ON MODELS FOR PROPAGATION OF COHERENTLY PRODUCED SYSTEMS THROUGH NUCLEI

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The general properties of models where the absorption of systems coherently produced in nuclei is small, because the absorption parameters are replaced by a non-diagonal absorption matrix are investigated. A new class of such models in which the absorption from large diagonal terms of the absorption matrix is partly cancelled by the off-diagonal terms is discussed. The dependence of the mass distribution of the coherently produced system on nuclear radius in this class of models is different from that predicted by Van Hove model. For certain values of parameters the model discussed here gives the mass distribution and absorption parameter independent of the nuclear radius, in apparent agreement with the data.

1. Introduction

Recent experiments on coherent production of (3π) systems on nuclei (further on called A-systems) reveal in particular the following two features of this process [1]–[5]:

i) The eikonal approximation [6] describes well the t-distributions, provided the absorption of the A-system in nuclear matter is put roughly equal to that of the pion;

ii) The effective mass distribution of the A-system shows no strong dependence on the radius of the target nucleus, except of course for the trivial dependence caused by the changing nuclear formfactor.

The smallness of the absorption coefficient for the A-system can be explained, when the absorption coefficient is replaced by an absorption matrix [7]–[10]. The most detailed analysis of this kind was described by Van Hove [9]. In Van Hove's model the effective mass distribution for the A-system shrinks with increasing nuclear radius. Gottfried [10] indicated the possibility of models in which the effective mass distribution of A-system broadens with increasing nuclear radius. For the moment these predictions

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cannot be precisely tested with the experimental data, because the experimental errors are still quite big and the analysis of the data is somewhat model-dependent. However, since the existing data are consistent with no change in the mass distribution, it seems interesting to construct models which explain the small absorption without requiring changes in the mass distribution. This is the purpose of the present paper.

In the following Section we recapitulate the main points of the eikonal approximation, mainly in order to introduce the necessary notation. In Section 3 non-diagonal absorption is described, Section 4 contains a description of our model and Section 5 the conclusions.

2. *Eikonal approximation*

In the eikonal approximation it is assumed that the incident pion travels across the nucleus along a straight line, like in geometrical optics a light ray in an absorption medium. One of the possible effects of absorption is that at some point along its path the π goes over into a (3π) system (further denoted A). The A-system travels from the point, where it was created along the same straight line determined by the incident pion momentum. It may either get in turn absorbed, or emerge from the nucleus.

Let us denote by b the collision parameter *i. e.* the distance of a given rectilinear path from the centre of the nucleus. We can calculate the effective path length from the formula

$$z(b) = \int_{-\infty}^{\infty} \varrho(b, x)dx, \tag{2.1}$$

where x is the coordinate along the path and $\varrho(b, x)$ denotes the nuclear density. According to the eikonal model at very high energies, where the longitudinal momentum transfer in the process $\pi \to A$ is negligible[1], the amplitude for the production of A (for an A-system emerging from the nucleus) in a collision with parameter b depends on z only. Denoting this amplitude by $f(z)$, we have for the differential cross-section (*cf. e. g.* [9])

$$\frac{d\sigma}{dt} = \left| \int_0^{\infty} bJ_0(b\sqrt{-t})f(z(b))db \right|^2. \tag{2.2}$$

Thus at high energies in order to describe the differential cross-section for the production of A on all the nuclei, large enough to make the eikonal model applicable, it is enough to know the single function of one variable $f(z)$. If we want to distinguish between A's produced in various states m, we can add a subscript m to $f(z)$ and to σ.

The function $f_m(z)$ satisfies a simple continuity equation (*cf.* [9]), which for the standard version of the model reads

$$\frac{df_m}{dz} = w(m)e^{-\lambda_\pi z} - \lambda(m, m)f_m(z) \tag{2.3}$$

[1] According to estimates from Ref. [11] for A with a mass below 1.32 GeV produced on an Ag nucleus the correction to the cross-section from taking into account the longitudinal momentum transfer is about 12% at 30 GeV/c incident momentum and decreases with increasing energy, or decreasing A mass.

with the initial condition

$$f_m(0) = 0. \tag{2.4}$$

On the right-hand side of Eq. (2.3) the first term gives the contribution of the pion going over into a A-system. Thus λ_π is the absorption coefficient for a pion propagating through the nucleus, $\exp(-\lambda_\pi z)$ is the amplitude for finding the pion at depth z and $w(m)$ gives the m distribution (further called mass distribution, *cf.* Ref. [9]) of the newly produced A-system. The second term is the standard absorption term.

Equation (2.3) can be easily integrated, but it is even simpler to work with function $F_m(z)$ satisfying the equation

$$\frac{dF_m(z)}{dz} = -\lambda(m, m)F_m(z), \tag{2.5}$$

$$F_m(0) = w(m). \tag{2.6}$$

This function describes the propagation of an A-system in state m starting from the moment, when it was produced. As easily checked by direct substitution

$$f_m(z) = \int_0^z e^{-\lambda_\pi x} F_m(z-x)dx. \tag{2.7}$$

On the other hand from (2.5) and (2.6)

$$F_m(z) = w(m)e^{-\lambda(m,m)z}. \tag{2.8}$$

Thus the logarithmic derivative of $F_m(z)$ gives immediately the absorption coefficient $\lambda(m, m)$.

Existing experimental data on the absorption of A-system in nuclear matter [1]–[5] indicate that indeed $F_m(z)$ can be written in the form

$$F_m(z) = w(m)e^{-\lambda_A^{\exp}(m)z}, \tag{2.9}$$

where

$$\lambda_A^{\exp}(m) \simeq \lambda_\pi = (4.5\ \text{fm})^{-1}/\varrho(0, 0) \tag{2.10}$$

is roughly independent of m.

This result is difficult to understand if the following two very natural assumptions are made

(*i*) The eikonal approximation is considered as the limit of the Glauber model, so that the elementary scattering act is scattering on the single nucleon, which proceeds as if the nucleons were free. Then (*cf. e. g.* [6])

$$\lambda_A(m, m) = \tfrac{1}{2}\sigma_{A_mN}; \quad \lambda_\pi = \tfrac{1}{2}\sigma_{\pi N}, \tag{2.11}$$

where σ_{A_mN} is the total cross-section for the scattering of the A-system in state m on a nucleon.

(*ii*) The A-system is a $(\pi\varrho)$ system. Then using Glauber's model for the scattering of a $(\varrho\pi)$ system on a free nucleon it can be proved that [1]

$$\sigma_{A_mN} \simeq 2\sigma_{\pi N}. \tag{2.12}$$

From (2.11) we thus obtain using (2.12) the following relation between the absorption parameters of the A-system and π:

$$\lambda_A^{\text{theor}} \simeq 2\lambda_\pi. \tag{2.13}$$

Combined with the experimental result (2.10) this implies that

$$\lambda_A^{\text{theor}} \neq \lambda_A^{\text{exper}}. \tag{2.14}$$

The source of this difficulty is in the assumptions that an A-system is a $(\varrho\pi)$ system and that its scattering off the free nucleon can be described by the Glauber model. If these assumptions are rejected *e. g.* by saying that the A-system is a resonance there is no problem [1]. However, it is interesting to look for other ways to avoid the contradiction (2.14). This is discussed in the next section.

3. Non-diagonal absorption

A way out of the difficulty described in the previous section is to propose a different interpretation of the experimental results (2.9) and (2.10). Two possibilities arise: either the absorption coefficient $\lambda_A^{\text{exper}}(m)$ is not related to the parameter $\lambda(m, m)$, *i. e.* formula (2.8) is not valid; or the parameter $\lambda_A^{\text{theor}}(m)$ is not related to the total cross-section of the A-system on a free nucleon, *i. e.* formula (2.11) is violated.

Van Hove [9] and Rogers and Wilkin [8] observed that such possibilities do indeed arise if one introduces non-diagonal absorption, *i. e.* if one replaces equations (2.3) and (2.5) by

$$\frac{df_m(z)}{dz} = w(m)e^{-\lambda_\pi z} - \sum_{m'} \lambda(m, m')f_{m'}(z) \tag{3.1}$$

and

$$\frac{dF_m(z)}{dz} = -\sum_{m'} \lambda(m, m')F_{m'}(z). \tag{3.2}$$

The initial conditions (2.4) and (2.6) and relation (2.7) remain unchanged. Also formula (2.11) remains valid under the same assumptions which were necessary to prove it in the diagonal case (*cf.* Appendix A). Specific models are obtained by fixing the matrix $\lambda(m, m')$.

Van Hove considered a class of models with non-diagonal absorption matrices and has shown that one may obtain a substantial reduction of the expected absorption parameter with respect to its value required by formula (2.13). The models considered by Van Hove have two essential properties:

a. The absorption matrix is a smooth and slowly varying function of the parameters m and m'.

b. The range of parameters m and m' in which $\lambda(m, m')$ is substantially greater than zero is of the same order as the range in which the probability amplitudes $w(m)$ for creation of A-systems are different from zero.

Furthermore, correspondence with the condition (2.11) can be obtained if one notices that in Van Hove's model [9]

$$\sum_{m'} \lambda(m, m') \simeq \tfrac{1}{2} \sigma_{A_m N}, \tag{3.3}$$

where $\sigma_{A_m N}$ refers to the scattering on a free nucleon.

It is easily seen that by the substitution

$$\lambda(m, m') \to \lambda_m \delta(m, m'). \tag{3.4}$$

Eqs (3.1) and (3.2) reduce to the diagonal case (2.3) and (2.5) with the condition (2.11) satisfied.

Apart from the basic result that absorption is strongly reduced, Van Hove's models have two other important properties:

(*i*) Condition (2.11) is not satisfied, as is obvious from formula (3.3). This is explained by observing that the interactions of the A-system in nuclear matter may have little to do with the scattering of an A-system on free nucleons.

(*ii*) The mass distribution $F_m(z)/\sum_m F_m(0)$ changes with distance z when the A-system travels in nuclear matter. This effect should lead to observable differences between the mass spectra obtained from scattering off different nuclear targets (apart from the trivial cut-off dependence implied by the nuclear form-factors); consequently it may perhaps serve as an experimental test of the Van Hove mechanism in future more accurate experiments.

The mechanism by which absorption is reduced in Van Hove's models can be seen by writting Eq. (3.2) in the form

$$\lambda_m(z) = -\frac{d}{dz} \{\log |F_m(z)|\} = \frac{\overline{F}_m(z)}{F_m(z)} \frac{1}{2} \sigma_{AN}, \tag{3.5}$$

where

$$\overline{F}_m(z) = \frac{\sum_{m'} \lambda(m, m') F_{m'}(z)}{\sum_{m'} \lambda(m, m')} \tag{3.6}$$

and $\lambda_m(z)$ is the effective absorption parameter for given m and z. Here we have used condition (3.3).

Let us first consider the effective absorption parameter $\lambda_m(z)$ for short distances $z \simeq 0$. Then formula (3.5) reads

$$\lambda_m(z = 0) = \frac{\overline{w}(m)}{w(m)} \frac{1}{2} \sigma_{AN}, \tag{3.7}$$

and we see that in the region where $w(m)$ is large: $\overline{w}(m) < w(m)$ and $\lambda_m(z = 0) < \frac{1}{2} \sigma_{\pi N}$.

Thus in the central region of the A-bump the absorption coefficient is reduced, whereas in the outside regions it is enhanced. Consequently, the A-bump shrinks as z increases.

At large distances the absorption parameter tends to zero as follows from the discussion given in Ref. [9].

These results show that

a) at large distances the absorption in Van Hove's model will be small independently of the normalization condition (3.3). This indicates that condition (3.3) may be relaxed [12] and still small absorption is obtained for a suitable set of $w(m)$. *E. g.* for $w(m) = F_m^{(0)}(z_0)$ where z_0 is a fixed large value and $F_m^{(0)}(z_0)$ are calculated with some original $w(m)$.

b) the effective absorption parameter changes with the distance z. Consequently the amplitude $F_m(z)$ cannot be written in the exponential form (2.9) and the dependence of the differential cross-section on the nuclear radius is altered with respect to the prediction of the simple eikonal model with diagonal absorption.

c) the effective absorption parameter depends not only on z but also on m. Thus the mass distribution changes when experiments are performed on different nuclei.

To investigate this last point in more detail we write Eq. (3.2) in the form

$$\frac{dF_m(z)}{dz} = \bar{F}_m(z)\frac{1}{2}\sigma_{AN} \tag{3.8}$$

and observe that, since $\lambda(m, m')$ is a slowly varying function of m and m', the function $\bar{F}_m(z)$ depends only weakly on m. Thus the rate of decrease is similar for all $F_m(z)$: their distribution sinks without changing shape. As a result negative values of $F_m(z)$ appear in the region where $w(m)$ are small, *i. e.* at the edges of the A-bump. The mass distribution changes in two ways. Firstly, the central peak shinks. Secondly, broad wings corresponding to the negative values of $F_m(z)$ appear, with minima between the central peak and the wings.

To summarize, in the Van Hove model the reduction of absorption is intimately related to changes in the shape of the mass spectrum. Furthermore, the attenuation of the produced A's is not exactly exponential in the travelled distance.

In the next section we discuss another class of models in which the reduction of absorption is not necessarily accompanied by these side-effects.

4. Quasi-diagonal absorption

In this section we discuss the main point of our paper which is to describe a mechanism for the absorption of the A-system in nuclei which has the following characteristics:

a. The conditions (2.11) and (2.12) are satisfied, *i. e.* the absorption of the A-system in nuclear matter can be determined from its interaction with a free nucleon.

b. Despite of this, the absorption parameter λ_A of the A-system need not be equal to twice that of the pion but can be made arbitrarily small and, in particular, equal to that of pion.

c. The absorption parameter λ_A depends practically neither of z nor on m, provided the mass distribution in the A-bump is a slowly varying function of m. Thus it should be

possible to describe the data for all nuclei by one value of λ_A. Furthermore, except for the effects of nuclear form-factors, the mass distribution in the A-bump is also the same for all nuclei.

The mechanism we discuss here is thus complementary to that proposed by Van Hove [9]. Point (c) above indicates that there are observable differences between these two mechanisms. It would be interesting to analyze the data with particular emphasis on these effects.

The mechanism can be illustrated on a model where Eq. (3.2) takes the form

$$\frac{dF_m(z)}{dz} = -\lambda_0 F_m(z) + \frac{1}{2}\lambda_1\{F_{m-1}(z)+F_{m+1}(z)\}, \tag{4.1}$$

$$F_0(z) = F_{N+1}(z) = 0, \qquad m = 1, \ldots, N. \tag{4.2}$$

Thus in this model the only non-zero elements of the λ-matrix are $\lambda(m, m) = \lambda_0$ and $\lambda(m, m\pm 1) = \frac{1}{2}\lambda_1$. The calculations are performed for N finite, but at the end the limit $N \to \infty$ is taken. In the limit therefore we have a continuous mass spectrum.

The properties (a)–(c) required from the model can be obtained also for any quasi-diagonal matrix $\lambda(m, m')$, for which only a finite number of off-diagonal elements close to the main diagonal is different from zero in each line. For simplicity we work with example (4.1).

From Eq. (4.1) we have the following expression for the absorption parameter $\lambda_A(m)$ of the A-bump in state m:

$$\lambda_A(m) = -\frac{d}{dz}\{\ln |F_m(z)|\} = \frac{\sum_{m'} \lambda(m, m')F_{m'}(z)}{F_m(z)} = \frac{\lambda_0 F_m - \frac{1}{2}\lambda_1(F_{m+1}+F_{m-1})}{F_m}. \tag{4.3}$$

Now, the point is that whenever $w(m)$ can be written as a smooth function of mass, the same can be expected for $F_m(z)$. Then the change of m by one unit should not affect much $F_m(z)$ and Eq. (4.3) can be rewritten approximately as

$$\lambda_A(m) \simeq \sum_{m'} \lambda(m, m') = \lambda_0 - \lambda_1. \tag{4.4}$$

Thus the absorption parameter λ_A is the sum of a diagonal and some off-diagonal elements of the absorption matrix. By choosing the off-diagonal terms negative (*i. e.* λ_1 positive) we obtain the reduction of absorption compared to the value obtained from the diagonal term alone. Thus the absorption can be made arbitrarily small inspite of the condition (2.11) applying to $\lambda_0 = \lambda(m, m)$.

Eq. (4.4) indicates also that the value of λ_A depends neither on m nor on z. Thus condition (c) is satisfied and we have

$$F_m(z) = w(m)e^{-(\lambda_0-\lambda_1)z}. \tag{4.5}$$

The arguments used here are only qualitative but we believe that they describe corectly the general behaviour of absorption. For the interested reader we give in Appendix B the exact solution of the set of Eqs (4.1) and verify explicitly the relation (4.5).

The same situation is obtained also for more complicated quasi-diagonal matrices $\lambda(m, m')$, provided the width of the central band of the matrix $\lambda(m, m')$ is small compared to the width of the A-bump. In this case we have several off-diagonal terms which reduce the large diagonal term in formula (4.4). This behaviour of $\lambda(m, m')$ is illustrated in Figure 1, where the schematic plot of $\lambda(m, m')$ as function of m' for fixed m is given. It may be con-

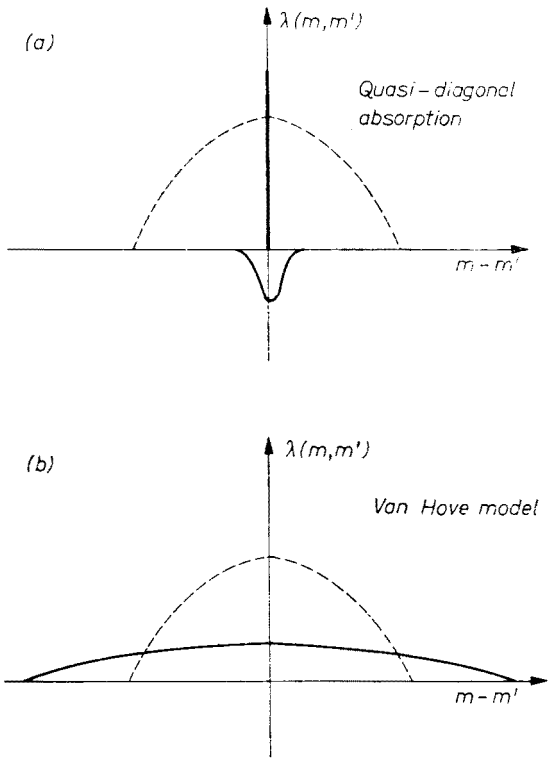

Fig. 1. The absorption matrix $\lambda(m, m')$ for fixed m as function of m' is represented by solid lines for the quasi-diagonal absorption (a) and for the Van Hove model (b). To fix the scale the mass distribution of the A-bump is indicated by the dotted lines

trasted with the one obtained in the Van Hove model, which is also shown in Figure 1. In this case there is no central peak, but the distribution is smooth and very broad.

We believe that, as argued by Bell [13] and Gottfried [10], the situation in which there is a sharp central peak of the elastic transition followed by a more diffuse distribution of the off-diagonal terms is likely to occur in the real world. For example, such structure is present if the scattering of the A-system from the nucleon is described by the Glauber model [10]. Although in the case the negative wings of the absorption matrix are too small to account for the observed reduction of the absorption parameter [1], it is important that qualitatively the effect is present. It seems to us likely that an improved calculation, taking into account the interaction between ϱ and π may well enhance the effect considerably.

We close the description of the model by the observation that in order to obtain cancellations in formula (4.3) for the absorption parameter λ_A, it is not necessary that the negative wings of the absorption matrix are very narrow. What is important, is the sum of all non-diagonal elements which should be negative and approximately equal to one half of the diagonal term. Thus qualitatively similar results can be obtained even for wings as broad as A-bump itself. In this case, however, similarly as in Van Hove model, the absorption parameter will depend on z and on m. Thus we should observe changes in the mass distribution and in the absorption parameter for different nuclei. It is interesting to note that the changes in mass distribution are opposite to those predicted in Van Hove's model: we expect a broadening of the central peak and the appearance of wings in the mass distribution without a dip between peak and wing. Thus the distribution simply broadens with increasing nuclear radius without developping any structure [10].

For completeness, let us mention another possibility of a mechanism which reduces absorption: for $\lambda_1 < 0$ cancellations are possible if $F_m(z)$ oscillates rapidly as function of m. For Eqs (4.1), (4.2) with

$$w(m) = (-1)^m \tag{4.6}$$

and λ_1 negative we have again

$$F_m(z) = e^{-[\lambda_0 - |\lambda_1|]z} w(m). \tag{4.7}$$

5. *Conclusions*

Following the discussion given by Van Hove [9] we have investigated the general properties of the models for the absorption of composite objects in nuclear matter in which the absorption is described by a non-diagonal matrix.

The main conclusions can be summarized as follows:

a) The presence of the large diagonal term corresponding to elastic transitions in the absorption matrix (δ-term in Van Hove's notation) does not necessarily lead to strong absorption of the composite object travelling through nuclear matter. Thus it may be possible to describe the scattering of this object from a free nucleon by the same amplitude which describes its scattering from a nucleon bound in nuclear matter.

b) We discussed in some detail a class of models in which the off-diagonal terms of the scattering matrix are negative and largely cancel the diagonal absorption term.

c) The properties of these off-diagonal terms can be deduced from studying the dependence of the mass distribution of the produced objects and of the absorption parameter on the nuclear radius. It is not difficult to construct models in which the mass distribution and the absorption parameter do not depend on the radius of the target nucleus in apparent agreement with existing data.

d) Property (c) can be used for an experimental distinctions between the models we discuss and the Van Hove model (in which the δ-term is not present). This is so because the dependence on the nuclear radius is very different in these two classes of models.

e) The structure of the absorption matrix discussed by us occurs in the Glauber model and is very likely to occur in its generalizations.

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APPENDIX A

In this Appendix we prove that relation (2.11) holds for non-diagonal absorption matrices, provided it holds in case when the absorption matrix is diagonal.

Let us denote the states which diagonalize the absorption matrix by $|\mu\rangle$. They are related to states $|m\rangle$ by

$$|\mu\rangle = \sum_m \Omega_{\mu m}|m\rangle, \tag{A.1}$$

where Ω is an unitary matrix. Also we have

$$\lambda_{m,m'} = \sum_\mu \Omega^+_{m\mu} \Lambda_\mu \Omega_{\mu m'}, \tag{A.2}$$

where Λ_μ are the eigenvalues of the absorption matrix.

According to our assumption, condition (2.11) holds for eigenvalues of the absorption matrix and we have

$$\Lambda_\mu = \tfrac{1}{2}\sigma_{A_\mu N}, \tag{A.3}$$

where $\sigma_{A_\mu N}$ is the total cross-section for scattering of an A-system in state $|\mu\rangle$ off a free nucleon. By the optical theorem we can write

$$\sigma_{A_\mu N} = \gamma\langle\mu|T|\mu\rangle, \tag{A.4}$$

where γ is a coefficient depending on the normalization of states $|\mu\rangle$ and $\langle\mu|T|\mu\rangle$ is the forward elastic amplitude.

To prove our statement, we observe that, since Ω is an unitary matrix, the normalization of states $|\mu\rangle$ and $|m\rangle$ is the same and we have also

$$\sigma_{A_m N} = \gamma\langle m|T|m\rangle, \tag{A.5}$$

where $\sigma_{A_m N}$ is the total cross-section for scattering of an A-system in state $|m\rangle$ off a free nucleon, and $\langle m|T|m\rangle$ is the corresponding forward elastic amplitude. Thus, using Eqs (A.2)–(A.5) we obtain

$$\lambda_{mm} = \sum_\mu \Omega^+_{m\mu}\Lambda_\mu\Omega_{\mu m} = \frac{\gamma}{2}\sum_\mu \Omega^+_{m\mu}\langle\mu|T|\mu\rangle\Omega_{\mu m} =$$

$$= \frac{\gamma}{2}\langle m|T|m\rangle = \frac{1}{2}\sigma_{A_m N}. \tag{A.6}$$

This completes the proof.

APPENDIX B

In this Appendix we present the solution of the system of equations (4.1). The method is standard (*cf.* Ref. [9]). Substituting

$$F_m(z) = \sqrt{\frac{2}{N+1}} \sum_{n=1}^{N} c_n(z) \sin\left(\frac{nm\pi}{N+1}\right) \tag{B.1}$$

into Eqs (4.1) we have

$$\frac{dc_n}{dz} = -\left(\lambda_0 - \lambda_1 \cos\frac{n\pi}{N+1}\right) c_n(z). \tag{B.2}$$

The initial condition (2.6) implies

$$c_n(0) = \sqrt{\frac{2}{N+1}} \sum_{k=1}^{N} w(k) \sin\left(\frac{nk\pi}{N+1}\right). \tag{B.3}$$

Solving Eqs (B.2) with the initial condition (B.3) and substituting into (B.1) we find

$$F_m(z) = \frac{2}{N+1} \sum_{k=1}^{N} w(k) \sum_{n=1}^{N} \sin\left(\frac{nm\pi}{N+1}\right) \sin\left(\frac{nk\pi}{N+1}\right) e^{-\left[\lambda_0 - \lambda_1 \cos\left(\frac{n\pi}{N+1}\right)\right]z}. \tag{B.4}$$

Since we have assumed that the mass distribution in the A-bump region is a slowly varying function of m, we can represent $w(k)$ by a finite Fourier series:

$$w(k) = \sum_{l=1}^{L} a_l \sin\left(\frac{lk\pi}{N+1}\right). \tag{B.5}$$

Substituting (B.5) into (B.4) and performing the summation over k we obtain

$$F_m(z) = \frac{N}{N+1} \sum_{l=1}^{L} a_l \sum_{n=1}^{N} \sin\left(\frac{nm\pi}{N+1}\right) \delta_{nl} e^{-\left(\lambda_0 - \lambda_1 \cos\frac{n\pi}{N+1}\right)z}. \tag{B.6}$$

For fixed L and $N \to \infty$, this formula gives

$$F_m(z) = \sum_{l=1}^{L} a_l \sin\left(\frac{ml\pi}{N+1}\right) e^{-(\lambda_0 - \lambda_1)z} = w(m) e^{-(\lambda_0 - \lambda_1)z}. \tag{B.7}$$

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