ON THE MULTIPLICITY OF NEUTRAL PIONS IN HIGH ENERGY COLLISIONS

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We study the experimental characteristics of the π^0 production in high energy π^-p and pp collisions. The observed features of charged multiplicity distribution and neutral pion production are described by a phenomenological model which assumes a Czyżewski-Rybicki formula for the total multiplicity distribution and the hypothesis of isospin independence.

The understanding of multiparticle production requires the knowledge of both charged and neutral particle multiplicities. Recently some new data on π^0 production became available, but the experimental information on neutral particles is still far less complete than on the charged ones.

1. *Average multiplicity of π^0's*

The average π^0 multiplicity for fixed number of charged particles produced has been determined in π^-p collisions at 9.9 [1], 25 [2] and 40 [3] GeV/c, and in pp collisions at 19 [4] and 205 GeV/c [5]. At the ISR energies corresponding to 500, 1000 and 1500 GeV/c only the overall average π^0 multiplicity has been measured [6]. In addition, at 1500 GeV/c the dependence of the average number of π^0 on the charged multiplicity has been measured within limited solid angle around $\theta = 20^0$ [7].

The aim of this paper is to describe the distribution of all produced particles, charged and neutral, in a way consistent both with the data on charged particles and the observed characteristics of π^0 production. In Fig. 1 we compare the energy dependence of the multiplicity of charged and neutral pions in pp and π^-p collisions (Ref. [1]-[7]). The average number of charged pions is calculated as $\langle n_{\pi ch}\rangle = \langle n_{ch}\rangle - \langle n\rangle_{protons}$, taking $\langle n\rangle_{pr} = 0.5$

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for $\pi^- p$ and $\langle n \rangle_{pr} = 1.4$ for pp collisions [9]. In the whole energy range studied the average number of charged pions is equal, within the errors, to twice the average number of π^0's, both for pp and $\pi^- p$ collisions.

A more detailed information is contained in Fig. 2, which shows the dependence of the average number of π^0's, on the number of negative particles produced in $\pi^- p$ and pp collisions at different incident energies. For lower energies the dependence is essentially flat, and becomes steeper with increasing energy.

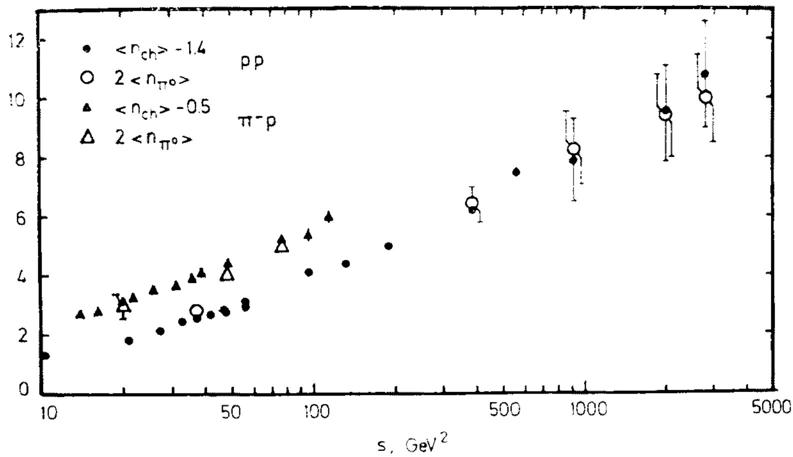


Fig. 1. Average number of charged and neutral pions as a function of ln s, for $\pi^- p$ and pp collisions

We have put $\langle n_{\pi^0} \rangle = an_- + b$ as a first approximation and fitted a and b parameters at each energy. Fig. 3 shows the dependence of the slope, a, on the average π^0 multiplicity (this variable is more convenient than s when comparing pp and $\pi^- p$ data). The last point in Fig. 3 represents the 1500 GeV/c ISR data taken within limited solid angle.

It is perhaps worth noticing that the slope, a, depends essentially on $\langle n_{\pi^0} \rangle$ — thus on the energy, and is similar for pp and $\pi^- p$ at similar energies.

2. *Models for the distribution of the number of neutral and charged particles produced*

The observed characteristics of π^0 and π charged production can be accounted for by the following empirical model. For the distribution of the number of particles produced $n_{tot} = n_{pr} - n_{\pi^\pm} + n_{\pi^0} + n_{neutrons}$, (neglecting K and Y production) we assume the Czyżewski-Rybicki (CR) formula:

$$P(n) = \frac{d}{D} e^{-d^2} \frac{d^{2d(n - \langle n \rangle + dD)/D}}{\Gamma[d(n - \langle n \rangle + dD)/D + 1]}, \quad n \geqslant 2,$$

where $P(n)$ is the probability for a given number of particles, D is the dispersion, $D = (\langle n^2 \rangle - \langle n \rangle^2)^{1/2}$, and d is a free parameter.

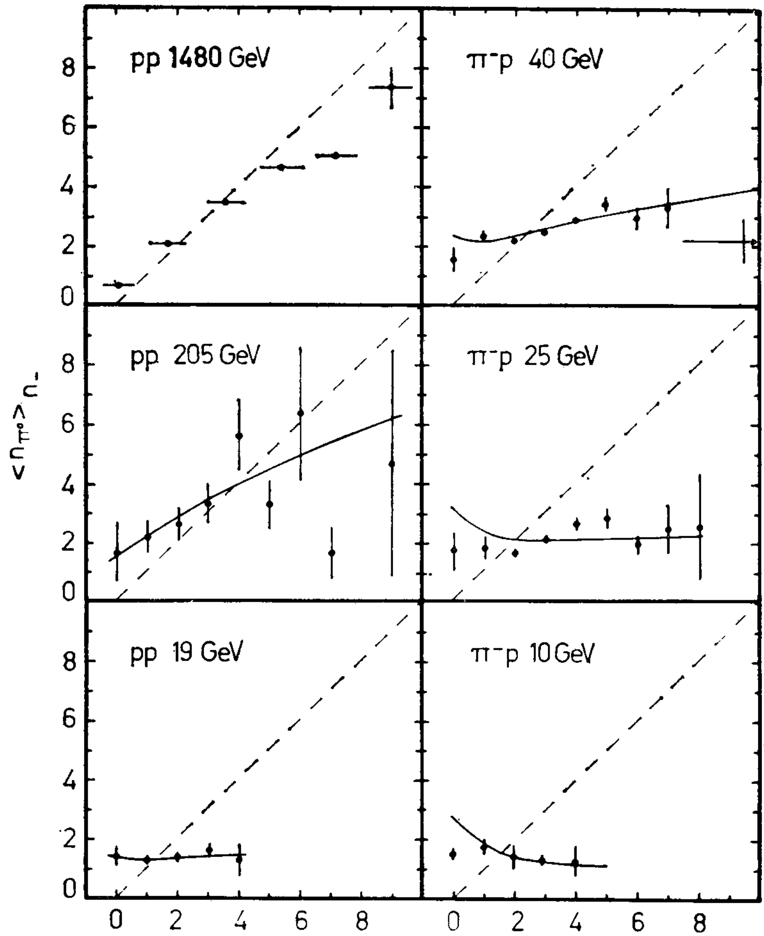


Fig. 2. Average number of neutral pions as a function of the number of negative particles produced in π^-p and pp collisions at different energies. Continuous lines are predictions of the CR+II model, described in the text. Dashed line corresponds to $\langle n_{\pi^0} \rangle = n_-$

We have put $d = 1.8$[1], as it was previously found to provide a good description of charged multiplicity distributions in π^-p and pp collisions at various energies [10]. Then the hypothesis of the isospin independence (II) [14] allows us to project the full multiplicity distribution onto the charged multiplicity distribution, fitting the parameters $\langle n_{tot} \rangle$ and D_{tot} so as to describe well the experimentally observed charged distribution. (We shall call this model CR+II.)

Table I illustrates how well are the charged distributions described by this method. For most of the data the probability of the fit is $> 5\%$, thus giving a reasonable description.

[1] This value was proposed by authors of Ref. [10]. In the latest version of the model [11] they take $d = 1.7$ for pp and $d = 2.2$ for π^-p. However, the formula is not very sensitive to the value of d.

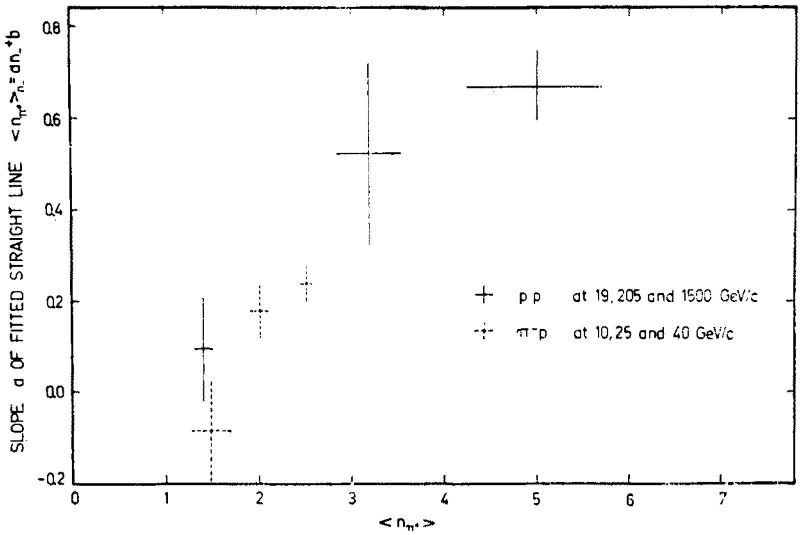


Fig. 3. Slope a of the dependence $\langle n_{\pi^0}\rangle = an_{-}+b$, shown as a function of the average number of π^0's produced in $\pi^- p$ and pp collisions. In the fit of the parameters a and b the last point (for highest n_-) is always omitted (two last points for 205 GeV pp)

TABLE I

	P_{lab} GeV/c	Ref.	$\langle n_{ch}\rangle$	$\chi^2_{prob.}$ for CR+II fit, %	$\langle N_{tot}\rangle$ from CR+II	D_{tot} from CR+II	$\langle n_{\pi^0}\rangle$ from CR+II	$\langle n_{\pi^0}\rangle$ exp.
pp	12.9	[8a]	3.66±.03	42	5.69+.05	1.62+.05	1.21	—
	18.0	[8a]	4.04±.03	2	6.25±.05	1.98±.05	1.40	—
	19.0	[4]	4.02±.02	39	6.17±.04	2.11±.04	1.39	1.36±.12
	21.1	[8a]	4.30±0.4	21	6.63±.06	2.22±.05	1. 3	—
	24.1	[8a]	4.47±.04	10	6.92±.06	2.31±.05	1.63	—
	28.4	[8a]	4.60±.04	0.8	7.03±.07	2.58±.05	1.68	—
	28.5	[8b]	4.58±.07	0.1	7.08±.09	2.41±.05	1.68	—
	35.0	[8c]	5.01±.07	46	7.74±.12	2.89±.14	1.92	—
	50	[8d]	5.32±.11	8	8.14±.19	3.49±.13	2.09	—
	69	[8d]	5.89±.07	5	8.92±.10	4.03±.07	2.37	—
	102	[8e]	6.38±.12	76	9.54±.26	4.59±.20	2.61	—
	205	[8f]	7.65±.17	24	11.68±.21	5.59±.16	3.31	3.19±.32
	303	[8g]	8.86±.16	10	13.15±.24	6.36±.21	3.80	—
$\pi^- p$	8.05	[8h]	3.36±.06	37	5.38±.04	1.47±.03	1.49	—
	9.9	[8i]	3.61±.06	62	5.74±.04	1.72±.03	1.61	1.48±.21
	16.2	[8j]	4.19±.04	12	6.62±.02	2.15±.02	1.9ʋ	—
	18.5	[8h]	4.40±.06	69	6.92±.05	2.14±.03	2.00	—
	25	[2]	4.85±.08	0.04	7.70±.04	2.47±.03	2.26	2.01±.10
	40	[3]	5.62±.04	10	8.74±.06	3.45±.06	2.60	2.51±.06
	50	[8k]	5.82±.14	98	9.03±.20	3.38±.16	2.70	—

With $\langle n_{tot}\rangle$ and D_{tot} fitted, we can calculate any projection of n_{tot} on n_{ch}, n_{π^0}. In particular we predict the average number of π^0's (see Table I) and the dependence of $\langle n_{\pi^0}\rangle$ on n_-, shown as continuous lines on Fig. 2. As it can be seen, the description of the data is rather good, giving a correct estimate of $\langle n_{\pi^0}\rangle$ and reproducing the increase of the slope of $\langle n_{\pi^0}\rangle$ *vs* n_- dependence. It is perhaps worth stressing that the fitting procedure did not use any experimental information on π^0's.

The model predictions for some other energies, for which the data on π^0's are not yet available are shown in Fig. 4.

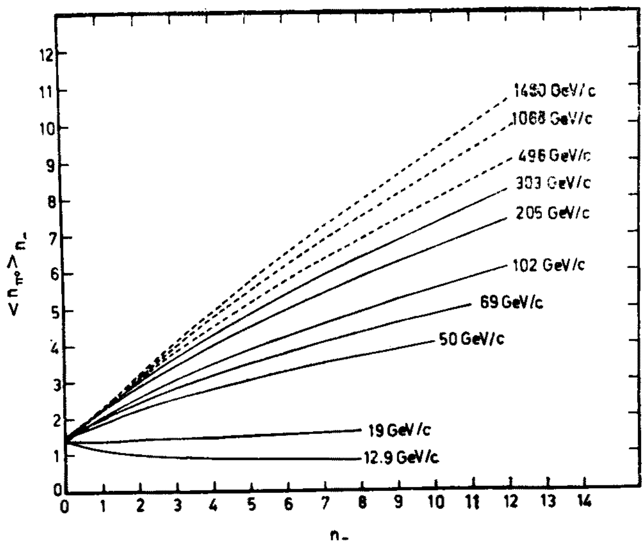


Fig. 4. The CR+II model predictions for the dependence of $\langle n_{\pi^0}\rangle$ on n_- for *pp* collisions at various energies. The dashed lines are for the energies at which the distribution of n_{ch} is not known and we have assumed that D_{ch} can be calculated from the Wróblewski formula (see text).

For the ISR energies there is no published data on the n_{ch} distribution, apart from $\langle n_{ch}\rangle$. We assumed that the dispersion of the charged multiplicity distribution D_{ch} may be obtained from the extrapolation of the Wróblewski empirical formula:

$$D_{ch} = A(\langle n_{ch}\rangle - 1), \quad A = 0.58 \pm .01,$$

found to describe well the charged particle data at lower energies [12]. These two parameters: $\langle n_{ch}\rangle$ and D_{ch} were used to calculate $\langle n_{tot}\rangle$, D_{tot} in the CR+II model. Thus we have complete information for both the charged and neutral particle multiplicity distributions at the ISR energy. We see from Fig. 4 that the dependence of $\langle n_{\pi^0}\rangle$ on n_- becomes steeper for higher energies. It flattens a little for the high values of n_- so that even for the highest energies used in calculation the slope is smaller than 1.

The discussed model does not pretend to give a detailed description of all cross-sections for exclusive reac.ion channels where the reliable data exist — see the Appendix.

The linear dependence of D_{ch} on $\langle n_{ch}\rangle$, found by Wróblewski, suggests that perhaps similar regularity exists for $\langle n_{tot}\rangle$ and D_{tot}. For the moment we can check it only with help of models. Fig. 5 shows D_{tot} *vs* $\langle n_{tot}\rangle$ calculated from the CR+II model. The *pp* data follow a straight line $D_{tot} = 0.68\ \langle n_{tot}\rangle - 2.26$, with $\chi^2/ND = 31/11$. For $\pi^- p$ there are fewer points and it is less clear whether they follow a straight line.

The proposed model is, of course, not unique. We have checked [3] that a Poisson distribution for n_{tot}, still often used, cannot be reconciled with the data if we assume isospin independence. It does not give the observed increase of $\langle n_{\pi^0}\rangle$ with n_- and the projection of n_{tot} on n_{ch} does not agree with experiment.

Another simple possibility of the description of the total particle multiplicity is to build it up from the experimental distribution of n_{ch} and the Poisson distribution of the number of π^0's for each n_{ch}, with $\langle n_{\pi^0}\rangle$ taken from experiment. The distribution of n_{tot} calculated from this model is compared with that resulting from the CR+II model in Fig. 6 (calculated for 205 GeV/*c* *pp* interactions). The shape of both distributions is similar. The experimental errors on $\langle n_{\pi^0}\rangle$ enter into the determination of the curve resulting from Poisson distributions of π^0's for each n_{ch}, and make the apparent two-bump structure insignificant.

3. *Two-particle correlations*

An insight into the problem of particle production mechanism can be gained from the study of the correlation parameters of particle distributions. A two-particle correlation parameter f_2 is defined as

$f_2 = \langle n(n-1)\rangle - \langle n\rangle^2$ for identical particles, and

$f_2 = \langle n_1 \cdot n_2\rangle - \langle n_1\rangle \cdot \langle n_2\rangle$ for different particles.

We have collected the data on particle multiplicities for various energies of $\pi^- p$ and *pp* collisions and calculated the f_2 parameters and their errors for pairs of particles with different charges, namely:

f_{--}, f_{0-} (whenever the data on $\langle n_{\pi^0}\rangle$ *vs* n_- were available), f_{+-}, $f_{\text{any two charged}}$ (we denote it by f_{atc}). These f_2 parameters are not independent; the following relations are easily proved:

$$f_{atc} = 2f_{+-} + f_{++} + f_{--}, \qquad f_{atc} = D_{ch}^2 - \langle n_{ch}\rangle,$$

$$f_{--} = \tfrac{1}{4} D_{ch}^2 - \tfrac{1}{2}\langle n_{ch}\rangle + \tfrac{1}{2} Q, \quad f_{++} = \tfrac{1}{4} D_{ch}^2 - \tfrac{1}{2}\langle n_{ch}\rangle - \tfrac{1}{2} Q, \quad f_{+-} = \tfrac{1}{4} D_{ch}^2,$$

$$f_{0-} = \sum_{n_-} n_- \langle n_{\pi^0}\rangle_{n_-} \cdot \sigma(n_-)/\sigma_{tot} - \langle n_{\pi^0}\rangle\,\langle n_-\rangle = f_{0+};$$

if $\langle n_{\pi^0}\rangle_{n_-} = an_- + b$ then $f_{0-} = a/4\ D_{ch}^2$; Q is the total charge of the initial state. The parameters f_{00} and $f_{\text{any two}}$ (denoted further by f_{at}) cannot be calculated from the available data. We can only predict their values on the basis of models for the total multiplicity distibution actual values of these quantities. For the sake of completeness we include these model-dependent f_2 values in Fig. 7.

The full set of f_2 parameters calculated from the available data is given in Table II, and presented in Fig. 7. From the previous discussion we have seen the merits of the CR+II model. From Table II we can check that the models predictions for f_2 agree well with the experimental points. This gives some support for the assumption that all f_2 parameters, including f_{00} and f_{at} parameters calculated from the model will be close to the

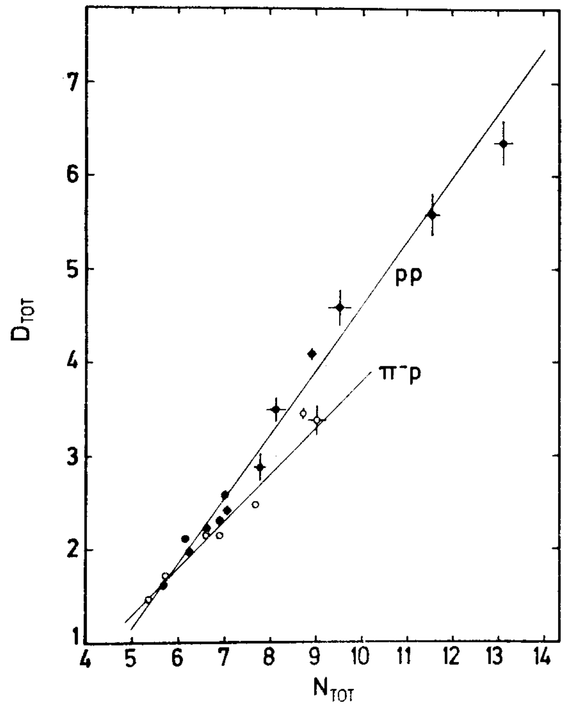


Fig. 5

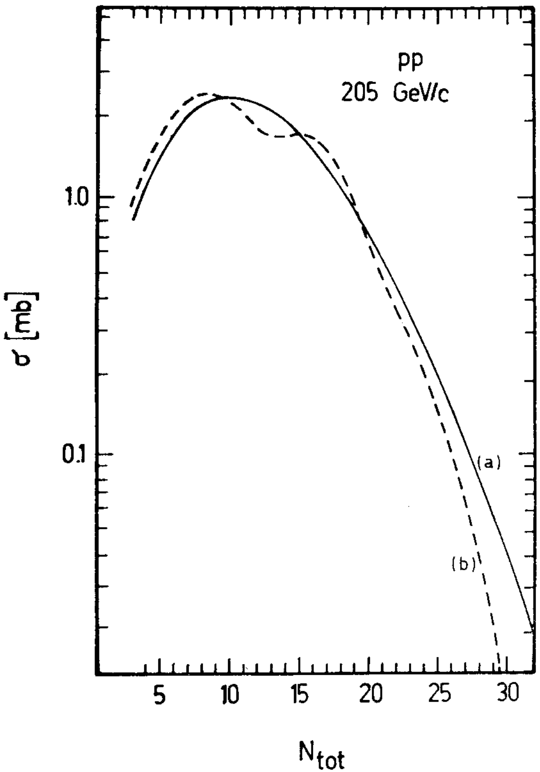
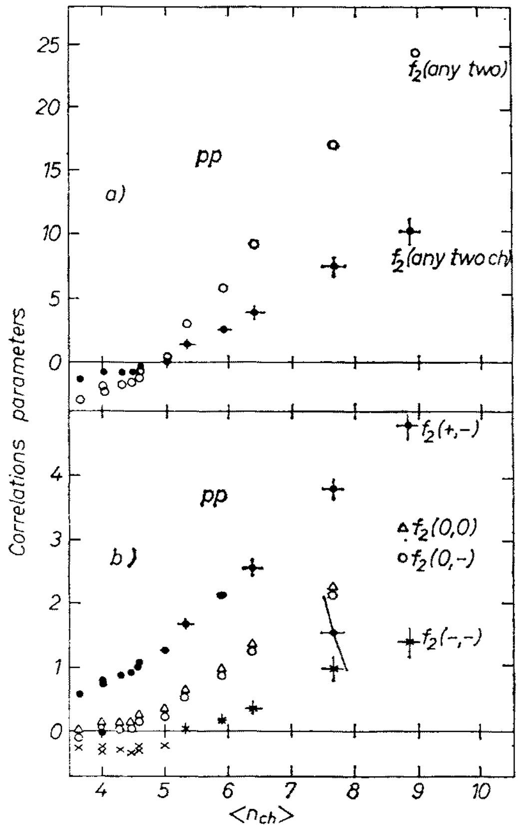


Fig. 6

Fig. 5. The dependence of $\langle n_{tot} \rangle$ on D_{tot} for $\pi^- p$ and pp collisions

Fig. 6. The distribution of the total number of particles produced in pp collision at 205 GeV/c; *a*) from the CR+II model, *b*) from the model with the Poisson distribution of n_{π^0} for each n_{ch} (see text)



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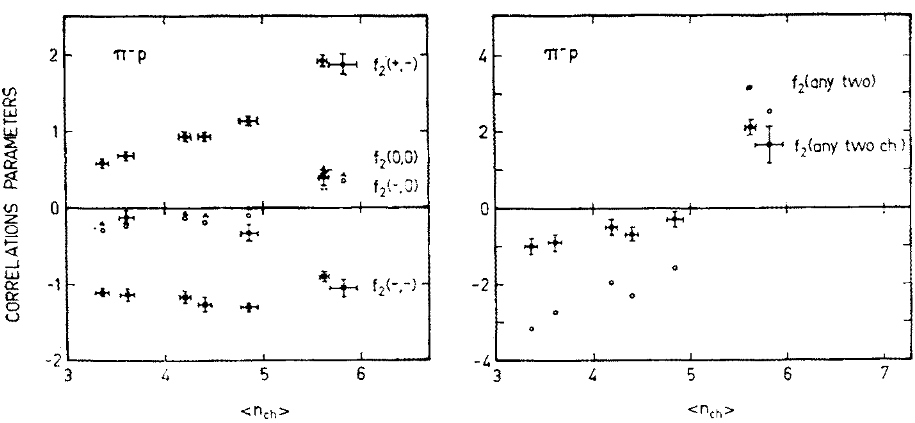

b

Fig. 7. The f_2 correlation parameters, defined in the text, as a function of $\langle n_{ch} \rangle$ for $\pi^- p$ and pp collisions. Full symbols correspond to experimental values. Open symbols are CR+II model predictions; a) for pp, b) for $\pi^- p$

From Fig. 7 we see that for sufficiently high energy all f_2 parameters increase with increasing energy. For $s > 70\ \mathrm{GeV}^2$ they become positive, which suggests the existence of dynamical correlations between particles [13].

The fact that $f_{+-} > f_{00}$ and $f_{0-} > f_{--}$ can be partly explained by the charge conservation. The charge conservation does not affect (0, 0) or (0, −) pairs, increases the correlation parameter for (+, −) pairs and decreases it for (−, −) pairs.

TABLE II

Correlation parameters f_2

	P_{lab} GeV/c	Any two charged		(+, −)		(−, −)		(−, 0)		(0, 0) model	Any two model
		exp.	model	exp.	model	exp.	model	exp.	model		
	12.9	−1.43±.06	−1.46	0.56±.01	0.55	−0.27±.02	−0.28	—	−0.11	−0.07	−3.09
	18.0	−1.15±.07	−1.16	0.72±.02	0.72	−0.30±.02	−0.30	—	−0.05	0.01	−2.44
	19.0	−0.95±.06	−1.00	0.77±.02	0.75	−0.24±.01	−0.25	0.01±.11	−0.00	0.06	−2.01
	21.1	−0.88±.08	−0.90	0.85±.02	0.85	−0.30±.03	−0.29	—	0.01	0.08	−1.86
	24.1	−0.89±.09	−0.82	0.90±.02	0.91	−0.34±.03	−0.32	—	0.02	0.10	−1.72
pp	28.4	−0.40±.10	−0.42	1.05±.02	1.03	−0.25±.03	−0.25	—	0.13	0.21	−0.76
	28.5	−0.65±.14	−0.69	0.98±.03	0.97	−0.31±.05	−0.31	—	0.06	0.13	−1.42
	35	−0.00±.22	0.04	1.25±.06	1.26	−0.25±.05	−0.24	—	0.23	0.32	0.23
	50	1.33±.30	1.19	1.67±.07	1.64	0.00±.09	−0.04	—	0.53	0.62	2.92
	69	2.49±.20	2.44	2.09±.04	2.09	0.15±.06	0.13	—	0.84	0.94	5.76
	102	3.75±.47	3.95	2.53±.12	2.58	0.34±.12	0.39	—	1.21	1.33	9.20
	205	7.47±.73	7.50	3.76±.16	3.81	0.95±.21	0.94	1.49±.62	2.09	2.22	17.15
	303	10.34±.92	10.76	4.80±.23	4.87	1.37±.24	1.51	—	2.90	3.04	24.49
	8.05	−1.01±.16	−1.04	0.58±.03	0.58	−1.09±.05	−1.10	—	−0.29	−0.21	−3.22
	9.9	−0.90±.20	−0.83	0.68±.04	0.69	−1.13±.06	−1.11	0.13±.10	−0.24	−0.16	−2.76
	16.2	−0.52±.17	−0.48	0.92±.03	0.93	−1.18±.05	−1.16	—	−0.15	−0.07	−1.97
$\pi^- p$	18.5	−0.71±.14	−0.64	0.93±.02	0.94	−1.28±.05	−1.26	—	−0.19	−0.11	−2.33
	25	0.33±.18	−0.31	1.13±.03	1.15	−1.30±.06	−1.30	0.32±.11	−0.10	−0.02	−1.56
	40	2.07±.19	1.81	1.93±.05	1.86	−0.89±.05	−0.95	0.42±.08	0.42	0.42	3.13
	50	1.64±.49	1.50	1.87±.13	1.83	−1.05±.12	−1.08	—	0.34	0.42	2.46

4. *Conclusions*

(*i*) We observe that with increasing incident energy the dependence of the average number of π^0's on the number of charged particles produced becomes more pronounced.

(*ii*) The observed features of both charged multiplicity distribution and neutral pion production can be described by a phenomenological model which assumes a Czyżewski-Rybicki formula for the total multiplicity distribution and the hypothesis of isospin independence.

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APPENDIX

In principle, the CR+II model should enable us to predict the cross-sections for all exclusive reaction channels, using the total cross-section as the only input data.

The most reliable information on the exclusive cross-sections exists for some four-constraint channels, although only for not too high energies. In the following table we compare the experimental cross-sections for such reaction channels (as quoted in the HERA Compilations, CERN 1972, or more recent original papers), with the model predictions. The comparison does not look conclusive. We stress again that the model was mainly thought of as means of describing some average characteristics of high energy reactions, not the details of all reaction channels.

TABLE AI

	Momentum	Reaction	σ_{exp} [mb]	σ_{model} [mb]
	8.04	$p\pi^+\pi^-\pi^-$	1.27 ± 0.07	1.37
	10.0	$p\pi^+\pi^-\pi^-$	1.01 ± 0.21	1.05
π^-p	16.0	$p\pi^+\pi^-\pi^-$	1.13 ± 0.05	0.65
	25.0	$p\pi^+\pi^-\pi^-$	0.90 ± 0.20	0.28
	10.0	$p2\pi^+3\pi^-$	0.42 ± 0.05	0.61
	16.0	$p2\pi^+3\pi^-$	0.25 ± 0.02	0.52
	19.0	$pp\pi^+\pi^-$	1.50 ± 0.20	1.25
	28.5	$pp\pi^+\pi^-$	1.25 ± 0.10	0.82
pp	19.0	$pp2\pi^+2\pi^-$	0.40 ± 0.20	0.70
	28.5	$pp2\pi^+2\pi^-$	0.46 ± 0.04	0.61
	28.5	$pp3\pi^+3\pi^-$	0.12	0.22

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