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ON THE NEW REDEFINITION OF THE INVERSION OPERATORS

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Following the general idea of a previous paper [5] the operations CP and T are defined in a rigorous manner by their actions on the one- and many- particle basis vectors describing free but physical (dressed) stable particles. It is shown that TCP invariance implies the possibility of a new redefinition of the inversion operators which restores the invariance under the full Poincaré group even in CP and T do not commute with the total Hamiltonian. The rigorous representatives of the space and time inversions have the forms ACP and AT respectively, where A is a suitable unitary operator.

1. *Introduction*

The full or extended Poincaré group Π_f is defined as the group of linear transformation of the space and time coordinates $x^\mu = (t, \boldsymbol{x})$:

$$x'^\mu = L^\mu_\nu x^\nu + a^\mu \tag{1}$$

which leave invariant the intervals

$$\Delta s^2 = \Delta x^\mu \Delta x_\mu \tag{2}$$

between any two events. Besides the continuous transformations π_g, which form the proper Poincaré group Π, the group Π_f contains the discrete operations of geometrical space and time inversion P_g and T_g:

$$P_g\colon\ t' = t, \quad \boldsymbol{x}' = -\boldsymbol{x}, \tag{3}$$

$$T_g\colon\ t' = -t, \quad \boldsymbol{x}' = \boldsymbol{x}, \tag{4}$$

and all the products of P_g, T_g and $\pi_g \in \Pi$. Obviously

$$P_g^2 = 1, \quad T_g^2 = 1, \quad P_g T_g = T_g P_g. \tag{5}$$

The rules of multiplication of the continuous Poincaré transformations π_g by P_g and T_g

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can be easily obtained from the corresponding relations for the 10 generators $j_g^{\mu\nu}$, p_g^{μ} of Π:

$$P_g p_g^{\mu} P_g^{-1} = \varepsilon(\mu) p_g^{\mu}, \quad P_g j_g^{\mu\nu} P_g^{-1} = \varepsilon(\mu)\varepsilon(\nu) j_g^{\mu\nu}, \tag{6}$$

$$T_g p_g^{\mu} T_g^{-1} = \varepsilon(\mu) p_g^{\beta}, \quad T_g j_g^{\mu\nu} T_g^{-1} = -\varepsilon(\mu)\varepsilon(\nu) j_g^{\mu\nu}, \tag{7}$$

where

$$\varepsilon(\mu) = \begin{cases} +1 & \text{for } \mu = 0, \\ -1 & \text{for } \mu = 1, 2, 3. \end{cases} \tag{8}$$

It is to be noted that both P_g and T_g commute with the generator p_g^0 of time translations. If we take T_g antilinear instead of linear, the relations (5), (6), (7), together with the well known commutation relations between the generators themselves, reproduce all the essential (local) group properties of Π_f.

For many years there were practically no doubts that Π_f (and not Π alone) is the group of rigorous relativistic invariance valid for classical and quantum physics. The experimental discovery in 1956 of several P-violating slow processes raised the first serious doubts concerning the inclusion of space inversions in the group of strict relativistic invariance. However, it was soon realized that the same processes which violate P, violate simultaneously the invariance under particle-antiparticle conjugation C. The violation of P and C is such that the invariance under the product CP remains valid. Taking CP as the correct quantum representative of P_g one could save Π_f as the relativistic invariance group [1].

Unfortunately, in 1964 it was shown that in some rare decay modes of neutral kaons the product CP seems to be violated [2]. So the operation CP is most probably no rigorous quantum representative of P_g, although it is much better than P alone.

Strictly speaking the experiment shows that the final decay products of the longlived kaon K_L^0 are not in an eigenstate of CP. One should emphasize the fact that in the case of ordinary slow processes the P-violating amplitude is of the same magnitude as the P-conserving amplitude. This is the result of the maximal violation of P by the weak interactions which are responsible for the slow processes. On the other hand in the case of K^0 decays the admixture of the CP-violating amplitude is small. In principle there is still a possibility that CP is not violated, but K_L^0 is not an eigenstate of CP. In fact there are some models [3] which explain all the experimental results without CP-violation, but assume — for example — the existence of a new particle, *e.g.* a neutral boson which has only weak interactions. However, the assumption of CP-conservation is not very popular among physicists, because the respective models are rather artificial or purely phenomenological possibilities. In either case it is difficult to explain the equality of masses and lifetimes (or mass distributions) in all the different decay channels which suggests rather one unstable particle.

Therefore, most physicists are rather inclined to accept the fact that CP is indeed violated, although the situation is not quite clear yet. Quite obscure are also the consequences of the CP-violation. For example T. D. Lee and G. C. Wick argue [4] that if CP is not conserved, then it cannot be defined in a rigorous manner. According to these authors one can define nonconserved discrete operations P, C, CP, T only formally and approximately for certain model theories described by some truncated Hamiltonians H_k. For a suitable approximate model theory with the Hamiltonian H_k one can define the corresponding operators P_k, C_k, T_k, that commute with H_k but do not commute with the exact Hamiltonian H. From the formal point of view the procedure of Lee and Wick is selfconsistent, howevei, from the physical point of view one can raise serious objections. For example the transformation properties of the physical particles with respect to the mentioned approximate operators remain unspecified. The same refers to the commutation relations of P_k, C_k, $(CP)_k$ and T_k with the generators of the rigorous Poincaré group of relativistic invariance. Moreover, the conclusion of Lee and Wick that the inversion operators cannot be defined rigorously if they do not commute with H, seems also very strange. We may ask the following question which brings out the inherent contradiction: How can we say that CP is violated if it is not well defined?

2. *The definition of* $P' = CP$

We shall assume in the following discussion that CP is not conserved in the sense that it doesnot commute with $H = p^0$. In order to clarify the situation let us first investigate the possible ways of defining an operator O in quantum theory.

The first possibility consists in prescribing the commutation relations of O with a suitable fundamental set of operators. The most convenient fundamental set of operators contains all the charges and the ten generators of Π. However, in the case of CP just its commutation relations with these generators are questioned so they cannot be used as defining relations.

In a quantum field theory one can define the discrete operators we are interested in, in terms of suitable independent quantum fields. However, the exact quantum field theory is not known yet. On the other hand the use of approximate model theories is not satisfactory for several reasons mentioned above in connection with the work of Lee and Wick who use just this method.

Another method of defining an operator O consists in prescribing its action on all basic vectors $|k\rangle$ which span the Hilbert space $\mathscr{H}$ of quantum states. This method has the adventage of being exact, unique, and very close to the procedure applied at the analysis of the experimental data. The vectors $|k\rangle$ need not belong to the space $\mathscr{H}$ but the set of all $|k\rangle$ should be essential and complete. As the basis vectors let us take the vectors $|\boldsymbol{p}_i\lambda_i q_i\rangle$ which describe the free physical particle i having definite momentum $\boldsymbol{p}_i$, helicity λ_i and charges q_i, and all the direct products of such vectors. The word particle is used here in the Wigner's sense, *i.e.* for systems which may be also composite (non-elementary) but have definite values m_i, s_i of mass and spin.

In order to avoid unnecessary repetitions of essentially the same formulae and arguments we shall restrict our discussion for the moment to the operator $P' = CP$. Its action on the one-particle basis vectors can be defined as follows [5]:

$$P'|\boldsymbol{p}_i\lambda_i q_i\rangle \overset{\mathrm{df}}{=} \eta_i'|-\boldsymbol{p}_i-\lambda_i-q_i\rangle, \tag{9}$$

where η_i', is the conventional phase factor of modulus one, which is proportional to the intrinsic CP-parity $\eta_i = \pm 1$ [6]. We can always choose these phases so as to have

$$P'^2 = (-1)^{2J}, \tag{10}$$

where J is the total angular momentum. The equation (10) reflects the well known fact that the intrinsic P-parity of a fermion (half-integer J) is not equal but opposite to that of the corresponding antifermion. Once the action of P' on single particle states is specified, there is no problem with states comprising many free particles. In fact we have then:

$$P' \prod_i |p_i \lambda_i q_i\rangle \overset{\mathrm{df}}{=} \prod_i \eta_i |-p_i - \lambda_i - q_i\rangle. \tag{11}$$

It is to be noted that no new phase factors are introduced in (11).

The assignment of the phase factors η_i' to all the stable particles may be partly experimental and partly enforced by suitable conventions [6]. We assume that the defining relations (9) and (11) can be imposed in a consistent manner on all the basic vectors describing free stable particles without creating nonexisting states or otherwise contradicting the experimental data. At the moment there is no experimental evidence against the validity of this assumption but, of course, it must be checked more carefully. We shall come back to this point later.

A closer investigation of the procedure adopted at the analysis of the experimental data reveals that in principle one is using there just the above definition, though sometimes in a not quite consistent manner. Some inconsistencies may emerge if, for example, apart from giving the prescription (9), (11) of the action of P' on a complete set of basis vectors, one requires an independent transformation law for special linear combinations of these basis vectors which correspond to metastable states or resonances. This may be a serious source of troubles which we can avoid only by restricting our defining relations to free *i.e.* noninteracting stable particles which form a complete set. From the theoretical point of view the possibility of a consistent definition of P' given by (9) and (11) corresponds to the following division of the total Hamiltonian [8]:

$$H = H_{\mathrm{free}} + V. \tag{12}$$

Here H_{free} describes free (*i.e.* noninteracting) but physical (*i.e.* dressed and bound not bare) particles and V is responsible for the scattering of the physical particles. In the quantum field theory one uses a different splitting

$$H = H_0 + H', \tag{13}$$

where H_0 describes free bare particles and H' is responsible for both: dressing and mutual interactions. Our definitions (9) and (11) may be justified by the following properties of the Hamiltonian:

$$[P', H_{\mathrm{free}}] = 0, \quad [P', V] \neq 0. \tag{14}$$

At any rate this is a sufficient condition for the validity of (9) and (11). The construction of H_{free} and V from H_0 and H' is quite complicated but a general prescription is known in literature (see *e.g.* [8]). In the following discussion we shall not need the explicit form of the Hamiltonian.

3. *A possible redefinition of the inversion operators*

Suppose that the set of the basis vectors for which the defining relations (9–11) hold, is complete. Then P' is obviously an unitary operator satisfying

$$P'^{\dagger} = P'^{-1}, \quad P'^2 = (-1)^{2J}. \tag{15}$$

Let us investigate the P'-transforms of the ten generators $p^{\mu}, j^{\mu\nu}$ of Π which act in the Hilbert space of quantum states. Since P' is unitary we can always write

$$P'p^{\mu}P'^{-1} = \varepsilon(\mu)A^{-1}p^{\mu}A, \quad P'j^{\mu\nu}P'^{-1} = \varepsilon(\mu)\varepsilon(\nu)A^{-1}j^{\mu\nu}A. \tag{16}$$

Because the operation of attaching the sign factors

$$p^{\mu} \to \varepsilon(\mu)p^{\mu}, \quad j^{\mu\nu} \to \varepsilon(\mu)\varepsilon(\nu)j^{\mu\nu} \tag{17}$$

can always be taken unitary, the operator A is also unitary. If P' does not commute with H' neither does A which represents the residual effect of P'. However, it can easily be seen that the unitary operator

$$P'' = AP' = ACP \tag{18}$$

not only commutes with H but also restores the group relations

$$P''p^{\mu}P''^{-1} = \varepsilon(\mu)p^{\mu}, \quad P''j^{\mu\nu}P''^{-1} = \varepsilon(\mu)\varepsilon(\nu)j^{\mu\nu}, \tag{19}$$

required for an operator representing the space inversion. The same effect has

$$P''^{-1} = P'^{-1}A^{-1}. \tag{20}$$

The condition

$$P''^2 = AP'AP' = P'^2 \tag{21}$$

will be satisfied if

$$P'AP'^{-1} = A^{-1}. \tag{22}$$

A similar procedure can be applied to the time inversion operator T. The result can be summarized as follows. If T doesn't commute with H but its action on a complete set of stable, free-particle states can be defined in the usual manner, we can introduce a new antiunitary time inversion operator $T'' = TB^{-1}$ which satisfies the group properties (6). Because of the still unfought TCP invariance we can put $B = A$ and hence

$$T'' = TA^{-1}. \tag{23}$$

The condition

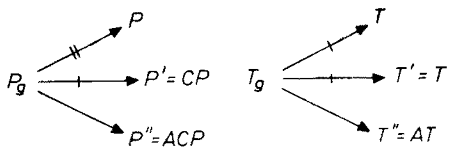
$$T''^2 = TA^{-1}TA^{-1} = T^2 = (-1)^{2J} \tag{24}$$

will be satisfied provided that

$$TAT^{-1} = A^{-1}. \tag{25}$$

The squares of the new quantum representatives P'', T'' of P_g and T_g are not exactly unity but ± 1 as in the case of CP and T, in agreement with the fact that for half-integer spin the quantum representations of Π_f are not faithful but only representations up to a sign factor.

In this way we have proved that under the conditions stated above one can construct new operators P'', T'' which are exact representatives of the geometrical space and time inversion respectively. We can visualize the situation with the help of the following picture.



(26)

In other words, a second redefinition of both inversion operators is then possible with the help of the same operator A. The single or double vertical dash indicates weaker or stronger violation of the respective approximate symmetry.

4. *The basic properties of the operator A*

The operator A has to satisfy the following relations

$$[p^\mu, A] \neq 0, \quad P'AP'^{-1} = A^{-1}, \quad TAT^{-1} = A^{-1}. \tag{27}$$

Since P' does not commute with H, neither does A. Thus the operator A cannot be a Poincaré invariant but it can be still a Lorentz invariant. At any rate there is no experimental evidence against such a simplifying assumption. (It would be wrong if for example P' were not commuting with the angular momentum $\boldsymbol{J}$, but there is no experimental evidence for such a drastic assumption.)

There are two distinct possibilities

$$\text{(a)} \quad A^{-1} = \pm A, \qquad \text{(b)} \quad A^{-1} \neq \pm A. \tag{28}$$

In the first case the operator A is itself some sort of "inversion". This type of operator has been used in 1957 at the first redefinition of the space inversion operator when P was replaced by CP. In fact $C^2 = 1$ and thus the operator C belongs to the class (a). At the moment it seems that there is no other nontrivial discrete operaton of type (a) which could be used for the second redefinition of inversions. However, its existence cannot be excluded.

Let us, therefore, investigate more carefully the second possibility. A unitary Lorentz invariant operator A of the second type, which satisfies all the conditions (27), must have the form

$$A = \exp(iW), \tag{29}$$

where

$$W^\dagger = W, \quad [W, p^\mu] \neq 0, \quad [T, W] = 0,$$
$$P'W + WP' = 0, \quad [W, j^{\mu\nu}] = 0. \tag{30}$$

In order to avoid the parity degeneracy of the physical vacuum $|0\rangle$ we must require

$$W\,|0\rangle = 0. \tag{31}$$

Obviously there are many solutions satisfying all the conditions (30) and (31). *E.g.* an operator of the form

$$W = QF(m, s, j_{\mu\nu}j^{\mu\nu}) \tag{32}$$

is satisfying all these conditions. Here Q is one of the charges, or an odd function of the charges, and F is a Lorentz invariant operator which is however not Poincaré invariant.

The actual form of W can be found either on the basis of some theoretical arguments or from the experimental data. Suppose that we know the Hamiltonian H and its CP-even and CP-odd parts

$$H = H_+ + H_-, \tag{33}$$

where

$$P'H_+P'^{-1} = H_+, \quad P'H_-P'^{-1} = -H_-. \tag{34}$$

It can easily be seen that

$$\exp(iW)H\exp(-iW) = H_+ - H_-. \tag{35}$$

Both W and H_- are odd with respect to P'. Separating the even part from the odd part we obtain two separate equations

$$[W, H_-] + \frac{i}{2}[W[W, H_+]] + \ldots = 0, \tag{36}$$

$$2H_- + i[W, H_+] + \frac{i^2}{2}[W[W, H_-]] + \ldots = 0. \tag{37}$$

The Eqs (36) and (37) will be satisfied up to the third order (*i.e.* in the zeroth, the first and the second order) if

$$[W, H_+] = i2H_-. \tag{38}$$

Knowing H_+ and H_- we could solve this operator equation for W. The solution would be approximate but rather quite accurate. In fact we could expect some errors only in the third of higher orders in H_-. Of course the above perturbative method of determining W may be useful only if H_+ and H_- are known, which at present is not the case.

5. *Discussion of the assumptions*

Let us consider the possibility that our assumptions which imply the existence of A are not fulfilled.

The existence of a unitary operator A satisfying (16) was the consequence of the unitarity of $P' = CP$ which followed from the definitions (9)–(11) of the action of P' on the complete set of basis vectors. Internal inconsistencies due to choice of phases can be excluded by the fact that our basis consists of stable particles and by the superselection rules [6]. Therefore, our theorem about the existence of a unitary A satisfying (16) may be wrong only if the set of the stable, free particle states on which the definition (9)–(11) can be enforced is not complete. Because of the structure of our basis vectors, the incompleteness must appear already in the set of one-particle states. Thus we come to the conclusion that the assumptions of our theorem may be wrong only if there exists at least one stable particle for which the change

$$(p_i\lambda_i q_i) \rightarrow (-p_i - \lambda_i - q_i) \tag{39}$$

leads to non-existing states and thus is not allowed. In this case the Hilbert space of physical states is not invariant under CP. Such a situation would be analogous to that pertaining to the two-component neutrino, where the action of P alone leads to non-existing states of a righthanded neutrino or left-handed antineutrino.

It is to be noted that an additional change of sign of the linear momentum can be achieved with the help of a rotation through 180° around an arbitrary axis perpendicular to $\boldsymbol{p}_i$. However, the resulting transformation of the basic quantum numbers

$$(p_i\lambda_i q_i) \rightarrow (p_i - \lambda_i - q_i) \tag{40}$$

coincides with that provided by the operation TCP. Therefore, it seems that the existence of stable particles for which the transformations (39) and (40) are forbidden is necessarily connected with TCP violation and not only with CP violation. In other words: TCP invariance implies the existence of a unitary operator A satisfying the conditions (16) even if we have CP violation.

It is hard to believe that a stable massive particle with such properties will be discovered. However, one may look for it among massless fermions or neutrinos, where the situation is far from being clear. In fact, one can easily imagine that apart from the usual (electronic or muonic) neutrinos ν for which the products CP and TCP are allowed but P and C are not, there exists another type of neutrinos ν' for which, *e.g.* C and CT are allowed but P, CP and TCP are not. We can visualize such a situation by the following two tables:

usual neutrinos ν	CP violating neutrinos ν'
(λq) allowed	$(\lambda' q')$ allowed
$(-\lambda q)$ forbidden	$(-\lambda' q')$ forbidden
$(\lambda - q)$ forbidden	$(\lambda' - q')$ allowed
$(-\lambda - q)$ allowed	$(-\lambda' - q')$ forbidden

The primes in the second column indicate the possibility that the new neutrinos may have the helicities and the leptonic charges different from those of the usual neutrinos. Such neutrinos and the corresponding antineutrinos would have the same helicity. Of course the interactions of these new neutrinos would be different from those of the ordinary neutrinos. *E.g.* if we assume an independent conservation law for the new leptonic charge q', the lack of the corresponding electricically charged leptons implies that the new neutrinos may occur only in some neutral leptonic currents $\bar{\nu}'\Gamma\nu$ coupled to other suitable neutral currents. It is to be noted that the existence of such neutrinos is possible only in the case of violation of *TCP* invariance.

Another difficulty which cannot be a priori excluded consists in the following. We may find a unitary operator A satisfying (16) but not satisfying the additional conditions (22) and (25). Such an operator would not restore all the group properties of Π_f. Without having the explicit form of the operator A it is hard to discuss the ways of solving this possible difficulty. One can only point out that the opertor A is not defined completely by (16) but only up to an arbitrary Poincaré invariant unitary factor. One can use this freedom to find such a factor which makes A satisfy the required additional conditions.

It is possible that in the case of many-particle states the operator A may not factorize into a product of operators A_i referring to single particles but may depend on some overall properties of the system or on the interaction. This would mean that the new inversion operators P'' and T'' are somehow related to the dynamics of the system. In fact, already at the first redefinition when the space inversion operator was identified with *CP*, the factor *C*, which is definitely not geometrical, implies some dynamical character of the space inversion operator in quantum physics.

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