PARTIAL TRANSITIONS IN MUON CAPTURE BY COMPLEX NUCLEI. II. THE GAMMA-NEUTRINO CORRELATION

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(Received February 6, 1973)

The gamma-neutrino directional angular distribution is calculated for the muon capture in ^{10}B and in ^{15}N. The dependence on nuclear structure and induced pseudoscalar coupling is investigated.

A detailed analysis of the independent multipole amplitudes which describe the transition in ^{15}N to the first excited level of ^{15}C is also given.

1. Introduction

This paper is a second part of the work [1], where the muon capture rates of the partial transition in some light nuclei have been investigated.

It is clear that knowing only the capture rates we are not yet able to understand the role of the pseudoscalar interaction as well as that of the nuclear structure. Additional ndependent observables are needed for that purpose, such as the gamma-neutrino distributions in partial transitions

$$\mu^- + (A, Z) \rightarrow (A, Z-1)^* + \nu \rightarrow (A, Z-1) + \gamma. \tag{1}$$

Here we are interested in $\gamma-\nu$ correlations in the following transitions

$$^{10}\mathrm{B}(3^+) \xrightarrow{\mu} {}^{10}\mathrm{Be}(2^+, E = 5.96 \text{ MeV}) \xrightarrow{\gamma} {}^{10}\mathrm{Be}(j = 2^+), \tag{2}$$

$$^{15}\mathrm{N}(\tfrac{1}{2}^-) \xrightarrow{\mu} {}^{14}\mathrm{C}(\tfrac{2}{5}^+) \xrightarrow{\gamma} {}^{15}\mathrm{C}(j = \tfrac{1}{2}^+). \tag{3}$$

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2. *General considerations*

As usual for the weak vertex we adopt Morita and Fujii's Hamiltonian [2] and neglect all relativistic corrections to the muon wave function. The nuclear model for these reactions was discussed in detail previously [1, 3]. The kinematic formulae describing the muon capture processes $3 \xrightarrow{\mu} 2 \xrightarrow{\gamma} j$ and $1/2 \xrightarrow{\mu} 5/2 \xrightarrow{\gamma} j$ are taken from [4]. The general form of the directional gamma-neutrino distributions for these processes is as follows

$$W = 1+B_2 a_2 P_2(\boldsymbol{k} \cdot \boldsymbol{\nu})+B_4 a_4 P_4(\boldsymbol{k} \cdot \boldsymbol{\nu}). \tag{4}$$

Here the B_S term describes the nuclear radiation process, a_S describes the weak vertex and by $\boldsymbol{k}$ and ν are denoted the unit vectors of nuclear gamma and neutrino momenta, respectively. The existence of the conversion rate between the hyperfine (hf) K-shell levels in the μ-mesic atom causes the time dependence of capture rates and of the coefficients determining angular correlations [5]. The coefficients a_S in (4) may be presented in the following general form

$$a_S = \frac{pa_S^+ +(1-p)a_S^-}{pA^+ +(1-p)A^-}, \tag{5}$$

where p is the probability of occupation of the hf level in the μ-mesic atom with total spin $F_+ = J_i+1/2$. The formulae describing constants $a_S^\pm$ and $A^\pm$ in terms of the reduced nuclear matrix elements and weak form-factors are given in [4].

3. *Muon capture in* ^{10}B

The calculations for $b_S \equiv B_S\, a_S$ for process [2] for the different initial populations of the hf levels lead to the result that b_S practically does not depend on the nuclear parameter k (k is the intermediate coupling parameter), as may be seen in Fig. 1. The dependence of b_2 on the induced pseudoscalar coupling g_P/g_A (Fig. 2) practically vanishes. The calculations are made for two cases. For the first case (solid line), all nuclear matrix elements including the second forbidden terms are taken into account[1]. The second one (dashed line) corresponds to the nuclear model independent approximation (NMIA). In the last case only one matrix element $[1u-1u]$ is considered. This matrix element usually dominates for unique transitions and b_S are completely independent in the nuclear model in this approach. A difference between curves calculated in NMIA and for the case with all matrix elements is evident. This fact confirms the significant role of nuclear structure in the description of the correlation characteristics.

4. *Muon capture in* ^{15}N

4.1. Multipole amplitudes

The reaction (3) is fully described by four independent multipole amplitudes [4]. The two amplitudes which correspond to the contribution of the multipole $I = 2$ are

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[1] The multipole amplitudes of the higher order are neglected.

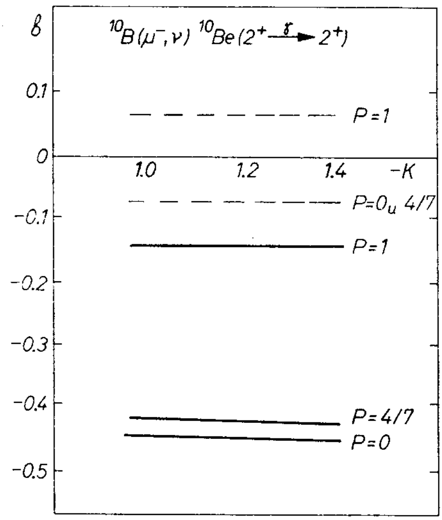

Fig. 1. Dependence of the $\gamma-\nu$ correlation coefficients on the model parameter k in the transition $^{10}B(\mu^-, \nu)\ ^{10}Be(2^+ \xrightarrow{\gamma} 2^+)$. Solid lines correspond to b_2 and dashed lines to b_4. (Pure $M1$ gamma radiation is assumed)

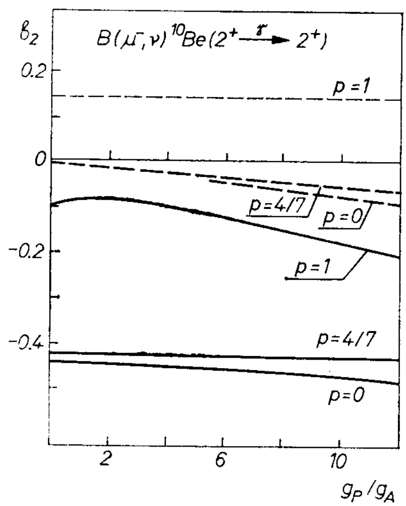

Fig. 2. Dependence of the b_2 coefficient in $\gamma-\nu$ correlation on the ratio g_P/g_A in the transition $^{10}B(\mu^-, \nu)$ $^{10}Be(2^+ \xrightarrow{\gamma} 2^+)$. Solid lines correspond to the case when all matrix elements are included and dashed lines to the NMIA (Pure $M1$ gamma radiation is assumed)

denoted by $M_2 \equiv M$ and $P_2 \equiv P$ and the remaining, which correspond to the second forbidden terms with $I = 3$, by $A_3 \equiv A$ and $V_3 \equiv V$. Here I denotes the total momentum of the lepton field $|J_f - J_i| \leqslant I \leqslant |J_f + J_i|$. In terms of the effective coupling constants and the nuclear matrix elements we have

$$M = G_A\left(\sqrt{2}\,[112]-\frac{2}{\sqrt{3}}\,[132]\right)-g_V\sqrt{\frac{10}{3}}\,[122p],$$

$$P = (G_A-G_P)(\sqrt{2}\,[112]+\sqrt{3}\,[132])+g_A\sqrt{15}\,[022p],$$

$$A = \sqrt{\frac{21}{2}}\,G_A[133]+g_V(\sqrt{3}\,[123p]-\tfrac{3}{2}\,[143p]),$$

$$V = -\sqrt{21}\,G_V[033]+g_V(\sqrt{3}\,[123p]+2[143p]). \tag{6}$$

The induced pseudoscalar contribution is strictly forbidden for A and V amplitudes and moreover, because we neglect the relativistic corrections to muon wave functions, the amplitude M does not depend on g_P. The capture rate may be expressed in terms of these four independent amplitudes [4] and the angular distributions depend on the three ratios P/M, A/M and V/M in sharp contrast with the capture rate. In Tables I and II there are given values of these ratios *versus* the nuclear model parameter V_0 (connected with k mentioned above). From Table I we see that ratios A/M and V/M connected with second forbiddenness are almost unsensitive to the nuclear model parameter V_0. A significant independence of the ratio P/M on the model (Table II) is apparent. At the same time this ratio depends strongly (and obviously strictly linearly) on the induced pseudoscalar interaction. The complete experiment may in principle determine all the independent amplitudes, giving at the same time maximum information about the nuclear structure of the involved nuclei and about the weak vertex.

TABLE I

Amplitude ratios independent on the induced pseudoscalar coupling, in the transition $^{15}N(\mu^-, \nu)\,^{15}C(5/2^+)$

	$V_0 = -50$ MeV	$V_0 = -40$ MeV	$V_0 = 0$ MeV
V/M	0.156	0.149	0.129
A/M	−0.156	−0.143	−0.102

TABLE II

Amplitude ratios P/M in the transition $^{15}N(\mu^-, \nu)\,^{15}C(5/2^+)$

g_P/g_A	$V_0 = -50$ MeV	$V_0 = -40$ MeV	$V_0 = 0$ MeV	NMIA
0	0.896	0.889	0.867	0.884
7	0.600	0.593	0.571	0.585
12	0.388	0.381	0.360	0.371

4.2. The gamma-neutrino angular correlation

The values of coefficients a_2 and a_4 are shown in Figs 3 and 4 and are given for statistically occupied hf levels in the μ-mesic atom. As it was shown in [6], in the range $g_P/g_A \leqslant 10$ the angular correlation coefficients a_S are practically independent on time. The numerical values of a_S are weakly associated with the nuclear parameter but they are apparently different for the NMIA and for the case when all matrix elements are taken into account. This difference is only due to the contribution of the second forbidden terms. The dependence of the angular correlation coefficients on the g_P/g_A is practically linear in the considered range.

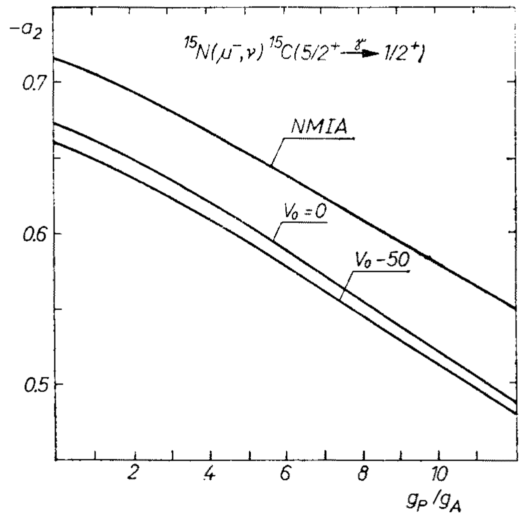

Fig. 3. Dependence of the coefficient a_2 on g_P/g_A in the transition $^{15}N(\mu^-, \nu)\,^{15}C(5/2^+ \xrightarrow{\gamma} 1/2^+)$

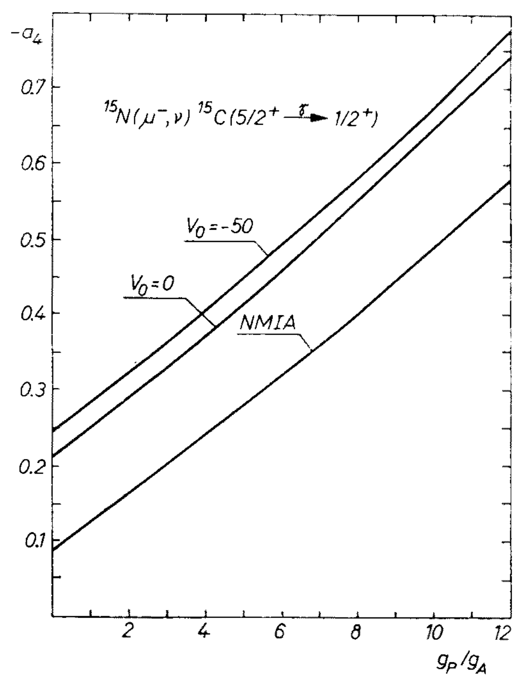

Fig. 4. Dependence of the coefficient a_4 on g_P/g_A in the transition $^{15}N(\mu^-,\nu)\,^{15}C(5/2^+ \xrightarrow{\gamma} 1/2^+)$

5. *Conclusions*

The gamma-neutrino angular distributions strongly depend on the interference between the allowed and the second forbidden terms. However complete experimental data which may in principle determine all the independent multipole amplitudes are needed as a source of the rich information on the weak vertex as well as on the nuclear part of the problem.

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