Vol. B4 (1973) ACTA PHYSICA POLONICA No 4

FIT OF THE ANALOG RESONANCES IN ^{46}Ti

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(Received January 4, 1973)

Calculations concerning some of the resonances in the excitation curve ^{45}Sc(p, p) in the energy range of incident protons $E_p = 1.6$–1.8 MeV are presented. Four resonances are considered as analog ones in the compound nucleus ^{46}Ti.

1. Introduction

The reasons of measuring the analog resonances in the isobaric pair ^{46}Sc — ^{46}Ti were considered in [1], where qualitative results were given. In this report we give the results of quantitative calculations concerning the fit of some of the resonances in the excitation curve given in [1], p. 67.

The measurements in the excitation energy range above 13 MeV of the analog nucleus ^{46}Ti have not given good results [2]. From the excitation curve in [2] it was impossible to distinguish separate resonances which could be recognized as analog ones. The reason is that the level density was already too high and $T/D > 1$.

We have chosen such an energy range of incident protons, for the level density in the compound nucleus not to be too high, however this energy is already a significant part of the Coulomb barrier. As we know from the spectroscopic data of the parent nucleus ^{46}Sc [3], this energy range ($E_p = 1.5$–2.2 MeV) corresponds to the strong (*i. e.* with a high value of spectroscopic factors) levels of ^{46}Sc nucleus.

Table I gives the excitation energy and the l- and S-values of the parent nucleus ^{46}Sc in the considered energy range of the parent nucleus.

Fig. 1 shows the excitation curve of the reaction ^{45}Sc(p,p), ([1], p. 67). The curve in this figure was calculated by averaging the curve IV. 3 from [1] with a Gaussian weighting function.

Although the corresponding resonance shapes are generally evident here, remnants of the fine structure and fluctuations in the data are also present after averaging, and they obscure the simple patterns in some cases. The arrows show the expected energy range

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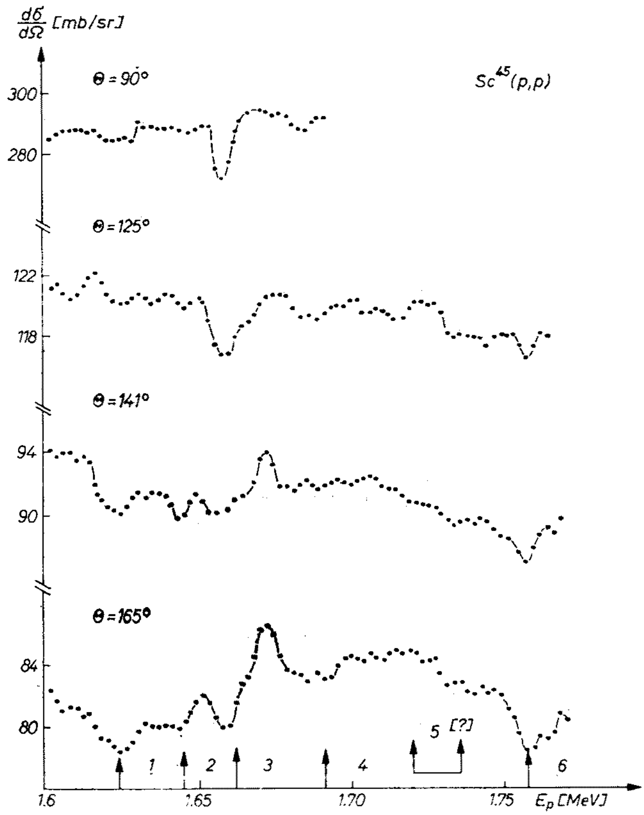

Fig. 1. Excitation curve of the reaction $^{45}Sc(p, p)$

where analog resonances should occur. As we can see, only resonances No 1, 2, 3 and 6 are clear and these resonances have been included in the analysis. The resonances No 4 and 5 are too weak to be taken into the calculation.

To analyze the resonances we have developed a special subroutine described in Section 2.

TABLE I

Spectroscopic parameters of the parent nucleus levels

No of the level [3]	E_x (MeV)	l	$\sim S$	No of the anal. res.
46	2.716	1	0.24	1
47	2.733	1	0.04	2
48	2.780	0	0.014	3
49	2.813	1	0.035	4
50	2.837	0	0.004	5
51	2.862	1	0.31	6
52	2.897	1	0.04	

2. *Calculations*

The analysis of the compound nucleus resonances formed in elastic scattering is performed by using the formula of the single resonance level given by Blatt and Biedenharn [4]. The expression for the differential cross-section $d\sigma$ for the scattering into the solid angle element $d\Omega$ at an angle Θ to the incident beam is

$$d\sigma = |f(\Theta)|^2\, d\Omega, \tag{2.1}$$

where the scattering amplitude $f(\Theta)$ consists of three parts:

$$f(\Theta) = f_{\mathrm{C}}(\Theta)+f_{\mathrm{CH}}(\Theta)+f_{\mathrm{Rl}}(\Theta). \tag{2.2}$$

In this formula the first term $f_{\mathrm{C}}(\Theta)$ represents the scattering amplitude for the pure Rutherford scattering

$$f_{\mathrm{C}}(\Theta) = -z \operatorname{cosec}^2 (\tfrac{1}{2}\, \Theta) \exp\left[-2i\eta \ln \sin (\tfrac{1}{2}\, \Theta)\right], \tag{2.3}$$

where

$$z = Z_a Z_x e^2/2Mv^2 \tag{2.4}$$

and

$$\eta = Z_a Z_x e^2/\hbar v; \tag{2.5}$$

Z_a and Z_x are the atomic numbers of the incident particle and target nucleus, respectively, and M is the reduced mass for the relative motion in the centre-of-mass system.

The second term $f_{\mathrm{CH}}(\Theta)$ in the expression (2.2) is the difference between the scattering amplitude from a charged hard sphere and $f(\Theta)$:

$$f_{\mathrm{CH}}(\Theta) = i\lambda\!\!\!^{-} \sqrt{\pi} \sum_{l'=0}^{\infty} \sqrt{2l'+1}\, \exp(2i\Psi_{l'})\left[1-\exp(2i\Phi_{l'})\right] Y_{l',0}(\Theta). \tag{2.6}$$

The third term $f_{\mathrm{Rl}}(\Theta)$ is the nuclear resonance scattering amplitude

$$f_{\mathrm{Rl}}(\Theta) = i\lambda\!\!\!^{-} \sqrt{(2l+1)\pi}\, \exp\left[2i(\Psi_l+\Phi_l)\right] \frac{\Gamma_{\alpha l} \exp(i\beta)}{\left[(E-E_0)^2+(\tfrac{1}{2}\,\Gamma)^2\right]^{1/2}}\, Y_{l,0}(\Theta). \tag{2.7}$$

The factor $\lambda\!\!\!^{-}$ is the de Broglie wavelength of the incident particles, Φ_l and Ψ_l are the additional phase shifts and

$$\beta = \arctan\left[(E-E_0)/\tfrac{1}{2}\,\Gamma\right]. \tag{2.8}$$

In the expression (2.7) $\Gamma_{\alpha l}$ are the partial widths for capture into the resonance *via* an l-wave in the α channel, T is the total width of the resonance, and E_0 is the resonance energy.

In the calculation a correction for the finite nuclear size was neglected, because in the energy range considered it should not play any significant role. Hence we may write the expression (2.1) for $d\sigma$ as follows:

$$d\sigma = \left[\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Coul}} + \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Interf}} + \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Res}}\right] d\Omega, \tag{2.9}$$

where

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Coul}} = z^2 \operatorname{cosec}^4\left(\frac{1}{2}\Theta\right). \tag{2.10}$$

The purely resonant term $\left(\frac{d\sigma}{d\Omega}\right)_{\text{Res}}$ is in our case negligible (less than 1 %) and the observed elastic scattering anomaly is caused mainly by the interference term

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Interf}} = \frac{\lambda\!\!\!^{-}_a z(2J_0+1)}{(2I+1)(2i+1)} \sum_{l=l_{\min}}^{J_0+I+i} \frac{\Gamma_{al}}{[(E-E_0)^2+(\frac{1}{2}\Gamma)^2]^{1/2}} \times$$

$$\times \operatorname{cosec}^2(\tfrac{1}{2}\Theta)\sin[2\eta \ln \sin(\tfrac{1}{2}\Theta)+2\Psi_l+2\Phi_l+\beta]P_l(\cos\Theta)d\Omega. \tag{2.11}$$

In this formula the spin of the incident particle is denoted by i, the spin of the target nucleus is I and J_0 is the spin of the compound resonance state. The quantity $l_{\min}$ is defined by:

$$\begin{aligned} l_{\min} &= J_0-(I+i) && \text{if } J_0 > I+i, \\ l_{\min} &= 0 && \text{if } |I-i| \leqslant J_0 \leqslant I+i, \\ l_{\min} &= |I-i|-J_0 && \text{if } J_0 < |I-i|. \end{aligned} \tag{2.12}$$

Using the above expression (2.9) for the differential cross-section, the calculations of resonance parameters Γ_l, T and E_0 for a given l were performed by a non-linear least square method.

TABLE II

Results of the calculations

No of the resonance	$E_r^{?}$ (MeV)	Γ_p (keV)	l
1	1.626	3.7	1
2	1.645	0.9	1
3	1.660	1.0	0
6	1.760	0.8	1

3. *Results*

Fig. 2 shows the Breit-Wigner fits to $^{45}Sc(p,p)$ analog state resonances given by our subroutine. The agreement of the experimental results with theory is satisfactory. One can see, that overshots of the resonances show larger differences as compared with experiment than the central parts of resonances. This is so because of the influence of the resonances in the vicinity of analog resonances. This influence makes the fitting procedure impossible, if the distance between the resonances is smaller than the half-width of the neighbouring resonances.

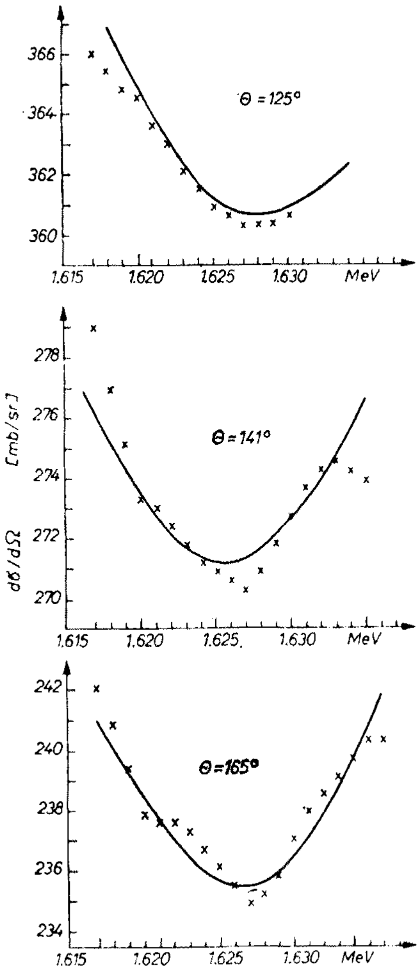

Fig. 2a. Breit-Wigner fits to $^{45}Sc(p, p)$ analog resonance No 1 (Table II)

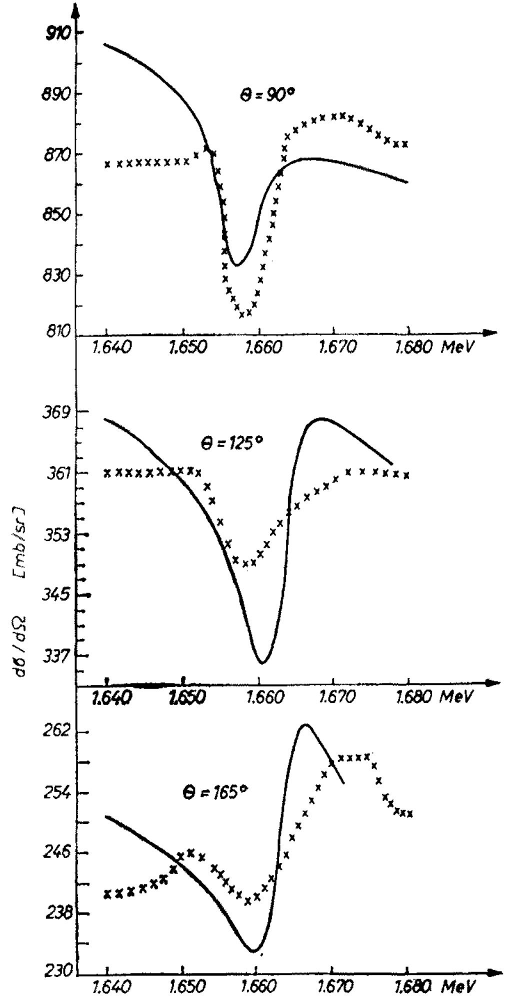

Fig. 2b. Breit-Wigner fits to $^{45}Sc(p, p)$ analog resonance No 3 (Table II)

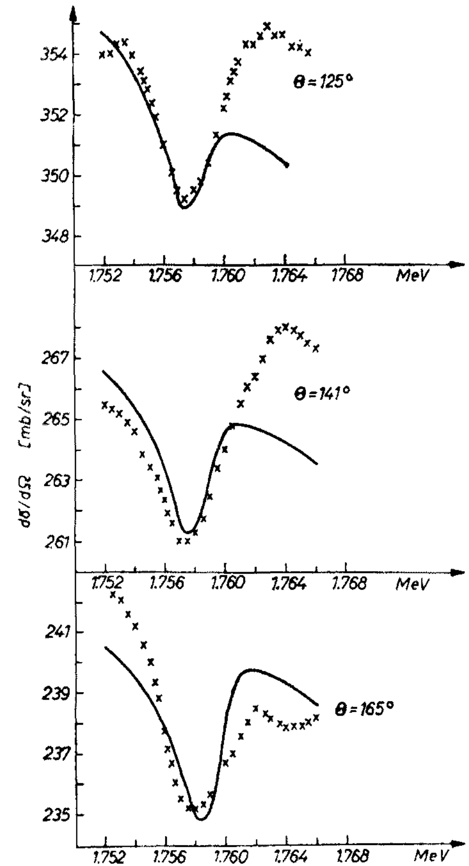

Fig. 2c. Breit-Wigner fits to $^{45}Sc(p, p)$ analog resonance No 6 (Table II)

For our calculations we are able to obtain spectroscopic data on the basis of four analog resonances.

We have also performed a calculation of the spectroscopic factor for the dominant resonance No 3 (with $l = 0$), according to the equation

$$S_{(p,p)} = (2T_0+1)\frac{\Gamma_p}{\Gamma_{sp}}, \tag{3.1}$$

where $2T_0 = N-Z = 4$ for ^{45}Sc, Γ_p — is the measured proton partial width and Γ_{sp} — proton single particle width.

We have taken $\Gamma_{sp} = 2P_l\gamma_{sp}$, where P_l is the penetrability (calculated with the aid of the graphs in the work of Marion and Young [5]), and $\gamma_{sp} = \hbar^2/m\,R^2$, with $R = 1.4\,A^{1/3}$, is the reduced width. The value of S for this resonance is 0.12.

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