CORRELATION INTEGRALS FOR CHARGED AND NEUTRAL PIONS IN INCLUSIVE PRODUCTION AND THE TWO-COMPONENT PICTURE

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The existing data on multiplicity distribution of charged and neutral pions produced in high-energy interactions are analysed in terms of correlation integrals. The two-component picture of particle production is used to explain the data. The model is shown to describe correctly the two- and three-particle correlations between charged pions as well as the two-particle correlations between charged and neutral pions. The possible detailed versions of the model differing at highest energies are discussed.

1. Introduction

It was recently pointed out by many authors [1–6] that the analysis of the multiplicity distribution in inclusive reactions in terms of Mueller's [7] correlation integrals provides a very useful tool in the investigation of the production mechanism. The analysis is experimentally much simpler than the double differential measurement of correlation function. Certainly, there is less information contained in integrals than in the function itself; the general features and in particular the energy dependence may be however recognised just from the integrated correlations.

We intend to discuss always the definite sort of particles — negative or neutral pions — since only in this case the correlation integrals have the simple intuitive meaning independent of the conservation laws. Experimental difficulties force us to adopt the rough assumption that all the negative particles produced are pions. Since we are dealing with quantities integrated over whole phase space, this assumption seems to be reasonably well satisfied. We use mainly pp data, which cover the most extended energy range. A short discussion of other interactions is included. For the production of neutral pions the $\pi^- p$ data are also used.

In the next section we summarise the data on two-particle correlation integrals for charged pions and explain them in terms of two-component model. The value of using the

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correlation integrals instead of moments of distribution for model testing is shown in a simple example. The energy dependence of the average multiplicity is also discussed.

In the third section we discuss three-particle integrals and different possible versions of two-component model which may describe them. The fourth section is devoted to the analysis of correlations between neutral and charged pions. The procedure used to compare $\pi^- p$ and pp data is described in the Appendix. We conclude with the last section.

Throughout the paper we use the two-component model of particle production, based on the ideas of Feynman [8] and Wilson [9] and described in Ref. [10], to which we refer for all details and derivations of formulae. A short summary of the model is given in the second section.

2. Two-particle correlations between charged pions

The two-particle correlations provide a very useful test of many models, which describe satisfactorily one-particle distributions, but differ substantially in predictions of correlation effects. Unfortunately, the differential data exist for rather low energies [11] where the energy-momentum conservation law determines to the large extent the shape of the distribution functions (the Uncorrelated Jet Model describes correctly the large part of data [19]).

The existing high-energy data [12, 13] are obtained for all charged particles, instead of one definite kind, which makes the comparison with lower energy data very difficult. (Moreover, log tan θ variable is used instead of rapidity). Thus the correlation integrals are the most important source of information about energy dependence and relative size of correlation effects. As mentioned above, we concentrate on the negative pion distribution from proton-proton integration. The formula for two-particle correlation integral is [7]

$$f_2 = \iint dx_1 dx_2 (\varrho_2(x_1, x_2) - \varrho_1(x_1)\varrho_1(x_2)) = \langle n(n-1)\rangle - \langle n\rangle^2 = D^2 - \langle n\rangle, \qquad (1)$$

where x stands for any kinematic variable (*e. g.* rapidity), ϱ_1 and ϱ_2 are single and double distribution functions, n denotes the number of particles of given kind and $\langle\ \rangle$ the average over the distribution. Note that for pions of a given charge f_2 is always negative if there are no correlations but those coming from conservation laws; it was the original motivation of choosing $n = n_-$ rather than $n = n_{ch}$.

The experimental data for energies between 10 and 300 GeV/c [14–18] are shown on Fig. 1. We see the unambiguous increase towards positive values. Note that f_2 increases faster than linearly with increasing $\langle n\rangle$. The data of Fig. 1 were extensively studied by different authors [1–6]. It is known that they suggest the necessity of long-range correlations, ruling out the independent emission model [19], simple multiperipheral [20] and other short-range correlation models, in which f_2 should be linear in n starting from rather low energies. It was claimed [3] that the analysis of moment $\langle n(n-1)\rangle$ does not allow to discriminate between s behaviour (predicted by fragmentation model) and $\ln^2 s$ behaviour of absorptive multiperipheral or two-component model. As we see in Fig. 1 the analysis of f_2 as a function of $\langle n\rangle$ allows to rule out the possibility of fitting simultaneously $\langle n \rangle$ by $a \ln s + b$ and $\langle n(n-1) \rangle$ by $c\sqrt{s}+d$ (the corresponding $f_2(\langle n \rangle)$ is shown as a broken line). This is because the errors of f_2 are much less than those of $\langle n(n-1)\rangle$ and $\langle n \rangle^2$ due to statistical correlations between moments. It supports our opinion that choice of f_2 provides the best possibility of testing the models. Of course the data do not exclude the possibility of $\sqrt{s}$ term in f_2 at ultra high energy: this term however cannot be dominant up to 200–300 GeV. The solid curve is obtained from the two-component model [10]. In this model one assumes that the particles are produced either by "diffraction" or "pionisation". The first mechanism is responsible mainly for low and the second for high multiplicities. In the version we use [5] the "diffraction" is energy independent

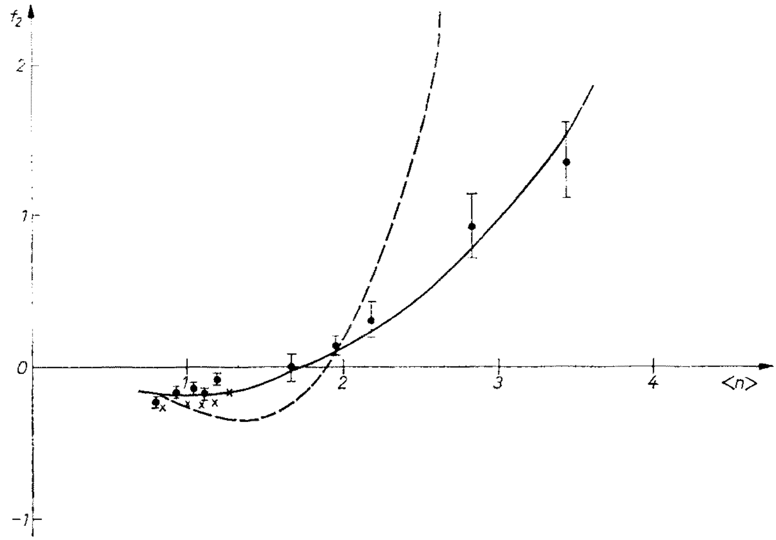

Fig. 1. Two-particle correlation integral for negative pions f_2 as a function of average multiplicity $\langle n \rangle$. The data are from Ref. [14–19], the x's denote the data of Ref. [14] in the normalisation of Ref. [11]. The solid curve is from two-component model [15], and the broken line results from fragmentation-type fit to $\langle n_{ch} \rangle$ and $\langle n_{ch}(n_{ch}-1)\rangle$ of Ref. [3]

and confined to two lowest multiplicities and "pionisation" is the independent emission of particles characterised by logarithmic increase of average multiplicity with energy and no correlations but those coming from conservation laws [19]. The only free parameters are the ratio of "diffractive" to "non-diffractive" part of total cross-section α_D/α_π and average multiplicity in "diffraction" $\langle n \rangle_D$. The relevant formula reads

$$f_2 = \alpha_D f_2^\pi + \alpha_\pi f_2^\pi + \alpha_D \alpha_\pi (\langle n \rangle_\pi - \langle n \rangle_D)^2 =$$

$$= -\alpha_D \langle n \rangle_D^2 + \alpha_\pi f_2^\pi + \frac{\alpha_D}{\alpha_\pi} (\langle n \rangle - \langle n \rangle_D)^2, \tag{2}$$

where f_2^π is a known function of $\langle n \rangle$ and $\alpha_D + \alpha_\pi = 1$.

As we can see, asymptotically $f_2/\langle n \rangle^2 \to \alpha_D/\alpha_\pi$, which enables us to fix this parameter; the value of $\langle n \rangle_D$ is chosen 0.33; the fit is not very sensitive to this value. It appears that the same values of parameters which fit formula (2) describe also quite satisfactorily the differential data [11] for correlation function at 21 GeV/c [5]. Since the model is certainly oversimplified, we regard this result mainly as the confirmation of conservation laws dominance. However, the asymptotic behaviour of $\bar{f}_2$ given by

$$\bar{f}_2 = \frac{f_2}{\langle n \rangle^2} = A + \frac{a}{\langle n \rangle} \tag{3}$$

seems to be in very good agreement with experimental data (as shown in Fig. 2) if $A = \alpha_D/\alpha_\pi = 0.28$ and $a = -0.48$[1]. This corresponds to 22% of diffraction in total inelastic cross-section in reasonable agreement with other estimations [42]. Note that the fit was performed without using the 300 GeV point [18] ($1/\langle n \rangle = 0.29$) and 10^4 GeV cosmic ray point

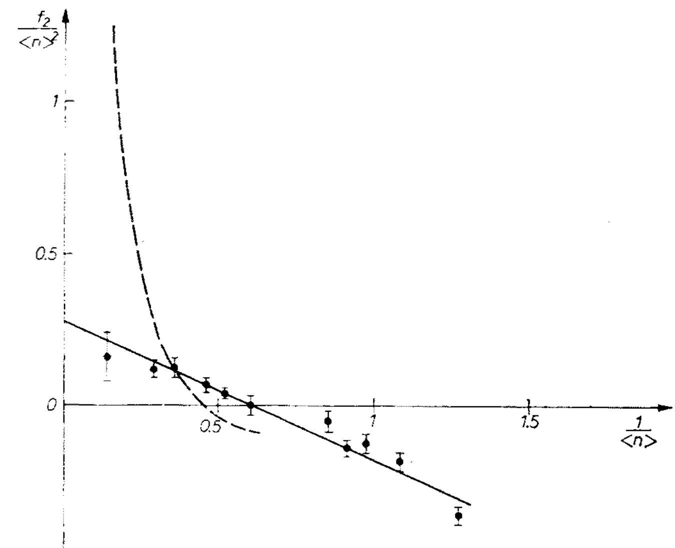

Fig. 2. Normalized correlation integral $\bar{f}_2 = \frac{f_2}{\langle n \rangle^2}$ as a function of $\langle n \rangle$. The solid line is a two-component model fit (3), and the broken line results from nova model [41] normalized to the 200 GeV point

[21] ($1/\langle n \rangle = 0.14$). The model provides the prediction at any energy to be compared with new data. This prediction is contrasted with fragmentation-type fit [41] which was normalised arbitrarily to 200 GeV point [17] ($1/\langle n \rangle \approx 0.35$). The rapid rise of $\bar{f}_2$ expected in this model seems to disagree with highest energy points.

In addition, let us discuss briefly the energy dependence of the average multiplicity of negative pions. In the two-component model one can use the simple formula

$$\langle n \rangle = \alpha_D \langle n \rangle_D + \alpha_\pi \langle n \rangle_\pi . \tag{4}$$

[1] These values are slightly different from those of Ref. [5]. This is because we used here to fit the corrected Serpukhov 50 and 70 GeV/c data [15], new NAL 100 GeV/c point [16] and different normalisation of Smith data [19] given in Ref. [22] (instead of [11]). This change is completely irrelevant for all our conclusions.

Asymptotically we have $\langle n \rangle_\pi \sim \ln s$, as mentioned above. Using the simple independent emission model (IEM) [19] for $\langle n \rangle_\pi$ we are also able to predict the energy dependence in subasymptotic region. Since α_D, α_π and $\langle n \rangle_D$ are fixed, we are left with definite prediction for $\langle n \rangle$. The obtained values are slightly too low. Since $\langle n \rangle_D$ is not well determined, we increased it to 0.7 to get the very good fit, shown in Fig. 3 as a solid line. Note that the IEM calculations are not normalised to fit formula (4); it is still certainly possible to improve the agreement. Our result is not strongly dependent on the value of $\langle n \rangle_D$. To show this, we calculated $\langle n \rangle$ using $\langle n \rangle_D = 0.3 \langle n \rangle$ (from the reasons to be explained in the next section).

The resulting broken curve is not very different from the previous prediction. Let us repeat once more that the IEM results are only approximate and the corresponding un-

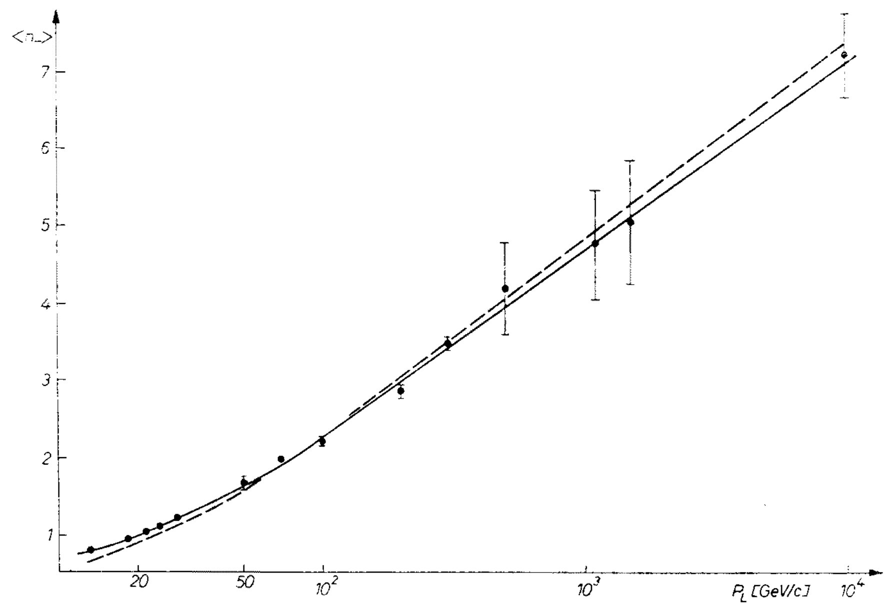

Fig. 3. Energy dependence of the average multiplicity. Solid (broken) curve results from two-component model with non-diffractive part described as independent emission [19] and $\langle n \rangle_D = 0.7$ ($\langle n \rangle_D = 0.3 \langle n \rangle$)

certainty is rather bigger than the experimental errors. Thus the agreement of such simplified two-component model with average multiplicity and two-particle correlation data is surprisingly good.

The reason for this may be the cancellation between two neglected effects: (*i*) energy dependence of "diffraction" suggested by observed rapid decrease of the low-multiplicity cross-section and (*ii*) possible positive dynamical correlations in the non-diffractive part of distribution. In other words, the first two terms in formula (2) are probably too negative, and the third positive term rises too rapidly, resulting in the correct net result. The energy dependence of "diffraction" may be helpful to avoid the unobserved double-peaking of multiplicity distribution at NAL energies while the existence of positive short-range dynamical correlation is strongly suggested by the observed maximum of correlation function at $y_1 = y_2$ instead of $y_1 = 0$ ($y_2 \neq 0$) for ISR energies [13]. However, the formula (3) is not affected by the possible modifications of model including these effects, although the meaning of A may change. It will be discussed in the next section.

We conclude that the two-component model describes correctly the energy dependence of average mutiplicity and the behaviour of two-particle correlation integral in the whole investigated energy range. It should be contrasted with the fragmentation-type asymptotic parametrisation [3], which seems not to work below 200 GeV/c.

3. *Three-particle correlations*

There exists no triple-differential data on particle production. Thus our knowledge of three-particle correlations is confined to the twice-integrated correlation function [22] at rather low energy, (showing practically only the conservation laws effects) and to the correlation integral. The energy dependence of three-particle correlation integral provides much clearer test of models than two-particle case, since now even the sign of predicted correlations is different. As pointed out by Le Bellac [6] the leading term in two-component model is negative (and proportional to $\ln^3 s$), whereas in fragmentation model [23–26] asymptotically f_3 is positive and proportional to s. The formula defining f_3 reads [7]

$$f_3 = \langle n(n-1)(n-2)\rangle - 3f_2\langle n\rangle - \langle n\rangle^3 = \langle n(n-1)(n-2)\rangle - 3\langle n\rangle\langle n(n-1)\rangle + 2\langle n\rangle^3. \quad (5)$$

It is easy to derive the formula decomposing f_3 in two-component model. By the procedure analogous to that used for f_2 [10] we get

$$f_3 = \alpha_D f_3^D + \alpha_\pi f_3^\pi + 3\alpha_D\alpha_\pi(\langle n\rangle_\pi - \langle n\rangle_D)(f_2^\pi - f_2^D) + \alpha_\pi\alpha_D(\alpha_\pi - \alpha_D)(\langle n\rangle_D - \langle n\rangle_\pi)^3 =$$

$$= \alpha_D f_3^D + \alpha_\pi f_3^\pi + 3\alpha_D(\langle n\rangle - \langle n\rangle_D)(f_2^\pi - f_2^D) - \frac{\alpha_D}{\alpha_\pi}\left(1 - \frac{\alpha_D}{\alpha_\pi}\right)(\langle n\rangle - \langle n\rangle_D)^3. \quad (6)$$

As we can see, the last dominant term is indeed negative if $\langle n\rangle > \langle n\rangle_D$ and $\alpha_\pi > \alpha_D$. Moreover, the asymptotic value of $f_3/\langle n\rangle^3$ is fixed by previous analysis of two-particle correlations. The experimental data are shown on Fig. 4. Up to 200 GeV they show the expected decrease towards big negative value. As pointed out by Berger [3] they are difficult to explain in fragmentation-type model — the only simple explanation states that the energy is too low to study the asymptotic behaviour. The 300 GeV point deviates strongly from previous smooth behaviour. It is not clear, if this is simply the statistical deviation allowed by very large error, or the indication of beginning rise predicted by fragmentation model. To discuss it more quantitatively, let us consider the auxilliary function

$$\frac{f_3}{\langle n\rangle^3} = B + \frac{b}{\langle n\rangle}. \quad (7)$$

In the framework of the simple two-component model considered in the previous Section, the value of B may be calculated from formula (6) and appears to be equal to -0.20.

The data up to 200 GeV/c may by fitted by formula (7) with this value of B and $b = 0.30$, the 300 GeV/c point, however, deviates very strongly (by 5 standard deviations). This suggests again the energy dependence of $\langle n \rangle_D$[2]. We can see easily that this correction makes the two- and three-particle fits compatible. Indeed, let us take $\langle n \rangle_D = \beta \langle n \rangle$. The form of the fits (3) and (7) does not change, the meaning of A and B is however different. We have

$$A = (1-\beta)^2 \frac{\alpha_D}{\alpha_\pi}, \tag{8}$$

$$B = -(1-\beta)^3 \frac{\alpha_D}{\alpha_\pi}\left(1 - \frac{\alpha_D}{\alpha_\pi}\right). \tag{9}$$

Adopting the value of $\beta = \frac{1}{4}$ as suggested by $\langle n \rangle_D$ value fitting the correlation function at 21 GeV/c, we get from fitted A the value of -0.03 for B. It is in quite good agreement with all the values of $f_3/\langle n \rangle^3$ above 25 GeV/c, suggesting $b \approx 0$ in formula (7). Both fits and data are shown in Fig. 5.

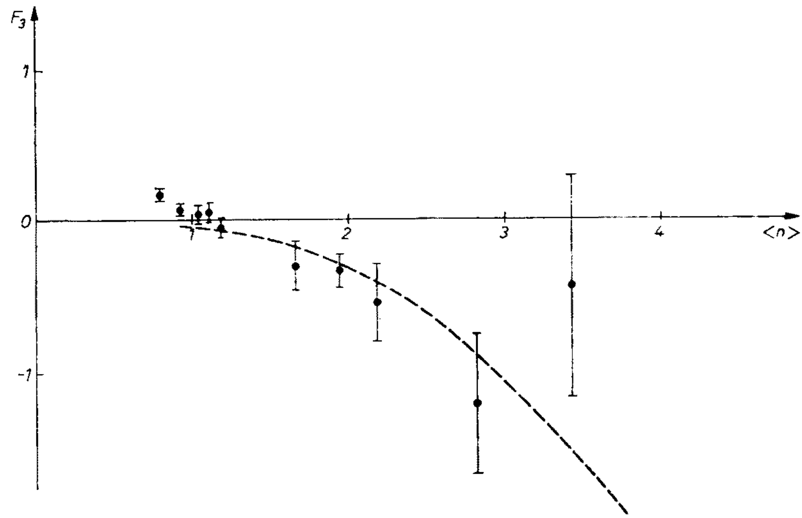

Fig. 4. Three-particle correlation integral f_3 as a function of $\langle n \rangle$. The $-0.04 \langle n \rangle^3$ line is drawn to guide the eye

The simple description of data presented above is certainly in disagreement with initial ideas of almost constant "diffraction" part of interaction. If the further experiments will confirm the relatively strong energy dependence of low multiplicity cross-section, it will be however necessary to adopt this change. It should be stressed that these experimental results are much more embarrassing for "pure" diffractive fragmentation models, where the energy-independent distribution at moderate multiplicities is the basic feature. The

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[2] The simplest explanation is obviously to say that the energy is not high enough to neglect further terms in (7). Since we have at the moment no reliable models for $f_2^{\pi,D}$ and $f_3^{\pi,D}$ we cannot prove or disprove this statement. Thus we discuss other possibilities.

two-component idea, however, is not necessarily connected with initially proposed identification of two competing mechanisms. Two different classes of multiperipheral interactions may be perhaps responsible for the observed experimental features.

The parametrisation above gives for α_D/α_π the new value of 0.64, resulting in 38 % of "diffraction" in total cross-section. It means that "pionization" is still dominating. This result is valid unless β is bigger than $\frac{1}{2}$. If this condition is not fulfilled, the asymptotic sign of f_3 may be reversed. It should be stressed that the formulae (3) and (7) are not the most general cases in the two-component model. They were derived assuming no long-range

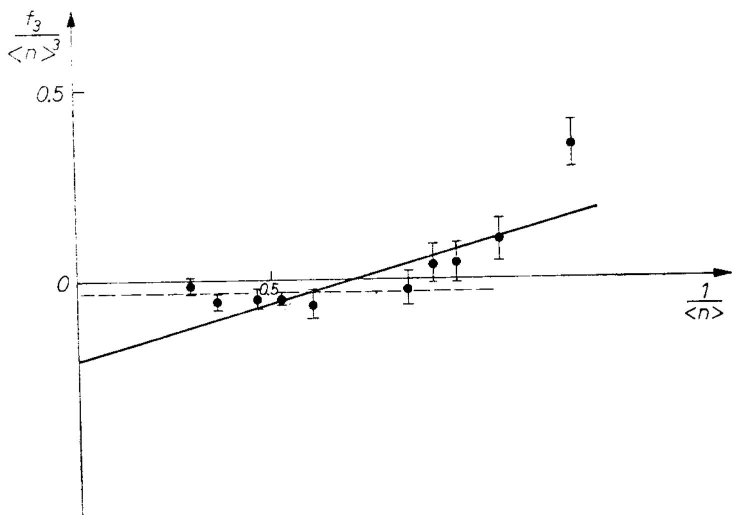

Fig. 5. Normalized correlation integral $f_3/\langle n\rangle^3$ as a function of $\langle n\rangle$. Solid (broken) line is a fit of two-component model with $\langle n\rangle_D = 0.33$ ($\langle n\rangle_D = 0.33\ \langle n\rangle$)

correlations within each particular mechanism. If we assume the nova-type description of the "diffraction" part of interaction, we get $\langle n\rangle_D \sim \ln s$ (as before), but $f_2^D \sim \sqrt{s}, f_3^D \sim s$, and the corresponding terms will dominate asymptotically formulae (2) and (6) even if "diffraction" will form only a small part of total cross-section. In this case, the asymptotic predictions of the model for correlation integrals may be the same as the predictions of fragmentation models. Note, however, that if energy is large enough, it must lead again (as in nova model) to the nonvanishing constant limits of low-multiplicity cross-sections and long $1/n^2$ tail for highest multiplicities. It is obvious that the existing experiments do not exhibit these features. We must wait for higher energy experiments to prove or disprove such possibility.

Another possibility may be connected with logarithmically decreasing "diffractive" cross-sections, yielding in average multiplicity $\langle n\rangle_D \sim \ln \ln s$. Since we have now at least two new free parameters, it is possible to fit f_2 and f_3 data preserving the asymptotic values of (3) and (6) fitted from two-particle distribution. This seems to be however not very attractive idea. In addition, the logarithmic decrease of low multiplicity channels is apparently experimentally too slow.

To conclude, we have shown that the three-particle correlation data up to 300 GeV are in strong disagreement with the asymptotic behaviour predicted by fragmentation models. The two-component picture is obviously more general that the simplified model used to describe two-particle correlations and may be easily adopted to fit the three-particle data.

The natural way of modifying the model seems to be to introduce the energy dependence in "diffractive" average multiplicity in agreement with other suggestions. It remains an open question, if there is any need of introducing energy dependence for ratio of "diffractive" and "non-diffractive" cross-sections and/or long-range correlations within "diffrac ive" mechanism. The eventual confirmation of sign change of f_3 may prove the necessity of such modifications.

Finally let us discuss shorthly the other existing data. The π^+p and K^+p results [27, 28] are roughly similar to the "low energy" (*i. e.* conventional accelerators) pp data [22]. The only major difference for π^+p case is the higher average multiplicity at the same energy resulting in stronger effect of conservation laws and, consequently, more negative f_2 (positive f_3). For the π^-p scattering there is a difficulty when comparing preceding reactions, since one of the observed negative pions may be the initial one (and not "produced"). Since the π^-p data are the best available for neutral particle distribution analysis, this problem is discussed in detail in the Appendix. It is possible to get the rough agreement between π^+p and π^-p data if we choose the simple procedure for "subtracting" the initial pion. The remaining two reactions, K^-p and $\bar{p}p$ are more difficult to compare, since it is no longer true that almost all negative particles are pions. Since there are no very high energy inclusive data for these reactions, we do not discuss them here.

4. *Correlations between charged and neutral pions*

The multiplicity distributions of neutral pions is much less known than that of charged pions due to the obvious experimental difficulties. There exist two high-energy measurements of π^0 distribution in pp interaction at 19 GeV/c [29] and 203 GeV/c [30]. The 12.3 GeV/c results [31] have no normalization, so we do not use them here. Let us note, however, that they are compatible with 19 GeV/c data except of the value of average π^0 multiplicity in 2-prong events, which seems to be overestimated in 19 GeV/c data. All the experiments give the values of average π^0 multiplicity at given number of charged particles, which will be donoted as $\langle n_0\rangle_{n_-}$. The data are shown at Fig. 6.

At 19 GeV/c and 12.3 GeV/c no clear dependence of $\langle n_0\rangle_{n_-}$ on n_- is seen, while at 200 GeV/c the increase of $\langle n_0\rangle_{n_-}$ with increasing n_- for moderate n_- is apparent. The π^-p data at 25 [32] and 40 GeV/c [33] indicate the intermediate shape (Fig. 7).

These results were used for comparison with different charge-distributions models for Poisson-type and $1/n^2$-type distributions for global multiplicities [34]. It was found that the observed increase is more natural for $1/n^2$-type distributions, although for the "$I = 1$ pair production" model both distributions explain qualitatively the data. The very poor accuracy of data does not allow to draw any more definite conclusions.

We investigate the π^0 data in more quantitative way. First, it should be stressed that the 203 GeV/c measurement does not prove the increase of $\langle n_0\rangle_n$ with n_-; in fact, the

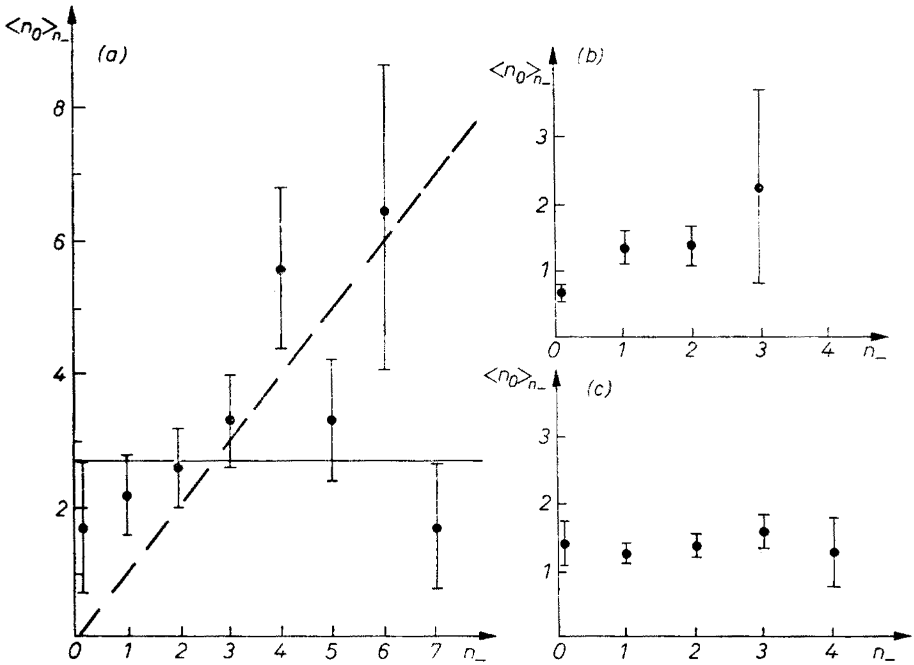

Fig. 6. Average multiplicity of neutral pions $\langle n_0 \rangle_{n_-}$ as a function of charged particle multiplicity in *pp* collisions; a) at 205 GeV/*c* [30]. The broken line is $\langle n_0 \rangle_{n_-} = n_-$ and the solid line $\langle n_0 \rangle_{n_-} =$ const., b) at 12.3 GeV/*c* [31] (with arbitrary normalization), c) at 19 GeV/*c* [29]

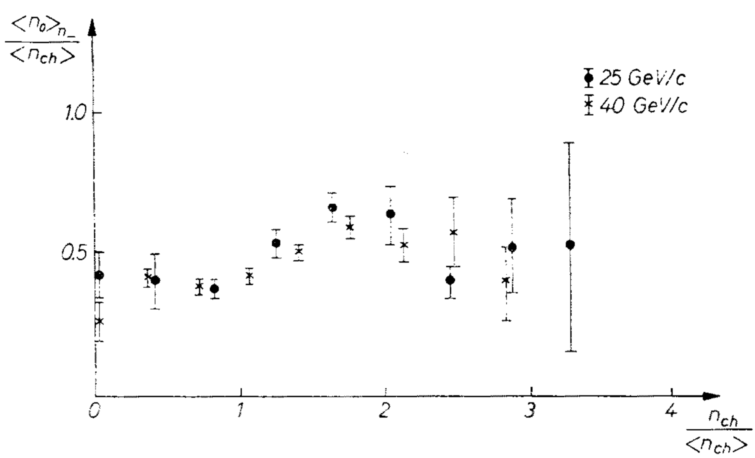

Fig. 7. Normalized average multiplicity of neutral pions $\frac{\langle n_0 \rangle_{n_-}}{\langle n_{ch} \rangle}$ as the function of normalized charged multiplicity $\frac{n_{ch}}{\langle n_{ch} \rangle}$ [44] in 25 GeV/*c* [32] and 40 GeV/*c* [33] $\pi^- p$ interactions

hypothesis $\langle n_0 \rangle_{n_-} = \text{const}$ (for all n_-) has higher confidence level, that proposed rough

$$\langle n_0 \rangle_{n_-} = n_- \tag{10}$$

behaviour for $n_- \leqslant 6$ (10% compared with 5%). So that data are certainly not conclusive. We may, however, use the correlation integrals to establish the gross features to be compared with charged pion data. The formula analogous to (1) reads

$$f_2^{0-} = \langle n_0 n_- \rangle - \langle n_0 \rangle \langle n_- \rangle. \tag{11}$$

Knowing $\langle n_0 \rangle_{n_-}$ for all n_- we can easily compute f_2^{0-}. If $\langle n_0 \rangle_{n_-}$ increases with increasing n_- for major part of distribution, f_2^{-0} is positive (positive correlations); if $\langle n_0 \rangle_{n_-}$ decreases, f_2^{-0} is negative. Independent emission of different charges corresponds of course to $f_2^{-0} = 0$. Before analysing the data, let us say what is expected in two-component model. The formula (2) remains almost unchanged

$$f_2^{0-} = \alpha_\pi f_2^{\pi 0-} + \alpha_D f_2^{D0-} + \alpha_D \alpha_\pi (\langle n_0 \rangle_\pi - \langle n_0 \rangle_D)(\langle n_- \rangle_\pi - \langle n_- \rangle_D). \tag{12}$$

We have no reason to expect any difference between $\langle n_0 \rangle$ and $\langle n_- \rangle$, so the last term coincides with that of formula (2). The "diffraction" part is assumed to be small, so the main difference may be in f_2^π. If we assume that the correlations within "pionization" are caused mainly by conservation laws, we get [35]

$$f_2^{\pi--} \simeq -\tfrac{1}{2} \langle n_- \rangle_\pi - C, \tag{13}$$

$$f_2^{\pi-0} \simeq -C \tag{14}$$

and, consequently

$$f_2^{-0} - f_2^{--} \simeq +\tfrac{1}{2} \alpha_\pi \langle n_- \rangle_\pi \simeq +\tfrac{1}{2} \langle n_- \rangle. \tag{15}$$

Thus the quantity

$$\delta = \frac{f_2^{-0} - f_2^{--}}{\langle n_- \rangle} \tag{16}$$

seems to be the most suitable for testing the model predictions. Let us stress that if the pions are emitted in pairs rather than independently, f_2^{--} may be almost independent of $\langle n_- \rangle$ and equal to the negative constant. So for the more realistic description of pionization we expect

$$0 < \lim_{n_- \to \infty} \delta < +\tfrac{1}{2}, \tag{17}$$

where two limiting cases correspond to "pure independent pair production" and "pure independent pion production".

Note that in all cases we expect $f_2^{-0} > f_2^{--}$. Since f_2^{--} increases as $\ln^2 s$ at high energy, so does f_2^{-0} and we obtain positive correlations — $\langle n_0 \rangle_{n_-}$ should increase with increasing n_-, as observed in experiment. The intuitive meaning of this result is obvious: if the number of charged pions is small, the event is probably "diffractive" and the average number of neutral pions is also small as compared with "pionization" events.

We have now however more quantitative test of the model. In Fig. 8 the quantity δ (16) is shown as a function of $\langle n_- \rangle$ for pp and "corrected" $\pi^- p$ data (see Appendix). We see that the condition (17) seems to be fulfilled, although data are too poor to draw definite conclusions. Note that the $\pi^- p$ data are not very sensitive for the applied correction for "initial" pion.

The prediction (17) should be contrasted with the prediction of most of models of Ref. [34], where $\delta \to 1$ for most cases if $1/n^2$ distribution is used. The value of 1 follows

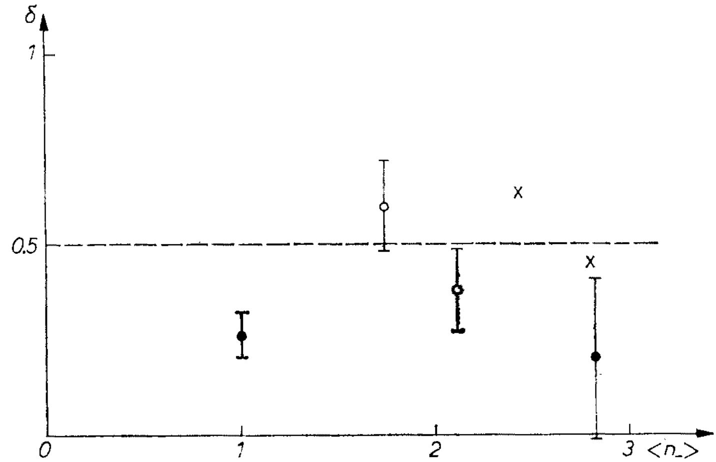

Fig. 8. The relative difference between correlation integrals $\delta = \frac{f_2^{-0} - f_2^{--}}{\langle n_- \rangle}$ as a function of average multiplicity. The full (open) circles are for pp ($\pi^- p$) interactions. Uncorrected $\pi^- p$ data are also marked as x's

also from the simple assumption (10), which seems to be the most natural way of realizing statistical charge distribution in fragmentation-type models. The finite-energy corrections may, however, diminish the value of δ, so the apparent disagreement between data and model is not very serious.

We conclude that the general features of correlations between neutral and charged pions are satisfactorily explained in the framework of two-component model. The model provides also rough predictions at any energy based on the previous analysis of charged pion distribution. The data are, however, certainly insufficient to discriminate between different classes of models.

It should be stressed that in our model the apparent absence of correlations between neutral and charged pions at low energy is accidental, and results from cancellations between negative energy-momentum conservation effect and positive term caused by two-mechanism interplay. It is an exact analogue of "Poissonian" distribution of negative pions at energies about 50 GeV. Here the cancellation occurs at lower energy, since the negative correlations are weaker (charge conservation law is not important). Thus the apparent "independence" of production of negative and charged pions occurs only in small range of energies. It is quite possible, however, that the correlations between negative and charged pions within one definite mechanism (pionisation) are negligible. The main effect for the global distribution at high energy is, as for charged pions, the two-mechanism interplay.

5. *Conclusions*

We have investigated the correlation integrals for charged and neutral pions produced in high energy inclusive processes. We have re-examined the previous analysis of two-particle correlations between charged pions and supplemented this by discussion of three-particle correlations and correlations between charged and neutral pions. The dependence of correlation integrals on average multiplicity appears to provide very useful test for different models of particle production. Due to statistical correlations between errors of different parameters, the correlation integrals allow for much better testing, than the alternatively used moments of distribution.

We discussed the data in terms of two-component model. We found that the over-simplified version of the model compatible with two-particle correlation data and the energy dependence of average multiplicity needs some improvements to fit the three-particle correlations. In particular, the energy dependence of "diffractive" part of interaction, suggested already by other experimental facts, seems to be welcomed. We discussed also other possible versions of the model.

The correlations between charged and neutral pions are easily understood in the framework of two-component model. They suggest again that the possible "dynamical" short-range correlations are dominated by the effects of two-mechanisms interplay. Some "pairing" of produced particles seems to be, however, very likely.

It should be stressed, however, that the meaning of two-component model in the framework of Mueller's generalised optical theorem [36] is far from being clear. Perhaps it may be related to the generalised Harari-Freund [37] decomposition in the dual model of production [38]. Another possible identification of two competing classes of events may be provided by absorptive multiperipheral model [39], the correspondence is however rather obscure. In both cases it seems to be difficult to keep the energy independent ratio of two mechanisms.

Nevertheless the possibility of simple description of many-particle distributions and their integrals in terms of single distributions and average multiplicity seems to be very attractive. Let us stress once more that in the first approximation we do not introduce any other correlation effects but these caused by conservation laws. All the dynamics is contained in the assumed decomposition into two different mechanisms of production. Thus we regard the model as the simplest possibility of explaining the data.

Finally, let us mention the similarity between the intuitive meaning of positive correlations in two-component and fragmentation model. In both cases the correlations are caused by two-step description of production. If the first pion (usually slow in CM system) was detected, the event was more likely non-diffractive (produced fire-ball was heavy) resulting in increasing expected multiplicity of further pions. The only major difference is in describing these high-multiplicity events as the separate class with strongly energy dependent cross-section in the two-component model, while in fragmentation model they form the tail of continuous distribution and have asymptotically constant cross-sections. This difference results in different asymptotic energy dependence of correlations.

We conclude that the present data seem to support the necessity of non-diffractive part of particle productions at the energies, up to 300 GeV. In fact, this non-diffractive part seems to be dominant. It remains an open question if asymptotically this picture will change. The further investigations of particle spectra and multiplicity distributions may help to resolve this problem.

APPENDIX

In the $\pi^- p$ reaction one pion is already present in the initial state, enlarging the number of observed pions in the final state. To see the effect of this contamination, let us assume for the moment that the initial pion cannot change sign during the interaction. Then for the produced pions we have

$$\langle n_- \rangle = \langle n_- \rangle_e - 1, \quad \langle n_0 \rangle = \langle n_0 \rangle_e,$$
$$f_2^{--} = f_{2e}^{--} + 1, \quad f_2^{-0} = f_{2e}^{-0}, \tag{A1}$$

where index e denotes the experimentally measured quantities, and unindexed parameters refer to the distribution of produced pions. The assumption above is certainly unrealistic, since we know that there are charge-exchange reactions. We can assume, however, that the fractions of cases in which initial particles survive (α) and those where their charge annihilates $(1-\alpha)$ is independent on the number of produced particles

$$\frac{\sigma(\pi^- p \to \pi^- p + n_{ch})}{\sigma(\pi^- p \to \pi^0 n + n_{ch})} = \frac{\alpha}{1-\alpha} \neq f(n_{ch}). \tag{A2}$$

This assumption is not quite obvious. In fact, it appears that using it we are usually able to determine uniquely α from the known cross-section $\sigma(\pi^- p \to \pi^0 n + 0_{ch}) = \sigma_{0p}$ and the condition of positivity for cross-sections. Since the results depend strongly on poorly measured σ_{0p} and change significantly from experiment to experiment, this derivation is however not very reliable. Moreover, the distribution of produced negative pions from 16 GeV/c $\pi^- p$ data [40] obtained after applying the discussed correction with α fitted from (A2) is not compatible with the distribution from $\pi^+ p$ data, suggesting the necessity of n_{ch} dependence in α. We hope however that this dependence is not very crucial. From (A2) we get

$$\langle n_- \rangle = \langle n_- \rangle_e - \alpha, \quad \langle n_0 \rangle = \langle n_0 \rangle_e - (1-\alpha),$$
$$f_2^{--} = f_{2e}^{--} + \alpha, \quad f_2^{-0} = f_{2e}^{-0} + \alpha(1-\alpha). \tag{A3}$$

The values of α obtained from (A2) at different energies and the value giving the distribution parameters compatible with π^+p data are all in the range 0.5–0.9. We choose for the comparison with *pp* data the average value of 0.7. It appears that the quantity used for neutral pion distribution

$$\delta = \frac{f_2^{-0} - f_2^{--}}{\langle n_- \rangle} = \frac{f_{2e}^{-0} - f_{2e}^{--} - \alpha^2}{\langle n_- \rangle_e - \alpha} \tag{A4}$$

depends rather weakly on α in the discussed range. Thus we believe the results corrected according to (A3) may be really compared with the *pp* data, increasing largely the amount of our information about neutral pion production.

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