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DIFFRACTIVE DISSOCIATION AS SHADOW SCATTERING

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It is pointed out that if the mechanism of diffractive production of particles is the same as that of elastic scattering, the diffractive dissociation can be calculated as shadow of non--diffractive processes. A general method of calculation is proposed. It uses the technique of the overlap matrix. A specific calculation in Uncorrelated Jet Model is performed. In this calculation the diffractive processes arise as a direct consequence of correlations induced in non-diffractive interactions by energy and momentum conservation. The most important prediction of the model is that the inclusive mass distribution of diffractive dissociation splits into non-scaling part describing the low-mass excitations and the approximately scaling part describing the high-mass excitations. The non-scaling part of the mass spectrum is dominated by single particle production and at large masses behaves as $d\sigma/d\mathcal{M}^2 \sim \mathcal{M}^{-6}$. The shape of the scaling part of the spectrum in the triple-Regge region is $d\sigma/d\zeta = (\zeta \log \zeta)^{-1}$ where $\zeta = \mathcal{M}^2/s$. The properties of exclusive diffractive channels are also discussed.

1. Introduction

One of the well-known features of strong interactions at high energies is a division of most of the channels into two classes:

(a) those with cross-sections dropping rapidly with increasing energy (non-diffractive channels);

(b) those with cross-sections varying slowly with energy (diffractive channels).

The best-known representative of class (b) is the elastic diffractive scattering. The existence of the diffractive inelastic channels was conjectured by Feinberg and Pomeranchuk (1956) and by Good and Walker (1960) and then identified experimentally (see *e.g.* a recent review by Lubatti (1972)).

There exists at present no fully satisfactory description of the diffractive production. However, all models agree with the basic idea of Good and Walker (1960) that the

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mechanism of diffractive dissociation is the same as that of elastic scattering, *i.e.* the absorption of incident and outgoing hadron waves[1].

In this paper we follow this point of view, and we investigate one of its straightforward consequences: that diffractive scattering and production is determined by the non-diffractive production of particles, which is the source of absorption. In other words, diffractive and non-diffractive production cannot be treated independently in consistent models of strong interactions at high energies.

This general idea was already widely explored in investigation of elastic scattering and proved to be a useful tool in the analysis of the properties of both elastic scattering and particle production (see *e.g.* Van Hove 1964, Michejda 1967, 1968).

The purpose of the present paper is to investigate this relation also for inelastic diffractive channels, with the hope that it may provide some interesting constraints on models for both diffractive and non-diffractive interactions. The main conclusion from our qualitative analysis is that such a program is indeed feasible and that many important properties of diffractive dissociation can be determined from the known or assumed properties of non--diffractive particle production.

Our basic tool is the unitarity condition which connects all channels at a given energy. We obtain the relation between the diffractive and non-diffractive channels from unitarity, using the method of overlap matrix developed by Białas and Van Hove (1965) and by Białas and Zalewski (1966). This is described in Section 2. We investigate further the obtained relationship by assuming the Uncorrelated Jet Model for non-diffractive particle production. The general formulation is described in Section 3 and some specific realization of the model in Sections 4 and 5. In Sections 6 and 7 we discuss briefly the general properties of diffractive production in our model. Finally, in Section 8 we summarize our conclusions.

2. *Diffractive dissociation and non-diffractive channels*

In this Section we would like to discuss to what extent diffractive dissociation can be estimated if the non-diffractive part of the interaction is known.

To begin with, we have to define what we mean by these two classes of interactions. It seems that the most striking feature which would help in identification of diffractive and non-diffractive parts of the scattering matrix is their energy dependence. Thus we propose to identify an "exclusive" process as diffractive if the energy dependence of its cross-section is similar to that of elastic scattering. Other processes, for which the cross--section drops with increasing energy according to a power law or faster are called non--diffractive. It is not clear whether there is in nature a clear separation between these two classes of processes and indeed the study of the "border line" between them is interesting. Nevertheless we feel that such a distinction makes sense, at least in the first approximation.

Thus our problem may be stated as follows: suppose we have a correct description

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[1] This is sometimes expressed by saying that both elastic scattering and diffractive dissociation are dominated by the exchange of the Pomeranchuk singularity.

of all channels with cross-sections that fall rapidly when the energy increases. Can we estimate the scattering amplitudes for other channels?

A similar problem was treated in a different context by Białas and Van Hove (1965) and by Białas and Zalewski (1966) and we follow their treatment here. Thus we assume that the production of particles at high energies is dominated by "non-diffractive" processes which have two basic characteristics:

(*i*) all exclusive processes tend to zero at high energies according to power law or faster. The inclusive single particle spectra show scaling behaviour;

(*ii*) the total non-diffractive cross-section, *i.e.* the sum of all exclusive cross-sections is a slowly varying function of the energy.

It is clear that such a non-diffractive interaction implies the absorption of the incident hadron waves. Our problem is to find how this absorption is reflected in the scattering amplitude, that is to say, what shadow effects are implied by it. We expect that shadow scattering contributes mainly to the elastic channel and a few other channels of low multiplicity. Thus, at reasonably high energies, most of the channels (high multiplicity channels) are practically not influenced by shadow corrections and, consequently, they are correctly described by the non-diffractive interaction. In the infinite momentum limit such a behaviour leads to complete separation of diffractive and non-diffractive channels. This property of diffractive dissociation (which should of course be checked *a posteriori* for consistency) allows us to find the shadow elastic and inelastic scattering by the method of Białas and Van Hove (1965). We repeat its basic ideas here.

Let us denote the T-matrix describing the non-diffractive interaction by T_N and the full T-matrix by T. Furthermore, let us denote the states which are not influenced by shadow scattering by $|N\rangle$ and the states to which shadow scattering contributes, by $|D\rangle$. Thus we have

$$\langle N|T_N|D\rangle = \langle N|T|D\rangle, \tag{2.1}$$

$$\langle D'|T_N|D\rangle \neq \langle D'|T|D\rangle. \tag{2.2}$$

Our problem is to calculate the energy-independent part of $\langle D'|T|D\rangle$ if $\langle N|T_N|D\rangle$ and $\langle D'|T_N|D\rangle$ are known. This can be done by using the unitarity condition. From the matrix element of the unitarity condition

$$i(T^\dagger - T) = T^\dagger T \tag{2.3}$$

between the two states $\langle D'|$ and $|D\rangle$ we obtain

$$i(\langle D'|T^\dagger|D\rangle - \langle D'|T|D\rangle) = $$
$$= \sum_{D''} \langle D'|T^\dagger|D''\rangle \langle D''|T|D\rangle + \sum_N \langle D'|T^\dagger|N\rangle \langle N|T|D\rangle. \tag{2.4}$$

With the abbreviations

$$\langle D'|S|D\rangle \equiv \langle D'|D\rangle + i\langle D'|T|D\rangle \tag{2.5}$$

and

$$\langle D'|F|D\rangle \equiv \sum_N \langle D'|T^\dagger|N\rangle \langle N|T|D\rangle = \sum_N \langle D'|T_N^\dagger|N\rangle \langle N|T_N|D\rangle \tag{2.6}$$

Eq. (2.4) can be written in the form

$$S^{\dagger}S = 1-F \tag{2.7}$$

in the subspace of $|D\rangle$ states.

The matrix $\langle D'|F|D\rangle$ is called the overlap matrix. It summarizes the effect of the non--diffractive channels on the diffractive channels. The general solution of Eq. (2.7) is (Białas and Zalewski 1966)

$$S = \Omega(1-F)^{1/2}, \tag{2.8}$$

where Ω is an arbitrary unitary matrix. The Ω matrix describes the transitions $D \rightarrow D'$ if there is no coupling to non-diffractive channels. Since we would like to study only this part of the transition $D \rightarrow D'$ which is induced by the presence of non-diffractive channels, we take[2]

$$\Omega = 1 \tag{2.9}$$

and, consequently

$$\langle D'|T|D\rangle \underset{E\rightarrow\infty}{\rightarrow} i\langle D'|\,(1-\sqrt{1-F})\,|D\rangle. \tag{2.10}$$

This formula represents the solution of our problem. Indeed, it gives explicitly the high--energy limit of the transition matrix elements $\langle D'|T|D\rangle$ in terms of the non-diffractive matrix elements $\langle N|T_N|D\rangle$. One obvious and important consequence of formula (2.10) is that diffractive and non-diffractive interactions cannot be treated entirely independently in a consistent model of strong interactions.

We see two drawbacks in this approach. Firstly it does not guarantee the unitarity of the whole S-matrix. Secondly, it assumes that the diffractive and non-diffractive channels can be separated, *i.e.* that there is no strong interference between them.

Finally, let us note that there exists also another possibility of calculating of the diffractive production from unitarity condition, the so-called unitarization procedure (Auerbach *et al.* 1972, Aviv *et al.* 1972, Baker and Blankenbecler 1972, Fulco and Sugar 1973, Neff 1973, Skard and Fulco 1973, Sugar 1973). In unitarization procedures both diffractive and non-diffractive amplitudes are calculated from the known Born terms. In contrast, in the overlap matrix approach presented here the diffractive production is calculated from the known non-diffractive interactions.

3. *Uncorrelated Jet Model*

The method of estimating the diffractive production presented in Section 2 is illustrated here using the Uncorrelated Jet Model for non-diffractive production. There are several reasons for this choice of the model. Firstly, it is interesting to understand how the correlations between particles produced non-diffractively influence the diffractive channels.

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[2] The assumption (2.9) is analogous to neglecting the real part of the elastic amplitude in calculation of elastic scattering.

For this the predictions of the Uncorrelated Jet Model are useful. Secondly, as was already shown by de Groot and Ruijgrok (1971) and by Sivers and Thomas (1972), the Uncorrelated Jet Model provides a reasonable description of the inclusive properties of non-diffractive particle production and, consequently, can perhaps be treated as a good first approximation to high-multiplicity events. Thirdly, the Uncorrelated Jet Model is the simplest model without short-range order. In models without short-range order the Pomeranchuk singularity is a cut rather than a pole (see *e.g.* Sivers 1972 for a discussion of this point). Thus the UJM is convenient for studying the properties of the Pomeranchuk cut. Finally, the calculations in the Uncorrelated Jet Model are relatively simple.

A fairly general discussion of the Uncorrelated Jet Model, applied to the high-energy proton-proton data was given recently by de Groot and Ruijgrok (1971) and by de Groot (1971). We follow rather closely their treatment, apart from some technical details. Thus we assume that our non-diffractive transition matrix T_N is given in the standard form of the Uncorrelated Jet Model:

$$T_N = \delta^{(4)}(P-\hat{P})T_n S_\pi, \tag{3.1}$$

where P is the total four momentum and $\hat{P}$ is the operator of the total four-momentum of the system of particles. Further

$$T_n = \int \frac{d^3k_A}{E_A}\frac{d^3k_B}{E_B}\frac{d^3k_C}{E_C}\frac{d^3k_D}{E_D}\,\psi_0(k_A, k_B, k_C, k_D)b^\dagger(k_C)b^\dagger(k_D)b(k_A)b(k_B) \tag{3.2}$$

is the operator responsible for the interaction between the nucleons. Here $b(\boldsymbol{k})$ and $b^+(\boldsymbol{k})$ are annihilation and creation operators of the nucleons with momentum $\boldsymbol{k}$, $\psi_0(k_A, k_B, k_C, k_D)$ is the corresponding transition amplitude [3]. The operator S_π describes the uncorrelated emission and absorption of mesons,

$$S_\pi = \exp\left(i\int \varrho^*(q)a(\boldsymbol{q})\frac{d^3q}{E} + i\int \varrho(q)a^\dagger(\boldsymbol{q})\frac{d^3q}{E}\right) =$$

$$= e^{-V/2}\exp\left(i\int \varrho(q)a^\dagger(\boldsymbol{q})\frac{d^3q}{E}\right)\exp\left(i\int \varrho^*(q)a(\boldsymbol{q})\frac{d^3q}{E}\right),$$

$$V \equiv V(k_A, k_B, k_C, k_D) = \int |\varrho(q)|^2\frac{d^3q}{E}. \tag{3.3}$$

Here $a^\dagger(\boldsymbol{q})$ and $a(\boldsymbol{q})$ are creation and annihilation operators of mesons with momentum $\boldsymbol{q}$. They satisfy the commutation relations

$$[a(\boldsymbol{q}), a^\dagger(\boldsymbol{q}')] = E\delta^{(3)}(\boldsymbol{q}-\boldsymbol{q}'). \tag{3.4}$$

Further, $\varrho(q)$ and $\varrho^*(q)$ are the probability amplitudes for creation and for annihilation. They may depend in general on nucleon momenta

$$\varrho(q) = \varrho(q; k_A, k_B, k_C, k_D). \tag{3.5}$$

[3] We limit ourselves to nucleon-nucleon interactions. The nucleon quantum numbers are neglected. Consequently we adopt the commutation relation $[b(\boldsymbol{k}), b^\dagger(\boldsymbol{k}')] = E\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')$.

Thus, in this model the meson wave functions depend on the nucleon momenta. However, the nucleon-nucleon transition amplitude depends neither on the momenta nor on the number of emitted mesons. The numerical factor $\exp(-V/2)$ can be absorbed into these transition amplitudes:

$$\psi_0 \to \psi = \psi_0 e^{-V/2}. \tag{3.6}$$

The overlap matrix elements can be written in the form

$$\langle q_1, \ldots, q_N, k_C, k_D|F|k_A, k_B\rangle =$$
$$= 2\delta^{(4)}(k_A+k_B-q_1- \ldots -q_N-k_C-k_D)\langle q_1, \ldots, q_N, k_C, k_D|A|k_A, k_B\rangle, \tag{3.7}$$

where the matrix elements $\langle q_1, \ldots, q_N, k_C, k_D|A|k_A, k_B\rangle$, to the second order in F, represent the amplitudes for diffractive scattering (see Eq. 2.10)

In Appendix B it is shown that

$$\langle q_1, \ldots, q_N, k_C, k_D|A|k_A, k_B\rangle =$$
$$= i^N \sum_{n=0}^{N} (-1)^n \sum_{\binom{N}{n}\text{combinations}} \varrho_f(q_{l_1}) \ldots \varrho_f(q_{l_n})\varrho_i(q_{l_{n+1}}) \ldots \varrho_i(q_{lN})\times$$
$$\times \Phi_{i\to f}(k_A+k_B-q_{l_{n+1}}- \ldots -q_{l_N}) \tag{3.8}$$

with

$$\Phi_{i\to f}(R) = \Phi(R; k_A, k_B, k_C, k_D) =$$
$$= \frac{1}{2}\int \frac{d^3k_1}{k_{01}}\frac{d^3k_2}{k_{02}} \psi^*(k_C, k_D, k_1, k_2)\psi(k_A, k_B, k_1, k_2)\times$$
$$\times \sum_{m=0}^{\infty} \frac{1}{m!}\int \delta^{(4)}(R-Q_1- \ldots -Q_m-k_1-k_2) \sum_{j=1}^{m} \frac{d^3Q_j}{E_j} \varrho_f^*(Q_j)\varrho_i(Q_j). \tag{3.9}$$

In this formula we distinguish between $\varrho_i(Q_j) = \varrho(Q_j; k_A, k_B, k_1, k_2)$ and $\varrho_f(Q_j) = \varrho(Q; k_C, k_D, k_1, k_2)$ which may be different in general case.

A special case of formula (3.8) is the elastic overlap function (Van Hove 1964)

$$\langle k_C, k_D|A|k_A, k_B\rangle = \Phi(k_A+k_B; k_A, k_B, k_C, k_D). \tag{3.10}$$

From this formula we see the physical meaning of the function $\Phi_{i\to f}(R)$ entering the general expression (3.8) for the overlap matrix: Φ is an elastic overlap function corresponding to the imaginary part of the generalized elastic amplitude describing the transition $(k_A, k_B) \to (k_C, k_D)$ at the total energy-momentum equal to R. It is important to realize that these generalized elastic amplitudes describe non-physical elastic scattering: indeed, the total energy-momentum vector R need not be equal to k_A+k_B and/or k_C+k_D. Consequently, these amplitudes cannot be determined directly from experimental data and must be estimated from model calculations based on the formula (3.9).

The general structure of the basic formula (3.8) can be conveniently represented by the diagrams in Figure 1 for the production of 0, 1 and 2 particles.

It is seen that the elastic amplitude is given by the elastic overlap function. Next, the single pion production amplitude is the difference of two terms. The first term may be interpreted as the creation of the meson followed by off-shell absorptive elastic scattering

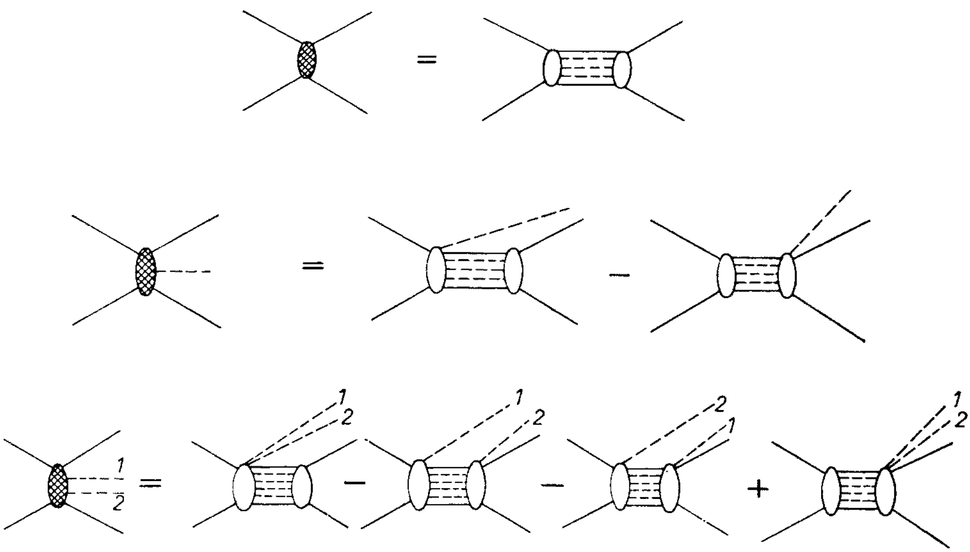

Fig. 1. Graphical representation of the structure of the diffractive shadow amplitudes

of nucleons. The second term represents the off-shell absorptive elastic scattering of nucleons followed by the creation of the meson. This structure is identical with that suggested by Good and Walker (1960). Then the two meson diffractive production amplitude is given by four terms corresponding to absorptive nucleon scattering and meson creation in four possible orderings. This simple structure is rather suggestive and may be more general than the model used for its derivation.

To summarize, we have shown that the diffractive production of particles in the Uncorrelated Jet Model is fully determined by

a) the probability amplitudes for creation of mesons and

b) the generalized elastic amplitudes for scattering of two nucleons.

We think that this result is appealing and can perhaps serve as a starting point in phenomenological applications.

One important point is still to be noticed. The contribution of the diffractive amplitude (3.8) to the inelastic scattering $N \neq 0$ vanishes if the generalized elastic amplitudes $\Phi(R; k_A, k_B, k_C, k_D)$ do not depend on R and if

$$\varrho_f(Q) = \varrho_i(Q) \equiv \varrho(Q). \tag{3.11}$$

Indeed, in such a case we can write

$$\langle q_1, \ldots, q_N, k_C, k_D|F|k_A, k_B\rangle =$$
$$= i^N \varrho(q_1), \ldots, \varrho(q_N)\Phi(k_A, k_B, k_C, k_D) \sum_{n=0}^{N} (-1)^n \binom{N}{n}. \tag{3.12}$$

However, for $N \neq 0$

$$\sum_{n=0}^{N} (-1)^n \binom{N}{n} = 0. \tag{3.13}$$

Thus, we see that in Uncorrelated Jet Model without energy-momentum conservation there is no diffractive production if $\varrho(Q)$ does not depend on nucleon momenta. In fact, after removing the δ-function from Eq. (3.9), function $\Phi_{i\to f}(R)$ does not depend on R and, as shown above, all shadow contributions vanish for $N \neq 0$. The inelastic shadow (*i.e.* diffractive) scattering is then generated by long-range correlations between nucleons and pions caused by energy and momentum conservation.

The diffractive production may be generated even without energy and momentum conservation if the meson functions ϱ depend on nucleon variables k. Indeed, neglecting the energy momentum conservation, we obtain from Eqs (3.8) and (3.9) the following formula for diffractive amplitude[4]

$$\langle q_1, \ldots, q_N, k_C, k_D|A|k_A, k_B\rangle = i^N\Phi(k_D+k_B) \prod_{n=0}^{N} (\varrho_i(q_n)-\varrho_f(q_n)). \tag{3.14}$$

In the following Sections we discuss a few examples of possible behaviour of the meson wave functions and generalized elastic amplitudes, as well as their consequences for the diffractive production of particles.

For convenience we collect here some formulae for the cross-sections, in terms of the amplitudes used in this paper.

The total inelastic cross-section:

$$\sigma_{\text{tot}} = \frac{8\pi^2}{Mk_{\text{lab}}} \langle k_A, k_B|A|k_A, k_B\rangle \tag{3.15}$$

(M is the nucleon mass).

Diffractive production of N pions ($N = 0, 1, \ldots$):

$$d\sigma = \frac{4\pi^2}{Mk_{\text{lab}}} \frac{1}{N!} \delta^{(4)}(k_A+k_B-k_C-k_D-q_1- \ldots -q_N)\times$$
$$\times |\langle k_C, k_D, q_1, \ldots, q_N|A|k_A, k_B\rangle|^2 \frac{d^3k_C}{E_C} \frac{d^3k_D}{E_D} \frac{d^3q_1}{E_1} \cdots \frac{d^3q_N}{E_N}. \tag{3.16}$$

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[4] We are indebted to Prof. Th. Ruijgrok for pointing out this formula to us.

Non-diffractive production of N pions:

$$d\sigma = \frac{4\pi^2}{Mk_{\text{lab}}} \frac{1}{N!} \delta^{(4)}(k_A+k_B-k_C-k_D-Q_1- \dots -Q_N)\times$$

$$\times |\langle Q_1, \dots, Q_N, k_C, k_D|\tau|k_A, k_B\rangle|^2 \frac{d^3k_C}{E_C} \frac{d^3k_D}{E_D} \frac{d^3Q_1}{E_1} \dots \frac{d^3Q_N}{E_N}. \tag{3.17}$$

Formula (3.16) contains, as a special case, the cross-section for elastic scattering. Matrix elements of the operators A and τ are given by formula (3.8) and by

$$\langle Q_1, \dots, Q_N, k_C, k_D|\tau|k_A, k_B\rangle = \psi(k_A, k_B, k_C, k_D)i^N \varrho_i(Q_1) \dots \varrho_i(Q_N) \tag{3.18}$$

4. *Non-diffractive processes*

Up to now, our discussion was fairly general and did not depend on special properties of the meson and nucleon wave functions. To proceed further, we have to specify them a little better. Fortunately, some of these properties are determined by the known experimental features of the inclusive distributions. To guarantee the scaling behaviour of the single particle spectra, the meson wave function ϱ will be taken as the function of transverse momentum and of the Feynman scaled longitudinal momentum[5]

$$\varrho(q; k_A, k_B, k_1, k_2) = \varrho(q_\perp, x), \tag{4.1}$$

where $\sqrt{s}$ is the total CMS energy available in the collision

$$s = (k_A+k_B)^2 \tag{4.2}$$

and

$$x = \frac{q_{||}}{\sqrt{s}}. \tag{4.3}$$

The transverse direction is measured in the centre of mass frame of the (k_A+k_B) system with respect to the direction of vector $\boldsymbol{k}_A$. It follows from Eq. (4.1) that

$$V \equiv \int \frac{d^3q}{E} |\varrho(q)|^2 = \lambda \log \frac{s}{s_0}, \tag{4.4}$$

where

$$\lambda = \int d^2q_\perp |\varrho(q\ \ , x = 0)|^2 \tag{4.5}$$

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[5] This is not the most general form of ϱ which assures scaling. We ignore the possible dependence on the nucleon variables (except for the direction of the incident nucleons) in order to simplify the discussion. No essential feature of the model is lost by this simplification.

and s_0 is a constant depending on the detailed shape of $\varrho(q)$. From Eqs (3.6) and (4.4) we see that the nucleon-nucleon scattering amplitude has the form

$$\psi(k_A, k_B, k_C, k_D) = \left(\frac{s}{s_0}\right)^{-\lambda/2} \psi_0(k_A, k_B, k_1, k_2). \tag{4.6}$$

We assume that the function ψ_0 factorizes

$$\psi_0(k_A, k_B; k_1, k_2) = \sqrt{E_A E_B E_1 E_2}\, \xi(w_A, w_1)\xi(w_D, w_2). \tag{4.7}$$

Here w_A, w_B, w_1, w_2 denote the four-vectors

$$w = (k_\perp, x, \varepsilon) \tag{4.8}$$

and

$$x = \frac{k_{||}}{\sqrt{s}}, \qquad \varepsilon = \frac{E}{\sqrt{s}}. \tag{4.9}$$

The form (4.7) is suggested by the requirement that the total cross-section is a slowly varying function of the incident momentum[6].

From Eqs (4.6) and (4.7) we see that the high-energy behaviour of the nucleon-nucleon amplitude is

$$\psi(k_A, k_B; k_1, k_2) \sim s^{1-\lambda/2}. \tag{4.10}$$

The same high-energy behaviour characterizes the amplitudes for non-diffractive particle production, as seen from Eq. (4.1) and (3.17). Thus we see that

$$\alpha = 1-\lambda/2 \tag{4.11}$$

plays a role of the trajectory of the leading Regge singularity which determines the non--diffractive nucleon-nucleon interaction[7].

5. *General properties of diffractive amplitudes at high energies*

Having specified the principal scaling properties of the nucleon and meson wave functions we can now discuss the high-energy behaviour of the diffractive amplitudes (3.8). We confine ourselves to the case of the forward scattering of the nucleons. The dependence on the transverse momenta of nucleons is much more complicated and requires rather

[6] The discussion of this point is given by de Groot and Ruijgrok (1971). These authors include also a logarithmic factor in the nucleon-nucleon amplitude, in order to obtain the constant total cross-section. We neglect this factor since it does not make any qualitative difference.

[7] It is interesting to note that our model is a specific example of the models of the high-energy interactions discussed recently by Harari (1972). Indeed, for $\lambda \to 0$ we have $\alpha \to 1$ *i.e.* vector exchange between two interacting nucleons. As λ (which plays the role of the πN coupling constant) increases, the exchanged trajectory decreases. The Pomeranchuk singularity stays always at $\alpha_P = 1$ and its position has nothing to do with the value of λ. This mechanism is caused by the fact that the part of the S-matrix responsible for the production of mesons (S_π given by Eq. (3.3)) is represented by a unitary operator.

involved arguments, as is already known from the work on elastic scattering (Michejda 1967, 1968, de Groot and Ruijgrok 1971).

If the nucleons scatter forward, then according to our assumption (4.1)

$$\varrho_f(q) = \varrho_i(q) \equiv \varrho(q) \tag{5.1}$$

and calculations simplify considerably. Using Eq. (3.8) we have

$$\langle q_1, \ldots, q_N, k_C, k_D|A|k_A, k_B\rangle = i^N \varrho(q_1) \ldots \varrho(q_N) F_N\{\Phi\}, \tag{5.2}$$

where

$$F_N\{\Phi\} \equiv \sum_{n=0}^{N} (-1)^n \sum_{\binom{N}{n} \text{ combinations}} \Phi(k_A + k_B - q_{l_{n+1}} - \ldots - q_{lN}). \tag{5.3}$$

The further discussion is greatly simplified by observation that $F_N\{\Phi\}$ can be approximately written in the form

$$F_N\{\Phi\} = (-1)^N q_1^{\mu_1}, \ldots, q_N^{\mu_N} \left(\frac{\partial}{\partial R^{\mu_1}} \ldots \frac{\partial}{\partial R^{\mu_N}} \Phi(R) \right)_{R=k_A+k_B} +$$

$$+ \text{ higher order terms in } q_1^{\mu_1}, \ldots, q_N^{\mu_N}. \tag{5.4}$$

Since the generalized elastic amplitude is a function of $x \equiv q_{||}/\sqrt{s}$ and $x_0 \equiv q_0/\sqrt{s}$ rather than $q_{||}$ and q_0, the formula (5.4) is a good approximation for small $|x|$. In this approximation the diffractive amplitudes are determined entirely by the behaviour of the function $\Phi(R)$ in the vicinity of the point $R = k_A + k_B$.

In Appendix A we show that at high energies the generalized elastic amplitude can be written as

$$\Phi(R) = \sqrt{E_A E_B E_C E_D}\, s_0^{\lambda} H(\lambda) \int_0^{Z_+/2} dx_1 \int_0^{Z_-/2} dx_2 (Z_+ - 2x_1)^{\lambda-1} \times$$

$$\times (Z_- - 2x_2)^{\lambda-1} \int d^2k_{1\perp} d^2k_{2\perp} \xi^*(w_C, w_1)\xi(w_A, w_1) \times$$

$$\times \xi^*(w_D, w_2)\xi(w_B, w_2) \exp\left[-(R_\perp - k_{1\perp} - k_{2\perp})^2/\Omega^2\right]/\Omega^2, \tag{5.5}$$

where

$$Z_\pm = \frac{R_0 \pm R_{||}}{\sqrt{s}} \tag{5.6}$$

and

$$\Omega^2 = \lambda \kappa^2 \log\left(\frac{\overline{Q}^2}{\overline{Q}_0^2}\right). \tag{5.7}$$

Here

$$\overline{Q}^2 = (R_0 - k_{10} - k_{20})^2 - (R_{||} - k_{1||} - k_{2||})^2 \tag{5.8}$$

and $\overline{Q}_0^2$, $H(\lambda)$, κ and s_0 are known constants.

As was already pointed out in Section 3, the dependence of the generalized elastic amplitude (5.5) on R is a consequence of the energy and momentum conservation. This can be seen explicitly in formula (5.5). The factors $(Z_+ - 2x_1)$, $(Z_- - 2x_2)$ and the integration limits reflect the conservation of energy and longitudinal momentum. The factor

$$\Omega^{-2} \exp\left[-(R_\perp - k_{1\perp} - k_{2\perp})^2/\Omega^2\right] \tag{5.9}$$

reflects the conservation of the transverse momentum (see Appendix A and de Groot (1972) for a discussion of these points).

To obtain the formula for the diffractive amplitude we have to perform differentiation of Φ given by Eq. (5.5) with respect to R. The inspection of Eq. (5.5) shows that such a differentiation provides two types of contributions to the diffractive amplitude:

(a) those which at large s and fixed configuration of particles (*i.e.* fixed $x_1, \ldots, x_N$ and $q_1, \ldots, q_N$) behave like elastic amplitude, and

(b) those which at large s and fixed configuration of particles reveal the additional negative powers of

$$\Omega_0^2 = \lambda\kappa^2 \log(s/s_0). \tag{5.10}$$

The terms of the type (a) arise by differentiation with respect to Z_+ and Z_- (*i.e.* $R_{||}$ and R_0). Thus they describe this part of the diffraction dissociation which is induced by conservation of the total energy and longitudinal momentum. At high energies (neglecting the corrections of the order $\left(\log\left(\frac{s}{s_0}\right)\right)^{-1}$ these contributions can be written in the form

$$\langle q_1, \ldots, q_N, k_C, k_D|A_1|k_A, k_B\rangle = \sqrt{E_A E_B E_C E_D}\, \frac{\Phi_0}{\Omega_0^2} \times$$

$$\times 2^N |x_1| \varrho(q_1) \ldots |x_N| \varrho(q_N) h_+^{(k)}(x_C) h_-^{(N-k)}(x_D), \tag{5.11}$$

where

$$h_\pm^{(k)}(x) = \frac{\partial^k}{\partial Z_\pm^k}\left[h_\pm(Z_\pm, x)\right]\Big|_{Z_\pm = 1} \tag{5.12}$$

and $h_\pm(x)$ are given by Eqs (A. 8) and (A.9) of the Appendix A. Here N denotes the number of produced particles and k denotes the number of particles moving to the right, *i.e.* with $x > 0$. The characteristic feature of the amplitude (5.11) is that it vanishes for $x_i \to 0$, for any i. Thus it describes the production of fast particles, with large longitudinal momenta.

The terms of kind (b) arise by differentiating of the factor (5.9) with respect to $R_\perp$. Thus they describe this part of the diffraction dissociation which is induced by transverse momentum conservation. The leading terms of this kind are of the form

$$\langle q_1, \ldots, q_N, k_C, k_D|A_2|k_A, k_B\rangle = \sqrt{E_A E_B E_C E_D} \times$$

$$\times \frac{\Phi_0}{\Omega_0^2} \frac{2^{N/2}}{\Omega_0^N} h_+(Z_+ = 1, x_C) h_-(Z_- = 1, x_D) C(q_{1\perp}, \ldots, q_{N\perp}), \tag{5.13}$$

where

$$C(q_{1\perp}, \ldots, q_{N\perp}) = \sum_{j_l \neq j_m} (q_{j_{1\perp}} \cdot q_{j_{2\perp}}) \ldots (q_{j_{N-1\perp}} \cdot q_{j_{N\perp}}). \tag{5.14}$$

It is seen that, indeed, for fixed $x_1, \ldots, x_N, q_1, \ldots, q_N$, the amplitude (5.13) has additional factor Ω_0^{-N} which makes it vanish at high energies. However when integrated over momenta of final particles, the amplitude (5.13) gives contribution to the diffractive cross-section which at high energies behaves like that of the elastic scattering. Thus the amplitudes (5.13) cannot be ignored in the high-energy limit. The reason for such a behaviour is that the amplitude (5.13) does not vanish at small x_i. Consequently, the integral over phase--space increases logarithmically, providing compensation of the factor Ω_0^{-N} (see *e.g.* de Groot (1972) for the discussion of the high-energy behaviour of the phase-space integral). Thus the amplitude (5.13) describes the production of the slow particles.

To conclude this Section we emphasize again that the amplitude for diffractive production in our model is a sum of terms describing the production of fast particles (small missing masses to the nucleon) and those describing the production of slow particles (large missing masses to the nucleon)[8]. These terms have different origin: the first one reflects the conservation of total energy and longitudinal momentum, the second one originates from transverse momentum conservation. Also their properties are rather different. Some of them are discussed in next two Sections.

6. *Non-scaling part of the diffractively excited spectrum*

In this Section we list briefly some properties of the diffractive dissociation processes described by the amplitude (5.11). As already noted, this part of the diffractive amplitude describes the production of fast particles in the centre-of-mass frame.

(*i*) Factorization

The amplitude (5.11) satisfies the factorization condition

$$\langle q_1, \ldots, q_N, k_C, k_D | A_1 | k_A, k_B \rangle \langle k_C = k_A, k_D = k_B | A_1 | k_A, k_B \rangle =$$

$$\langle q_1, \ldots, q_k, k_C, k_D = k_B | A_1 | k_A, k_B \rangle \langle q_{k+1}, \ldots, q_N, k_C = k_A, k_D | A_1 | k_A, k_B \rangle. \tag{6.1}$$

Relation (6.1) implies that the study of the production of particles with, say, $x > 0$ is sufficient for the determination of the behaviour of the diffractive production of fast particles at high energies. It should be stressed, however, that this factorization property of the amplitude is valid only up to the terms which decrase as inverse powers of Ω_0^2, *i.e.* inverse powers of $\log(s/s_0)$. Thus one may expect important corrections to factorization even at quite high energies.

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[8] There are of course also mixed terms in which some particles are fast and others are slow.

(*ii*) Multiplicity distribution

From formula (5.11) we obtain the differential cross-section for production of N particles with $x > 0$

$$\frac{d\sigma_N}{d^2k_{\perp C}d^2k_{\perp D}}\bigg|_{k_{C\perp}=k_{D\perp}=0} = \frac{\pi^2 s}{Mk_{\text{lab}}}\frac{|\Phi_0|^2}{\Omega_0^4}\frac{4^N}{N!}\times$$

$$\times|h_+^{(N)}(x_C = \tfrac{1}{2} - x_1 - \dots - x_N)|^2 x_1|\varrho(q_1)|^2 \dots x_N|\varrho(q_N)|^2 \times$$

$$\times|h_-(x_D = -\tfrac{1}{2})|^2\delta^2(q_{1\perp} + \dots + q_{N\perp})dx_1, \dots, dx_N d^2q_{\perp 1}, \dots, d^2q_{\perp N}. \quad (6.2)$$

The multiplicity distribution of the diffractively produced system is obtained by integrating Eq. (6.2) over $dx_1, \dots, dx_N\ d^2q_{\perp 1} \dots, d^2q_{\perp N}$ with the conditions

$$x_i > 0 \text{ and } x_1 + \dots + x_N \leqslant \tfrac{1}{2}. \quad (6.3)$$

The main property of the multiplicity distribution obtained in this way is that it falls sharply at large N. It can be shown that for a wide class of the nucleon-nucleon probability amplitudes all moments of the multiplicity distribution exist, *i.e.* all sums $\sum_{N=0}^{\infty} \sigma_N N^k$ are finite. This means that the multiplicity distribution of diffractively produced fast particles does not change appreciably with increasing incident energy.

To illustrate this point we have calculated the multiplicity distribution for several examples of nucleon-nucleon scattering amplitudes $\xi(w_i, w_k)$ which determine the function $h(Z, x_c)$ by formula (A.9). The results of these calculations show that only single particle production is important. For $\lambda \simeq 2$ the production of two particles is down by more than one order of magnitude compared to single particle production[9]. This dominance of single pion production processes seems to be consistent with existing data: it is well known that in most cases the diffractively excited system with low mass consists of two particles: a pion and another particle or resonance[10]. Although in a simplified model we can discuss neither the resonance production nor their possible quantum numbers, we expect that the dominance of single pion production will hold also in more realistic models, thus providing an explanation of this important experimental fact.

(*iii*) Mass spectrum

From the previous discussion it is clear that the mass distribution is determined almost completely by the channel with only one pion produced. For this channel it is not difficult to transform the momentum distribution (6.2) into the mass distribution. The obtained

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[9] The value of parameter λ can be estimated from the energy dependence of the exclusive non-diffractive amplitudes (as seen from Eq. (4.10)) and from the high-energy behaviour of the average multiplicity of non-diffractive channels (de Groot and Ruijgrok 1971) by formula $\langle n \rangle = \lambda \log (s/s_0)$. The experimental data on energy dependence of exclusive cross-section (Hofmokl and Wróblewski 1970, Hansen, Kittel and Morrison 1971) indicate that $\lambda \simeq 2$ (de Groot and Ruijgrok 1971). This is not inconsistent with the recent data on energy dependence of the average multiplicity (see *e.g.* Antinucci *et al.* 1973).

[10] *E.g.* in pp collisions the most important diffractive channels at energies up to 30 GeV are pp+ → (Nπ)+p and p+p → (Δπ)+p.

formula shows that the mass distribution $d\sigma/d\mathcal{M}^2$ for the single pion production channel is energy independent, *i.e.* it does not scale. Furthermore, at large $\mathcal{M}^2$ we have

$$\frac{d\sigma}{d\mathcal{M}^2} \sim \mathcal{M}^{-6} \tag{6.4}$$

i.e. the distribution falls off rather sharply for high-mass excitations. The precise determination of the shape of the mass distribution requires further assumptions on the shape of the amplitude (5.11).

Finally, let us note that formula (6.4) describing the tail of the mass-spectrum is valid for all channels, with arbitrary number of the produced pions. This can be seen as follows. Noting that

$$2\mathcal{M}^2 \simeq \frac{M^2+k_\perp^2}{x_C} + \sum_{i=1}^{N} \frac{\mu^2+q_{\perp i}^2}{x_i}, \tag{6.5}$$

we see from Eq. (6.2) that the average value of $\mathcal{M}^2$ in any channel is finite[11]. Thus $d\sigma/d\mathcal{M}^2$ must fall faster than $\mathcal{M}^{-4}$. On the other hand, the average of $\mathcal{M}^4$ has a logarithmic divergence at $x_i = 0$ due to terms $(\mu^2+q^2)^2/x^2$. This indicates that indeed $d\sigma/d\mathcal{M}^2 \sim \mathcal{M}^{-6}$ for any multiplicity.

This result shows that the amplitude (5.11) is responsible for the diffractive excitation of low mass systems. It shows also that this part of the excitation spectrum does not scale. We remind the reader that this part of the spectrum originates from the conservation of energy and longitudinal momentum in non-diffractive interactions.

In closing this Section we would like to stress again that the results discussed here are valid only in the high-energy limit. The corrections are of the order $(\log s)^{-1}$ and thus may be important even at high energies.

7. *Diffractive production of high missing masses*

We review here the characteristic properties of the diffractive amplitude A_2 given by Eq. (5.13). This part of the amplitude originates from the transverse momentum conservation in non-diffractive interactions and describes the production of slow ("wee") pions, *i.e.* excitation of high-mass systems.

As already indicated the energy dependence of the integrated cross-section for the diffractive production of N wee pions

$$\sigma_N = \frac{\pi s}{Mk_{\text{lab}}} \int |\langle q_1, \ldots, q_N, k_C, k_D|A_2|k_A, k_B\rangle|^2 \times$$

$$\times\, \delta^{(4)}(q_1+ \ldots +q_N+k_C+k_D-k_A-k_B)\, \frac{d^3q_1}{E_1} \; \ldots \; \frac{d^3q_N}{E_N}\, \frac{d^3k_C}{E_C}\, \frac{d^3k_D}{E_D} \tag{7.1}$$

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[11] Provided $h^{(N)}(x_C = 0) = 0$. This condition follows from a realistic requirement that the nucleon wave function vanishes for slow nucleons in CMS.

is the same as that of the elastic cross section. Thus the diffractive production of wee pions provides a finite fraction of the total diffractive cross-section in the high-energy limit, and may therefore influence considerably the high-energy behaviour of the inclusive spectra. Furthermore we note that since the number of terms in Eq. (5.14) increases considerably with increasing N, the multiplicity distribution of "wee" pions is expected to extend to fairly large multiplicities.

For fixed (large) mass of the diffractively excited system (consisting of one nucleon and N wee pions) the cross-section behaves like

$$\frac{d\sigma_N}{d\mathcal{M}^2} \simeq \frac{\sigma_{\text{elastic}}}{(\log s)^N} N a_N \frac{(\log \mathcal{M}^2)^{N-1}}{\mathcal{M}^2}, \tag{7.2}$$

where a_N are constant coefficients depending on details of the model. Thus for fixed mass of the diffractively excited system the production of wee pions falls as inverse power of logarithm with respect to elastic cross-section. However, as already discussed, the integral of $d\sigma/d\mathcal{M}^2$ over $\mathcal{M}^2$ gives contribution depending on energy in the same way as elastic scattering. By comparing Eqs (6.4) and (7.2) we see that the high-mass tail of the diffractive excitation spectrum is dominated by production of wee pions. As energy increases, the region where the wee pion production is important moves towards higher masses.

One observation can still be made on the distribution of transverse momenta of wee pions: since the amplitude (5.13) contains products $(q_{j_1\perp} \cdot q_{j_2\perp}) \dots (q_{j_{N-1}\perp} \cdot q_{j_N\perp})$, the distribution of wee pions is additionally damped in the region of small transverse momenta. This damping has two effects. Firstly, it reduces significantly the cross-section for diffractive production of wee pions. Secondly, it makes the transverse momentum distribution of wee pions produced by diffractive mechanism different from those produced by non--diffractive mechanism.

To simplify the discussion, we consider only the case in which all pions are wee: $x_1 = x_{12} = \dots x_N = 0$. Formula (5.2) predicts, however, also the existence of mixed events, in which some pions are fast and other are slow. These events behave in the way similar to the one described above.

Until now we discussed only the behaviour of the individual channels described by the amplitude (5.13). It is, however, very interesting to consider also the inclusive diffractive process, *i.e.* the sum over all diffractive channels for production of wee pions. Using Eq. (7.2) we obtain the following formula for missing mass distribution of inclusive diffractive excitation in the triple Regge region

$$\frac{d\sigma}{d\mathcal{M}^2} = \frac{1}{\mathcal{M}^2 \log s} \sum N a_N \left(\frac{\log \mathcal{M}^2}{\log s}\right)^{N-1} =$$

$$= \frac{1}{\mathcal{M}^2 \log s} U'\left(\frac{\log \mathcal{M}^2}{\log s}\right). \tag{7.3}$$

Here

$$U'(z) = \frac{dU(z)}{dz} \text{ and } U(z) = \sum a_N z^N \tag{7.4}$$

is related to the generating function of the multiplicity distribution.

To study the scaling properties of the spectrum we introduce the scaling variable

$$\zeta = \frac{\mathcal{M}^2}{s}. \tag{7.5}$$

Distribution (7.3) can be written in the form

$$\frac{d\sigma}{d\zeta} = \frac{1}{\zeta \log s} U' \left(1 + \frac{\log \zeta}{\log s}\right) \tag{7.6}$$

and we see that its scaling properties are determined by the behaviour of the function $U'(z)$ in the vicinity of the point $z = 1$.

If the function $U'(z)$ is analytic at $z = 1$, all moments of the multiplicity distribution (determined by derivatives of $U(z)$ at $z = 1$) are finite in the high energy limit. In this case the missing mass distribution has the form

$$\frac{d\sigma}{d\zeta} = \frac{1}{\zeta \log s} \tag{7.7}$$

i.e. it does not scale. The violation of scaling is, however, only logarithmic.

If we would like the distribution (7.6) to scale, *i.e.* to be only a function of ζ[12], the function $U'(z)$ must have a simple pole at $z = 1$. In such a case the mass distribution takes the form

$$\frac{d\sigma}{d\zeta} = \frac{1}{\zeta} \ln \zeta. \tag{7.8}$$

Furthermore, all the moments of the multiplicity distribution tend to infinity at high energies. It can be shown that (barring factors like log log s) the correlation parameters behave like

$$f_n \sim (\log s)^n. \tag{7.9}$$

8. *Conclusions*

We have shown that the overlap matrix formalism can be used for the calculation of the diffractive production of particles. In this approach the diffractive production is generated as a shadow of non-diffractive interactions. This shadow is described by the inelastic elements of the overlap matrix.

To study the properties of the diffractively produced systems, we have calculated the overlap matrix in the Uncorrelated Jet Model. Our main conclusions can be summarized as follows.

a) In the particular version of the model which we consider there is no diffractive production without correlations in non-diffractive interactions. The energy and momentum conservation introduces the long-range correlations which in turn generate the diffractive

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[12] Such a behaviour is suggested by the recent data from CERN Intersecting Storage Ring (Albrow *et al.* 1972) and by the phenomenological analysis of Chan *et al.* (1972).

production. These energy and momentum conservation effects do not vanish even in the high-energy limit.

b) At high energies the conservation of transverse momentum is well separated from conservation of energy and longitudinal momentum. As a consequence, there are two types of diffractively produced systems. This is reflected in the two-component structure of the diffractively excited mass spectrum which splits naturally into an approximately scaling part and a non-scaling part.

c) The scaling component of the mass spectrum arises from transverse momentum conservation, and consists of slow ("wee") particles in the CM system. The energy and longitudinal momentum conservation generate the production of the fast particles in the CM system, which contribute to the non-scaling part of the mass spectrum.

The detailed properties of the diffractive channels depend in our model on the assumed properties of the meson and nucleon wave functions describing the non-diffractive interactions. We have taken the mesonic wave functions depending only on transverse momentum and scaled longitudinal momentum. The nucleon wave functions were assumed in the factorized form, depending on transverse momenta or on momentum transfer from initial to final nucleon.

For fixed mass of the excited system, the energy dependence of the cross-section for exclusive processes in which there are some slow (wee) particles in the CM system is different from that of elastic scattering. They fall down faster than the elastic cross-section at the rate $(\log s)^{-N}$ where N is the number of wee particles produced in the given channel. However, the cross-sections in these channels integrated over mass of the excited system depend on energy in the same way as the elastic scattering. The tail of the mass distribution in the channel with N wee pions is of the form

$$\frac{d\sigma_N}{d\mathcal{M}^2} \propto \frac{1}{(\log s)^N} \frac{(\log \mathcal{M}^2)^{N-1}}{\mathcal{M}^2} \tag{8.1}$$

which, integrated up to $\mathcal{M}^2 \sim s$, gives $\int \frac{d\sigma_N}{d\mathcal{M}^2} d\mathcal{M}^2 \sim$ const.

It is seen from Eq. (8.1) that the processes of production of wee pions correspond to the diffractive excitation of large missing masses.

The inclusive mass spectrum of the diffractively excited system (summed over all channels with wee pions) is consistent with the scaling property suggested by recent ISR experiments (Albrow *et al.* 1972) and by phenomenological analyses (see *e. g.* Chan *et al.* 1973): depending on the details of the model, the inclusive missing mass spectrum may either scale exactly, or only up to a factor of $1/\log s$.

If the inclusive mass distribution scales exactly, the model predicts that in the triple-Regge region its shape is of the form

$$\frac{d\sigma}{d\zeta} = \frac{1}{\zeta \log \zeta} \tag{8.2}$$

where ζ is the scaling variable, $\zeta = \dfrac{\mathcal{M}^2}{s}$.

In the Regge language such a behaviour corresponds to the exchange of the singularity with intercept $\alpha = 1$, *i. e.* it describes the so-called triple-pomeron vertex. The logarithmic factor indicates that this singularity is a cut rather than a pole, as indeed expected in the models without short range order (see *e. g.* Sivers (1972), for recent discussion and other references).

Other characteristic features of these processes are discussed in Section 7.

The diffractive production of fast particles is characterized by the amplitudes which have the same energy dependence as the elastic amplitude, in contrast to the behaviour of the wee particles production which was discussed up to now. Its most important properties can be summarized as follows.

(*i*) It factorizes into parts describing right-moving and left-moving particles up to the terms of the order $1/\log s$.

(*ii*) The amplitude is a function of transverse momenta and scaled longitudinal momenta of all produced particles. It vanishes as x_i for $x_i \to 0$, where x_i is the scaled longitudinal momentum of the particle. This property implies that no wee particles are produced and that the integrated cross-section for production on N fast particles has the same energy dependence as elastic scattering.

(*iii*) The mass distribution of the diffractively excited system of fast particles shows a large (non-resonant) bump at small masses, followed by a sharp drop. For large masses, *i. e.* in the triple-Regge limit the spectrum behaves as

$$\frac{d\sigma}{d\mathcal{M}^2} \sim \mathcal{M}^{-6}. \tag{8.3}$$

This asymptotic behaviour at large $\mathcal{M}^2$ is independent of multiplicity. Formula (8.3) shows that the considered processes contribute to the non-scaling part of the diffractively excited mass spectrum.

(*iv*) It can be shown under rather general conditions that all moments of the multiplicity distribution are finite and energy independent. In fact, most of the diffractive production of fast particles goes into single pion production. This feature seems attractive, because it may perhaps serve as qualitative explanation of the experimental evidence that, for small missing mass, the diffractive dissociation proceeds (in most of the known cases) *via* two-body intermediate steps (see *e. g.* Morrison 1970).

In closing this Section, let us observe that the general picture of the high-energy interactions described in this paper is the same as in the so-called two-component models of particle production (*cf.* Wilson 1970, Fiałkowski and Miettinen 1972, Frazer *et al.* 1972, Harari and Rabinovici 1972, Quigg and Jackson 1972). However, in our approach the diffractive and non-diffractive components are closely related through the unitarity condition.

Another important feature of our model is that the non-diffractive component does not have the property of short range order. In the absence of short-range order the leading (Pomeranchuk) singularity in diffractive scattering is a cut rather than a pole and consequently our results are different from those obtained by other authors (Frazer and Snider 1973, Kajantie and Ruuskanen 1973).

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APPENDIX A

High-energy behaviour of the generalized elastic amplitudes

To determine the high-energy behaviour of $\Phi_{f\to i}(R)$, we use the known formula for the asymptotic behaviour of the sum of phase-space integrals (de Groot 1971, 1972, and Bassetto, Toller and Sertorio 1971)

$$C_\lambda(Q) = \sum_{m=0}^{\infty} \frac{1}{m!} \int \delta^{(4)}(Q-Q_1- \dots -Q_m) \prod_{j=1}^{m} \frac{d^3Q_j}{E_j} |\varrho(Q_j)|^2$$

$$\underset{\overline{Q}\to\infty}{\to} \frac{H(\lambda)}{\Omega^2} \overline{Q}^{2(\lambda-1)} \exp(-Q_\perp^2/\Omega^2), \tag{A1}$$

where

$$\overline{Q}^2 = Q_0^2 - Q_{||}^2, \tag{A2}$$

$$H(\lambda) = \frac{2}{\Gamma(\lambda)^2 \bar{\mu}^\lambda} \tag{A3}$$

and

$$\Omega^2 = \lambda\kappa^2 \log \frac{\overline{Q}^2}{\overline{Q}_0^2}. \tag{A4}$$

Here $\overline{Q}_0^2$, $\bar{\mu}$ and κ^2 are constants depending on the shape of the transverse momentum distribution of pions.

Using Eqs (A1), (3.9), (5.7) and (5.8) we obtain

$$\Phi_{i\to f}(R) = \left(\frac{s}{s_0}\right)^{-\lambda} \sqrt{E_A E_B E_C E_D} \int d^3k_1 d^3k_2 \xi^*(w_C, w_1) \times$$

$$\times \xi(w_A, w_1) \xi^*(w_D, w_2) \xi(w_B, w_2) C_\lambda(R-k_1-k_2) =$$

$$= \sqrt{E_A E_B E_C E_D}\, s_0^\lambda H(\lambda) \int dx_1 dx_2 d^2k_{1\perp} d^2k_{2\perp} \xi^*(w_C, w_1) \xi(w_A, w_1) \times$$

$$\times \xi^*(w_D, w_2) \xi(w_B, w_2) \left[(Z_0-x_1-x_2)^2-(Z_{||}-x_1+x_2)^2\right]^{\lambda-1} \times$$

$$\times \exp\left[-(R_\perp-k_{1\perp}-k_{2\perp})^2/\Omega^2\right]/\Omega^2. \tag{A5}$$

Here $x_{1,2} = |k_{1,2\parallel}|/\sqrt{s}$, s_0 is a constant and $Z = R/\sqrt{s}$. Thus the generalized elastic amplitude can be written in the form

$$\Phi_{i\to f}(R) = \sqrt{E_A E_B E_C E_D}\, s_0^\lambda H(\lambda) \int_0^{Z_+/2} dx_1 \int_0^{Z_-/2} dx_2 (Z_+ - 2x_1)^{\lambda-1} \times$$

$$\times (Z_- - 2x_2)^{\lambda-1} \int d^2k_{1\perp} d^2k_{2\perp} \xi^*(w_C, w_1)\xi(w_A, w_1)\xi^*(w_D, w_2)\xi(w_B, w_2) \times$$

$$\times \exp[-(R_\perp - k_{1\perp} - k_{2\perp})^2/\Omega^2]/\Omega^2. \tag{A6}$$

In the high-energy limit we have $\Omega \to \infty$ and the leading term simplifies considerably to give the factorized form

$$\Phi_{i\to f}(R) = \sqrt{E_A E_B E_C E_D}\, \frac{s_0^\lambda H(\lambda)}{\Omega_0^2} h_+(Z_+, x_A, x_C) h_-(Z_-, x_B, x_D) +$$

$$+ \text{ terms of the order of } \Omega_0^{-4}, \tag{A7}$$

where

$$h_+(Z_+, x_A, x_C) = \int_0^{Z_+/2} dx (Z_+ - 2x)^{\lambda-1} \int d^2k_{1\perp} \xi^*(w_C, w_1)\xi(w_A, w_1) \tag{A8}$$

and

$$h_-(Z_-, x_B, x_D) = \int_0^{Z_-/2} dx (Z_- - 2x)^{\lambda-1} \int d^2k_{2\perp} \xi^*(w_D, w_2)\xi(w_B, w_2) \tag{A9}$$

and Ω_0 is given by Eq. (5.19)

$$\Omega_0^2 = \lambda\kappa^2 \log\frac{s}{s_0}. \tag{A10}$$

APPENDIX B

Calculation of the overlap matrix

In this Appendix we derive the formulae (3.7)–(3.9). Starting from Eq. (3.1) we replace the δ-function of energy and momentum conservation by its Fourier representation

$$\delta^{(4)}(\hat{P} - P) = \frac{1}{(2\pi)^4}\int d^4x e^{i(\hat{P}-P)\cdot x} \tag{B1}$$

and express the total four-momentum operator $\hat{P}$ by creation and annihilation operators of pions and nucleons

$$\hat{P}_\mu = \int \frac{d^3k}{E} k_\mu (a^\dagger(k)a(k) + b^\dagger(k)b(k)). \tag{B2}$$

Using (B1) and (B2), the T_N matrix can be written in the form

$$T_N = \frac{1}{(2\pi)^4} \int d^4x \tilde{T}_N(x) e^{-iP\cdot x}. \tag{B3}$$

The Fourier transform $\tilde{T}_N(x)$ factorizes into nucleon and meson parts:

$$\tilde{T}_N(x) = \tilde{T}_n(x)\tilde{S}_\pi(x), \tag{B4}$$

where

$$\tilde{T}_n(x) = \exp\left(i\int \frac{d^3k}{E}\, k\cdot x b^\dagger(\boldsymbol{k})b(\boldsymbol{k})\right) T_n \tag{B5}$$

and

$$\tilde{S}_\pi(x) = \exp\left(i\int \frac{d^3k}{E}\, k\cdot x a^\dagger(\boldsymbol{k})a(\boldsymbol{k})\right) S_\pi. \tag{B6}$$

The overlap operator can now be written as

$$F \equiv T_N^\dagger T_N = \frac{1}{(2\pi)^8}\int d^4x d^4x' e^{-i(P\cdot x - P'\cdot x')} \times$$
$$\times\, \tilde{T}_n^\dagger(x')\tilde{T}_n(x)\tilde{S}_\pi^\dagger(x')S_\pi(x), \tag{B7}$$

where P and P' are the total four-momenta of initial and final state, respectively.

The most complicated part of the derivation is the calculation of the operator $\tilde{S}_\pi^\dagger(x')S_\pi(x)$. The crucial step in this calculation is the identity

$$\exp\left(-i\int\frac{d^3k}{E}\,\varrho^*(k)a(\boldsymbol{k})\right)\exp\left(i\int\frac{d^3k}{E}\,k\cdot(x-x')a^\dagger(\boldsymbol{k})a(\boldsymbol{k})\right)\times$$
$$\times\exp\left(i\int\frac{d^3k}{E}\,\varrho^*(k)a(\boldsymbol{k})\right) =$$
$$= \exp\left(i\int\frac{d^3q}{E(q)}\,\varrho^*(q)a(\boldsymbol{q})\,(e^{-iq\cdot(x-x')}-1)\right)\exp\left(i\int\frac{d^3k}{E}\,k\cdot(x-x')a^\dagger(\boldsymbol{k})a(\boldsymbol{k})\right) \tag{B8}$$

which can be derived from the general formula

$$e^{iA}Be^{-iA} = \sum_{k=0}^{\infty}\frac{i^k}{k!}A^k\{B\} \tag{B9}$$

where A and B are any operators and

$$A^0\{B\} = B, \quad A^k\{B\} = [A, A^{k-1}\{B\}]. \tag{B10}$$

Using Eq. (B8), the operator $\tilde{S}_\pi^+(x')\,\tilde{S}_\pi(x)$ can be transformed into

$$\tilde{S}_\pi^\dagger(x')\tilde{S}_\pi(x) = \exp\left(\int \frac{d^3k}{E(k)}\,|\varrho(k)|^2(e^{ik\cdot(x-x')}-1)\right)\times$$

$$\times\exp\left(i\int \frac{d^3q}{E(q)}\,\varrho(q)a^\dagger(q)\,(e^{iq\cdot(x-x')}-1)\right)\times$$

$$\times\exp\left(i\int \frac{d^3p}{E(p)}\,\varrho^*(p)a(p)\,(e^{-ip\cdot(x-x')}-1)\right)\exp\left(i\int \frac{d^3k}{E(k)}\,k\cdot(x-x')a^\dagger(k)a(k)\right). \tag{B11}$$

The last step is the calculation of the matrix element for production of N pions which gives

$$\langle q_1, \ldots, q_N|\tilde{S}_\pi^\dagger(x')\tilde{S}_\pi(x)|0\rangle = \exp\left(\int \frac{d^3k}{E(k)}\,|\varrho(k)|^2(e^{ik\cdot(x-x')}-1)\right)\times$$

$$\times\langle q_1, \ldots, q_N|\exp\left(i\int \frac{d^3q}{E(q)}\,(e^{iq\cdot(x-x')}-1)\varrho(q)a^\dagger(\boldsymbol{q})\right)|0\rangle =$$

$$= \exp\left(\int \frac{d^3k}{E(k)}\,|\varrho(k)|^2e^{ik\cdot(x-x')}\right)i^N\prod_{j=1}^{N}\varrho(q_j)\,(e^{iq_j\cdot(x-x')}-1). \tag{B12}$$

In this formula the first factor describes the elastic shadow scattering $N=0$ and the other factors are characteristic for the production. The product of the last factors is a sum of 2^N terms containing exponential depending on momenta of produced pions. Each of these terms has a simple interpretation which is presented graphically in Fig. 1.

Eq. (B12) is a special case of the general formula

$$\langle q_1, \ldots, q_N|\tilde{S}_\pi^\dagger(x')\tilde{S}_\pi(x)\,|Q_1, \ldots, Q_k\rangle = e^{-V}\exp\left(\int \frac{d^3k}{E(k)}\,|\varrho(k)|^2e^{ik\cdot(x-x')}\right)\times$$

$$\times\exp\,(iP_\pi\cdot(x-x'))\,\langle 0|\prod_{j=1}^{N}\,(a(\boldsymbol{q}_j)+i\varrho(q_j)\,(e^{iq_j\cdot(x-x')}-1))\times$$

$$\times\prod_{l=1}^{k}(a^\dagger(Q_l)+i\varrho^*(Q_l)\,(e^{-iQ_l\cdot(x-x')}-1))\,|0\rangle, \tag{B13}$$

where

$$P_\pi = \sum_{l=1}^{k} Q_l. \tag{B14}$$

Inserting Eq. (B12) into Eq. (B7) and performing integration over x and x' we obtain the formulae (3.7)–(3.9), where we additionally distinguish explicitly between probability amplitudes $\varrho(q)$ for the initial and final states.

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