\varepsilon$ values in $\hbar^2$MeV^{-1} in the SHE region

Z	N	ε=0.1		ε=0.4		ε=0.7	
		B_ε	$\tilde{B}_\varepsilon$	B_ε	$\tilde{B}_\varepsilon$	B_ε	$\tilde{B}_\varepsilon$
108	172	605.4	440.1	1057.7	941.2	1895.2	1696.5
	174	752.1	554.7	1144.6	982.3	2152.5	2245.8
	176	700.8	519.9	1137.9	965.4	1993.0	2118.1
	178	713.0	537.7	1049.9	816.8	1839.4	1846.5
	180	1703.5	1597.0	881.7	621.7	1607.7	1523.9
	182	1113.6	1055.6	956.9	685.1	1527.8	1397.6
	184	1089.1	1093.3	1049.0	767.3	1458.4	1418.6
	186	1024.2	1056.1	1002.7	802.3	1536.6	1506.5
	188	921.5	952.0	1128.6	945.9	1553.3	1595.9
112	172	587.0	412.2	1106.9	986.3	1862.5	1750.0
	174	775.3	542.5	1219.5	1046.7	2205.3	2389.1
	176	693.1	487.2	1215.9	1032.3	2008.9	2201.5
	178	660.7	467.5	1111.6	866.7	1834.6	1884.2
	180	2184.2	1755.3	889.6	635.3	1551.8	1487.4
	182	1151.1	1029.8	995.6	715.6	1427.2	1329.0
	184	1134.4	1075.9	1098.9	805.7	1350.1	1338.0
	186	1050.4	1021.4	1033.7	831.0	1422.1	1421.2
	188	921.5	892.7	1172.2	987.4	1439.6	1505.4
116	172	605.7	414.8	1131.3	1020.9	1863.3	1743.5
	174	882.5	584.1	1272.0	1106.6	2314.5	2491.2
	176	802.6	528.3	1277.6	1097.0	2069.3	2233.1
	178	756.0	492.5	1164.2	916.4	1859.7	1874.5
	180	2937.5	2096.4	897.6	646.3	1503.8	1395.9
	182	1381.4	1166.6	1038.6	760.8	1326.4	1207.9
	184	1383.8	1262.7	1158.3	854.0	1239.4	1210.2
	186	1296.9	1236.1	1079.5	871.7	1310.7	1296.2
	188	1162.5	1138.7	1235.4	1047.9	1331.5	1377.1
120	172	568.9	373.9	1138.3	947.6	1924.5	1786.9
	174	855.8	567.5	1309.3	1051.3	2491.3	2665.9
	176	726.6	485.7	1299.9	1032.8	2196.6	2324.9
	178	611.6	401.3	1128.9	822.8	1954.5	1932.1
	180	3256.6	2364.9	736.3	492.3	1521.9	1361.5
	182	1290.5	1111.3	923.6	608.8	1291.2	1145.3
	184	1294.4	1202.8	1062.9	708.3	1194.0	1144.7
	186	1180.2	1131.1	957.0	695.5	1269.8	1238.5
	188	1009.5	972.7	1145.7	873.6	1296.0	1324.0

Fig. 3 displays the differences in B_ε and $\tilde{B}_\varepsilon$ and the fission barriers for the cases when $|\Delta(\log T_{SF})| = 0.1$. The nuclei $Z = 116$ and $Z = 120$ have $\Delta(\log T_{SF}) = 0.1$, the other two have $\Delta(\log T_{SF}) = -0.1$. Fig. 4 gives analogous dependences for the cases when $\Delta(\log T_{SF}) > 2.5$. The nuclei $^{188}116$ and $^{178}120$ have $\Delta(\log T_{SF}) = 2.6$; these values for $^{174}116$ and $^{176}118$ are 2.7 and 2.9 respectively. If we remember that the $\log T_{SF}$ values in this region depend fairly strongly on various other assumptions made in the calculation, *e. g.* whether G is proportional to the surface or not, on the value of the zero-point energy E in the formula (1.1), on the method of calculating *etc.* (see Ref. [3]), we may conclude

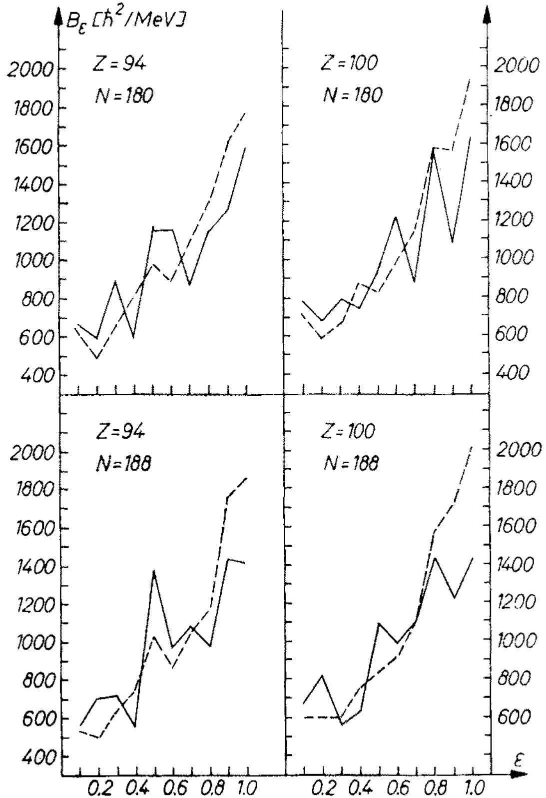


Fig. 2

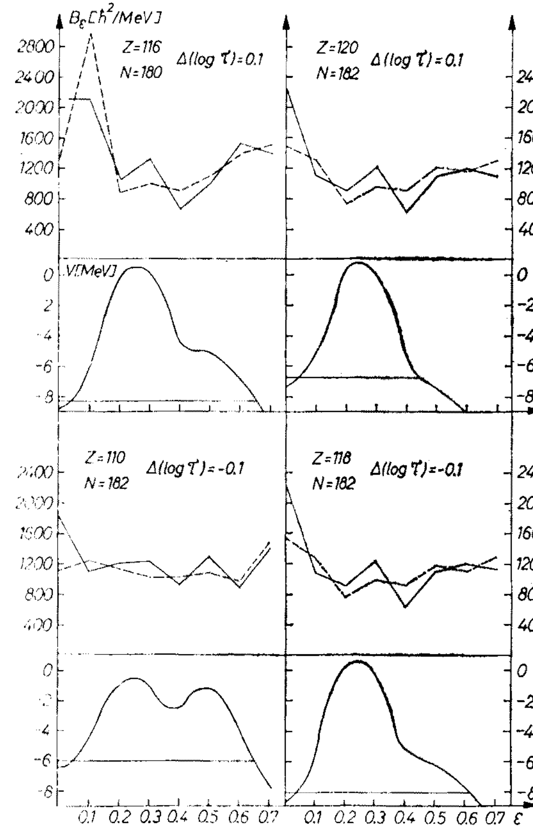


Fig. 3

Fig. 2. B values calculated in the R region (solid line represents $\tilde{B}_\varepsilon$ while the dashed one gives B_ε)

Fig. 3. Fisson barriers (lower part) and B values (upper part) in the SHE region. $\tilde{B}_\varepsilon$ is given by the solid lines, B_ε by the dashed ones

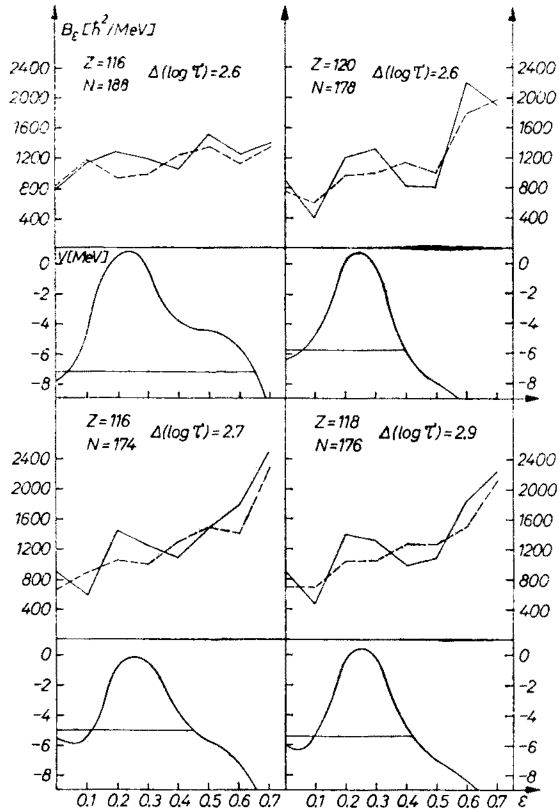


Fig. 4. For description, see Fig. 3

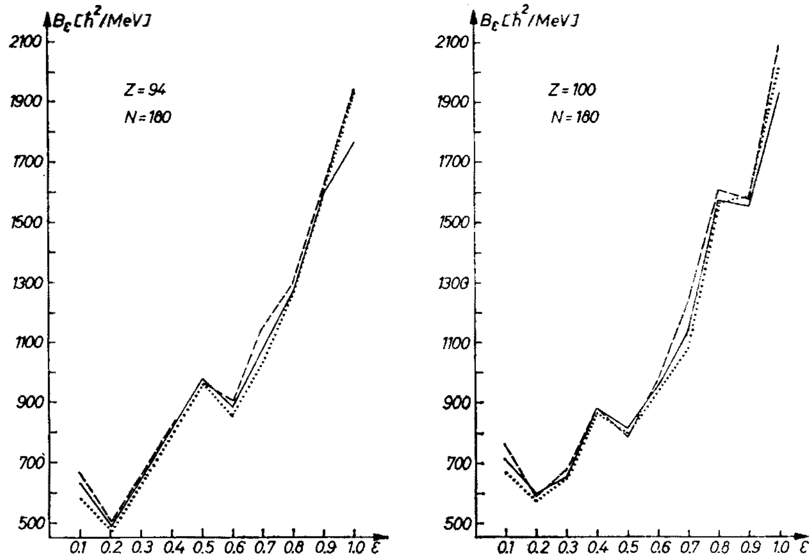
that at least in the SHE region the problem of choosing between the formulae (2.19) and (3.12) is not a crucial one in the determination of the spontaneous fission half-lives.

In the case of the formula (3.12) the tests were made concerning the influence of the volume-conserving term entering (4.3), *i. e.* $\tilde{B}_\varepsilon$ values were calculated with

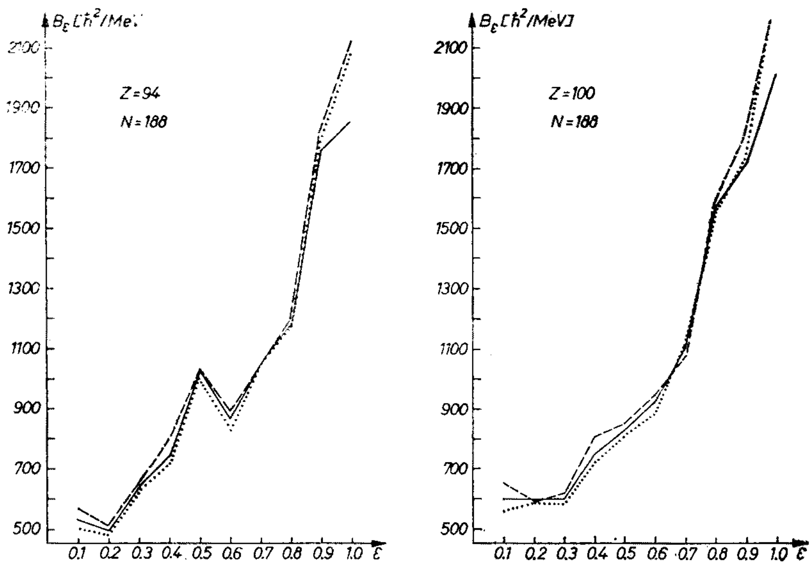
$$\frac{\partial}{\partial \varepsilon}\,\omega_0(\varepsilon, \varepsilon_4) = 0 \tag{5.3}$$

and compared with those with the full expression for $\partial V/\partial \varepsilon$. The results are illustrated in Fig. 7. One can see that the role of the term proportional to $\frac{\partial}{\partial \varepsilon}\,\omega_0(\varepsilon, \varepsilon_4)$ is small for all ε-values used in the calculation. It is quite negligible for small deformations ($\varepsilon \leqslant 0.4$) and for $\varepsilon = 1.0$ it does not exceed 10% of the true value of $\tilde{B}_\varepsilon$.

The sensitivity of both B_ε and $\tilde{B}_\varepsilon$ on the ε_4 values is far greater. Fig. 8 gives the ε_4-dependence of $\tilde{B}_\varepsilon$ (solid line) and B_ε (dashed line) for $\varepsilon = 0.4$, 0.6, 0.8 for four different nuclei. $\tilde{B}_\varepsilon$ changes oven more rapidly with ε_4 than B_ε; the differences are as great as 20% for B_ε and 35% for $\tilde{B}_\varepsilon$ as compared to the values with chosen as indicated in Fig. 1.

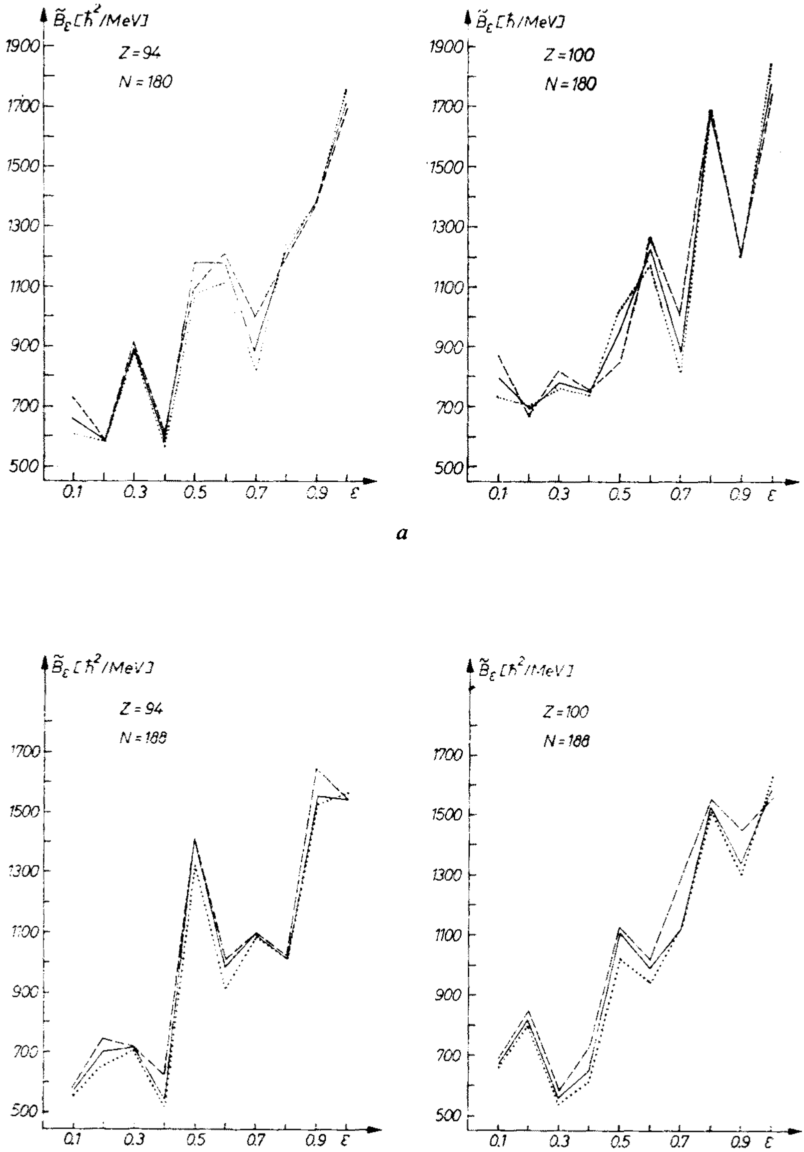


a



b

Fig. 5. B_ε values for three sets of $\varkappa$ and μ parameters as extrapolated to $A = 264$ (dotted line), $A = 280$ (solid line) and $A = 298$ (dashed line)



b

Fig. 6. $\tilde{B}_\varepsilon$ values for three sets of $\varkappa$ and μ parameters as extrapolated to $A = 264$ (dotted line), $A = 280$ (solid line) and $A = 298$ (dashed line)

Fig. 9 gives the dependence of B_ε and $\tilde{B}_\varepsilon$ on the proton number Z (for $N = 180$) and on the neutron number N (for $Z = 94$), for $\varepsilon = 0.2$, 0.5, 0.8. The general behaviour of B_ε and $\tilde{B}_\varepsilon$ is similar but $\tilde{B}_\varepsilon$ is a more sensitive function of both Z and N.

Finally the estimate of $B_{\text{phen}} = \text{const} \cdot A^{5/3}$ ($\hbar^2$ MeV^{-1}) was investigated on the basis of the values of B_ε and $\tilde{B}_\varepsilon$ obtained in this calculation. The results are listed in Table IV:

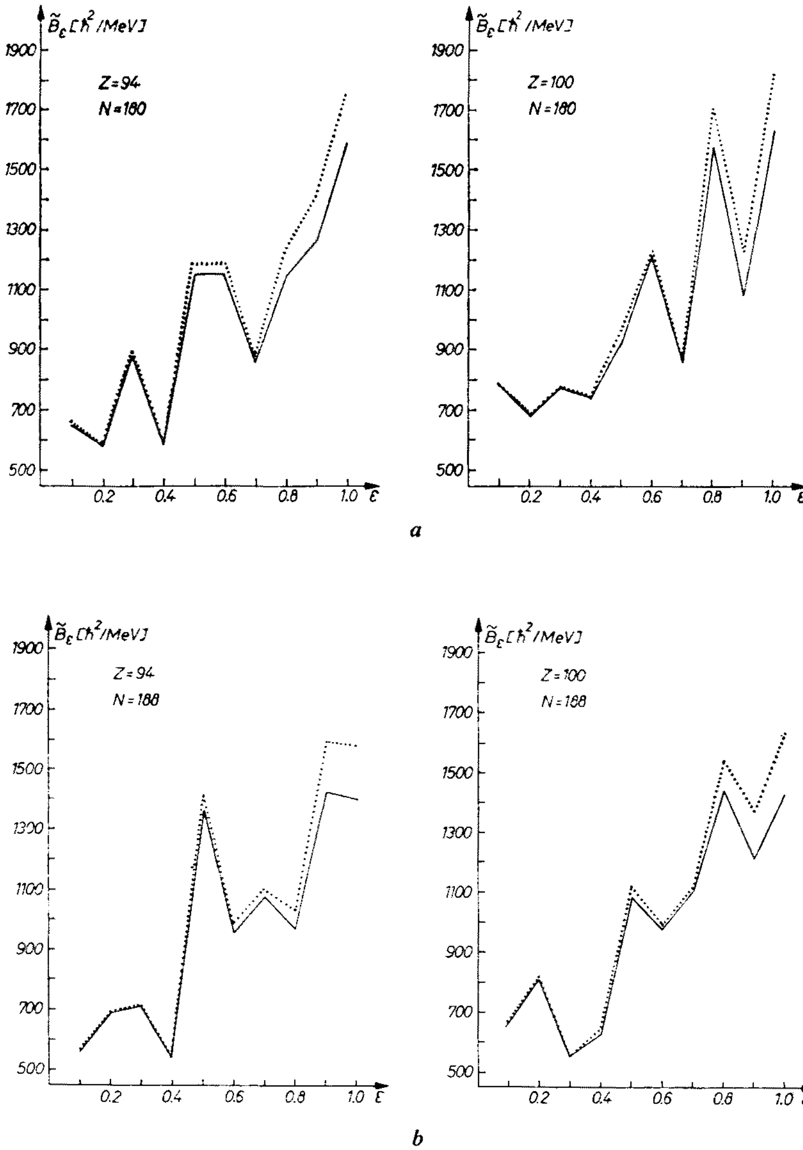


Fig. 7. The influence of the terms proportional to $\partial\omega_0/\partial\varepsilon$ on $\tilde{B}_\varepsilon$ values. Dotted line gives $\tilde{B}_\varepsilon$ calculated with

$$\frac{\partial\omega_0}{\partial\varepsilon} = 0, \text{ the solid one represents the true values of } \tilde{B}_\varepsilon$$

the first column gives the mass number A, the second — atomic number Z; the next three columns give $B_\varepsilon A^{-5/3}$ for $\varepsilon = 0.1$, 0.5, 0.8, the other three give $\tilde{B}_\varepsilon A^{-5/3}$. Fig 10 gives the values of $B_\varepsilon A^{-5/3}$ for $\varepsilon = 0.1$ (solid lines), $\varepsilon = 0.5$ (dashed lines) and $\varepsilon = 0.8$ (dotted lines). Fig. 11 gives the same for $\tilde{B}_\varepsilon$. For comparison the mean phenomenological value of $BA^{-5/3} = 0.054\ \hbar^2\ \mathrm{MeV}^{-1}$ (see Refs [7] and [1]) is given by the heavy solid line.

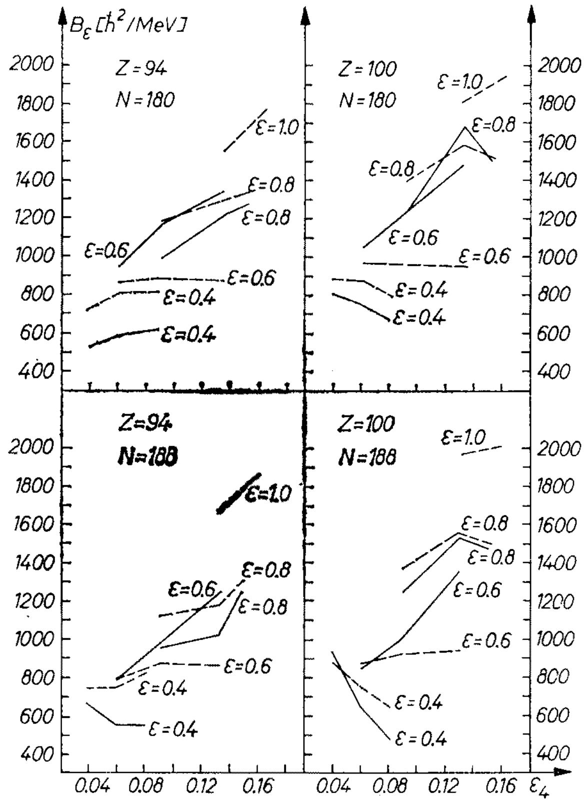


Fig. 8

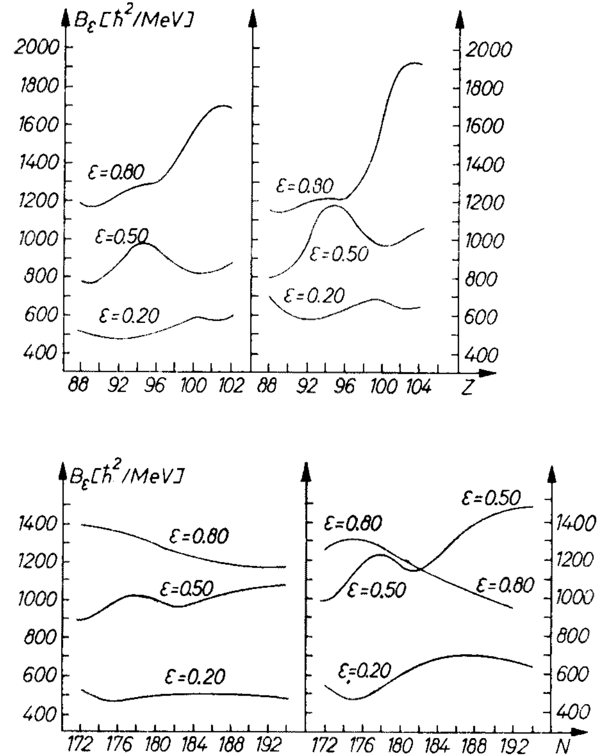


Fig. 9

Fig. 8. The dependence of B on the ε_4 deformation in the neutron rich region R. $\tilde{B}_\varepsilon$ is given by solid lines, B_ε by dashed lines

Fig. 9. The dependence of B on Z and N numbers. The upper part gives Z-dependence of $\tilde{B}_\varepsilon$ (left-hand side) and $\tilde{B}_\varepsilon$ (right-hand side) for $N = 180$. The lower part gives N-dependence of B_ε (left-hand side) and $\tilde{B}_\varepsilon$ (right-hand side) for $Z = 94$

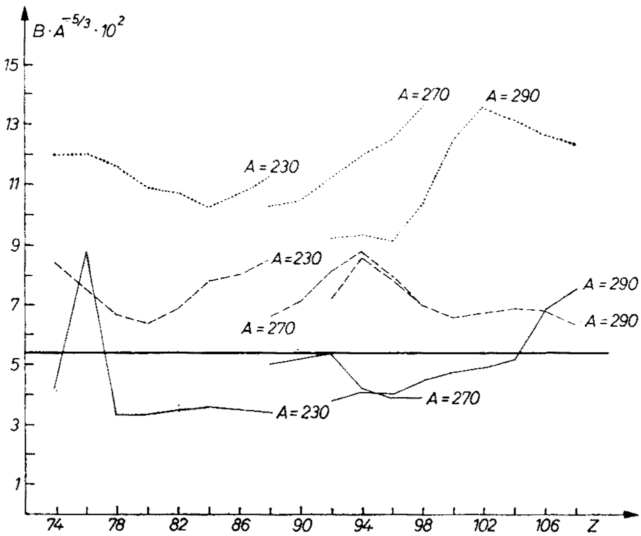


Fig. 10. $B \cdot A^{-5/3}$ values in the R region for B_ε for $\varepsilon = 0.1$ and $\varepsilon = 0.5$ and for $A = 230, 270, 290$

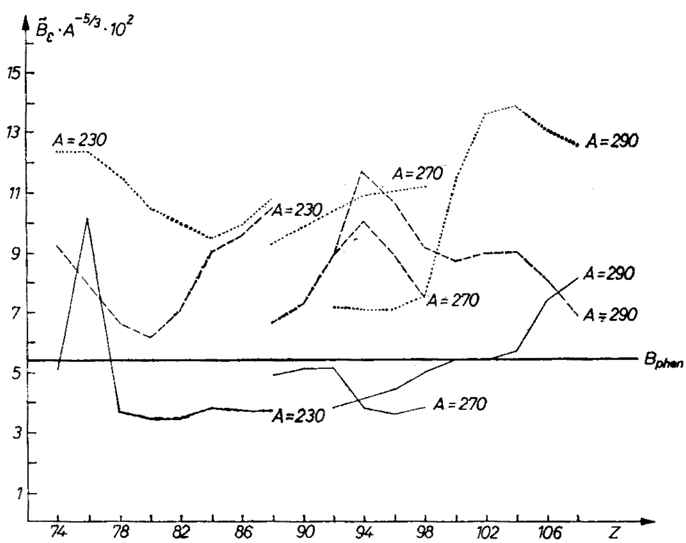


Fig. 11. $B \cdot A^{-5/3}$ values in the R region for $\tilde{B}_\varepsilon$ for $\varepsilon = 0.1$ and $\varepsilon = 0.5$ and for $A = 230, 270, 290$

TABLE III

Spontaneous fission life-times for the SHE region as calculated with B_ε and $\tilde{B}_\varepsilon$

Z	N	log T_{SF}		$\Delta(\log T_{SF})$
		B_ε	$\tilde{B}_\varepsilon$	
108	180	3.2	3.9	0.7
	182	9.9	10.3	0.4
	184	18.7	19.5	0.8
	186	10.1	11.4	1.3
	188	1.3	3.3	2.0
110	178	3.1	4.6	1.5
	180	11.1	11.6	0.5
	182	17.2	17.1	-0.1
	184	25.7	25.6	-0.1
	186	17.8	18.7	0.9
	188	10.1	11.8	1.7
112	174	-3.2	-1.9	1.3
	176	5.9	7.1	1.2
	178	12.8	13.7	0.9
	180	22.8	22.2	-0.6
	182	26.7	25.8	-0.9
	184	35.0	34.5	-0.5
	186	27.9	28.3	0.4
	188	20.9	22.2	1.3
114	172	-1.9	-0.6	1.3
	174	9.3	11.5	2.2
	176	21.2	22.9	1.7
	178	32.1	33.4	1.3
	180	40.7	42.6	1.9
	182	46.6	49.1	2.5
	184	56.3	60.8	4.5
	186	49.7	55.1	5.4
	188	43.8	50.2	6.4
116	174	-5.2	-2.5	2.7
	176	5.4	7.8	2.4
	178	12.7	14.4	1.7
	180	23.3	23.4	0.1
	182	26.3	26.6	0.3
	184	34.1	35.5	1.4
	186	27.2	29.1	1.9
	188	20.7	23.3	2.6
118	176	-4.6	-2.0	2.9
	178	1.6	3.4	1.8
	180	12.7	12.4	-0.3
	182	16.5	16.4	-0.1
	184	24.5	25.3	-0.2
	186	16.8	18.1	1.3
	188	9.1	10.6	1.5
120	178	-5.2	-2.6	2.6
	180	3.4	4.2	0.8
	182	6.6	6.7	0.1
	184	15.2	15.9	0.7
	186	4.4	5.6	1.2
	188	-1.5	0.4	1.9

TABLE IV

$B \cdot A^{-5/3}$ values (in $\hbar^2$MeV^{-1}) for B_ε and $\tilde{B}_\varepsilon$: $\varkappa$ and μ for $A = 280$

A	Z	$B_\varepsilon \cdot A^{-5/3}$			$\hat{B}_\varepsilon \cdot A^{-5/3}$		
		ε=0.1	ε=0.5	ε=0.8	ε=0.1	ε=0.5	ε=0.8
270	88	0.0504	0.0663	0.1027	0.0493	0.0667	0.0933
	90	0.0522	0.0713	0.1053	0.0508	0.0731	0.0976
	92	0.0541	0.0810	0.1133	0.0508	0.0890	0.1044
	94	0.0423	0.0877	0.1198	0.0382	0.1013	0.1088
	96	0.0391	0.0798	0.1249	0.0362	0.0892	0.1113
	98	0.0391	0.0703	0.1357	0.0379	0.0749	0.1163
280	88	0.0381	0.0677	0.0913	0.0370	0.0781	0.0744
	90	0.0411	0.0691	0.0907	0.0413	0.0816	0.0766
	92	0.0430	0.0742	0.0962	0.0448	0.0917	0.0813
	94	0.0463	0.0837	0.0994	0.0503	0.1076	0.0840
	96	0.0518	0.0775	0.1024	0.0585	0.0950	0.0883
	98	0.0574	0.0682	0.1145	0.0668	0.0782	0.0983
	100	0.0597	0.0686	0.1316	0.0658	0.0779	0.1317
	102	0.0639	0.0767	0.1428	0.0646	0.0900	0.1561
	104	0.0499	0.0777	0.1453	0.0408	0.0910	0.1626
290	96	0.0398	0.0790	0.0918	0.0436	0.1067	0.0708
	98	0.0446	0.0697	0.1041	0.0499	0.0924	0.0764
	100	0.0475	0.0659	0.1248	0.0537	0.0870	0.1123
	102	0.0486	0.0675	0.1357	0.0544	0.0894	0.1361
	104	0.0516	0.0692	0.1324	0.0571	0.0904	0.1386

6. *Summary*

The comparison was made between the two adiabatic approaches to the collective motion of the spontaneously fissioning nucleus. Each of them has its own shortcomings: in the first case it is the validity of the approximation in the region of nuclear shapes which is limited to the neighbourhood of the extremal points, in the other case it is the use of the potential deformation parameter instead of the density distortion in the expansion of the total energy and in the formula for B_ε.

Numerical results for the nuclei in the two investigated regions show that the differences in the values of the mass parameter B may be as large as 40% of the B_ε when compared with $\tilde{B}_\varepsilon$. On the other hand, when incorporated into the spontaneous fission life-time formula, both B_ε and $\tilde{B}_\varepsilon$ give relatively consistent estimates of the half-lives in the superheavy region.

Both B_ε and $\tilde{B}_\varepsilon$ were found to be nearly independent of the choice of the κ, μ parameters when those were extrapolated to the values of the mass number $A = 264$, 280, 298 respectively.

The dependence of B on ε_4 deformation parameter was considerable in both cases.

The comparison was made between the calculated B_ε values and the phenomenological value $B_{\mathrm{phen}} = 0.054\,\hbar^2$ MeV^{-1}, frequently used in the fission half-lives estimations.

The author would like to express her deep gratitude to Professor Z. Szymański for suggesting the problem and many valuable comments and discussions.

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