KAON CONDENSATE WITH TRAPPED NEUTRINOS AND HIGH-DENSITY SYMMETRY ENERGY BEHAVIOR

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Effects of the neutrino trapping and symmetry energy behavior are investigated in the framework of the chiral Kaplan–Nelson model with kaon condensation. Decrease in the condensation threshold during deleptonization if found to be generic regardless uncertainties in the nucleon–kaon interactions and symmetry energy. Quantitatively however, differences are shown to be important.

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1. Introduction

Neutron stars are born in core-collapse supernova explosions [1–6]. First minutes of their life is a period of rapid evolution [7]. This is protoneutron star (PNS) stage, where matter is transformed to final cold catalyzed matter [8] state: one of the greatest mysteries in astrophysics [9]. Among other matter with kaon condenstates, proposed by Kaplan and Nelson [10, 11], is very intriguing possibility. Unfortunately, existing experimental and observational data is still unable to select one, correct model [1]. Therefore, further investigation of effects present under particular assumptions on high-density matter model is in place.

Possibly, only nearby core-collapse supernova explosion will allow researchers to collect enough data, mainly in neutrino channel (*cf.* Fig. 3 in [12]), to find clear signatures of particular model. However, neutrinos are trapped in protoneutron stars. Therefore, it is required to study nuclear matter with two-parameter (baryon number n_B and lepton number Y_L) model at least.

Deleptonization in the first seconds after PNS is born causes decrease in lepton number from value typical for initial pre-supernova "Fe" matter $Y_{\rm Le} \sim 0.4$ [13] down to some numerically small value, *e.g.* $Y_{\rm Le} = 0$ in pure neutron matter model. Meanwhile neutrinos escape outer PNS region, but in central, high-density core we may assume quasistatic evolution of matter parameterized with decreasing $Y_{\rm Le}$.

We have dropped all finite temperature effects for simplicity, however they are potentially equally important [9]. Generally, effects of decreasing temperature are smaller than decreasing lepton number and act in similar direction on critical kaon condensation density.

Article is organized as follows: Sec. 2 presents general formalism including kaon condensate, neutrino trapping and nucleon interactions with symmetry energy. Sec. 3 discusses base results without symmetry energy and with no neutrino trapping. Sec. 4 presents effects of the neutrino trapping on kaon condensate alone, and Sec. 5 combines all three components: kaon condensate, neutrino trapping and various symmetry energy models.

2. Model with kaon condensate

Energy density for matter with kaon condensate is given by:

$$\begin{aligned} &\varepsilon(n_n,\, n_p,\, \theta,\, \mu_e,\, \mu_{\nu_e},\, \mu_K,\, \mu_\mu,\, \mu_{\nu_\mu}) \\ &= \varepsilon_{F_n} + \varepsilon_{F_p} + \varepsilon_{F_e} + \varepsilon_{F_{\nu_e}} + \varepsilon_{F_\mu} + \varepsilon_{F_{\nu_\mu}} + \varepsilon_{\rm int} + \varepsilon_{\rm kaon}\,, \end{aligned} \tag{1}$$

where variables on the left hand are: n_n, n_p — neutron and proton density, θ — amplitude of the kaon condensate, μ_i — chemical potentials. Right-hand has been decomposed into: $\varepsilon_{\rm F_i}$ — Fermi sea energies, $\varepsilon_{\rm Kaon}$ — kaon–nucleon interaction and $\varepsilon_{\rm int}$ — nucleon interaction including symmetry energy.

Nucleon part can be rewritten as:

$$\varepsilon(u,\, x) = \tfrac{3}{5}\, E_{\rm F}^0\, n_0\, u^{5/3} + u\, n_0\, (1 - 2\, x)^2\, {\rm S}(u)\,, \tag{2}$$

where $E_{\rm F}^0 = 36.885\,{\rm MeV}$. Standard parameterization ($n_0 = 0.16\,{\rm fm}^{-3}$) has been used:

$$\begin{aligned} n_p &= x\, n_B\,, && (3)\\ n_n &= (1-x)\, n_B\,, && (4)\\ n_B &= u\, n_0 && (5) \end{aligned}$$

and $S(u)$ is the *symmetry energy*.

Leptons contribute to the total energy:

$$\varepsilon_{F_i} = \frac{g_i\, {\mu_i}^4}{8\,\pi^2}\,, \tag{6}$$

where g_e=2 and $g_\nu = 1$ (see, however, footnote 2).

Generalization to other lepton families is trivial. However, we have not included μ and τ flavors, as their contribution is minimal [15, 17].

Contribution to the total energy from the kaon condensate can be computed from the Kaplan–Nelson chiral formalism [14, 15]:

$$\begin{aligned}\varepsilon_{\rm kaon} &= \frac{\mu^2 f^2 \sin^2\theta}{2} + (\cos\theta - 1) \\ &\times \left[n_B\, x\, \Sigma_{\rm Kp} + n_B\,(1-x)\,\Sigma_{\rm Kn} - f^2\, m_K^2\right]\,, \end{aligned} \tag{7}$$

where $m_K = 493,7\,\mathrm{MeV}$ and $f = 93\,\mathrm{MeV}$ is kaon mass and pion decay rate, respectively. Constants Σ_{Kp} and Σ_{Kn} define kaon–proton and kaon–neutron interaction strength. Due to large uncertainties they may be replaced with single quantity $\Sigma_{\rm KN}$ placed between 168 and 520 MeV. Some authors use $a_3 m_s$ instead:

$$\Sigma_{\rm KN} = -\frac{1}{2}\,(a_1 m_s + 2\,a_2 m_s + 4\,a_3 m_s)\,, \tag{8}$$

where $a_1 m_s = -67\,\mathrm{MeV}$, $a_2 m_s = 134\,\mathrm{MeV}$.

Kaon density is [9]:

$$n_K = \mu_K\, f^2 \sin^2\theta + n_B\,\left(\tfrac{1}{2}\,x + \tfrac{1}{2}\right)(1-\cos\theta)\,. \tag{9}$$

Minimalization of the energy must include conservation of the baryon number, electric charge and lepton numbers if the neutrinos are trapped:

$$n_n + n_p = n_B\,, \tag{10a}$$

$$n_p = n_e + n_{K^-} + n_\mu\,, \tag{10b}$$

$$x_i + Y_{\nu_i} = Y_{Li}\,. \tag{10c}$$

Baryon number density is a free parameter. Charged lepton fractions are denoted as $x_i \equiv n_i/n_B$, neutrinos as Y_{ν_i}. Conservation of the electric charge is ensured using kaon chemical potential as a Lagrange multiplier:

$$\tilde{\varepsilon} = \varepsilon - \mu(n_p - n_e - n_{K^-} - n_\mu)\,. \tag{11}$$

Electroweak and strong interactions allow reactions:

$$n \longleftrightarrow p^+ + e^- + \bar{\nu}_e\,, \tag{12a}$$

$$n \longleftrightarrow p^+ + K^-\,, \tag{12b}$$

$$n \longleftrightarrow p^+ + \mu^- + \bar{\nu}_\mu\,, \tag{12c}$$

and respective chemical potentials obey:

$$\mu_n - \mu_p = \mu_e - \mu_{\nu_e}\,, \tag{13a}$$

$$\mu_n - \mu_p = \mu_\mu - \mu_{\nu_\mu}\,, \tag{13b}$$

$$\mu_n - \mu_p = \mu_K\,. \tag{13c}$$

If neutrinos escape freely, one can put simply $\mu_{\nu_i} \equiv 0$ and we have left with only one independent chemical potential equal for all negative electric charge particles.

3. Kaon condensation without neutrinos

If neutrinos escape freely (old neutron star case) we have only one driving parameter: baryon density. Minimalized energy is:

$$\varepsilon(n_n,\, n_p,\, \theta,\, \mu_e,\, \mu_K) = \varepsilon_{F_n} + \varepsilon_{F_p} + \varepsilon_{F_e} + \varepsilon_{\mathrm{kaon}}\,. \tag{14}$$

Energy is minimalized numerically solving system of equations:

$$\frac{\partial\,\tilde{\varepsilon}(x,\mu,\,\theta)}{\partial\theta} = \frac{\partial\,\tilde{\varepsilon}(x,\mu,\,\theta)}{\partial x} = \frac{\partial\,\tilde{\varepsilon}(x,\mu,\,\theta)}{\partial\mu} = 0\,. \tag{15}$$

Functions $x(u)$, $\theta(u)$, $\mu(u)$ are immediate result of calculations. Other properties, *e.g.* $n_e(u)$, $n_K(u)$ and EOS can be then easily obtained. Condensation threshold is defined as a maximum density where still $\theta(u) = 0$. Numerical results were obtained for u up to 12 for three values $\Sigma_{\mathrm{KN}} = 168,\, 344,\, 520\,\mathrm{MeV}$[1]; covering entire considered range for this parameter.

Larger Σ_{KN} (*i.e.* smaller $a_3 m_s$) gives stronger kaon–nucleon interaction and lower condensation threshold. Both amplitude of the condensate and proton fraction exhibit asymptotic behavior. This is typical if symmetry energy is constant or growing with density. Chemical potential for kaons and electrons can reach large ($\mu < 100\,\mathrm{MeV}$) negative values if Σ_{KN} is large, and muon flavor will be produced. Overall contribution from muons is however small [15, 17]. As lepton number is not conserved electron/positron fraction can be relatively high. This behavior is completely changed with neutrino trapping, as we explain in the next section.

4. Kaon condensate and neutrino trapping

If neutrinos are trapped then properties of the dense matter are numbered by the two parameters: baryon density n_B and lepton number density Y_{Le}. In principle we have three separate lepton numbers, but initially only Y_{Le} is not identically zero. Therefore, due to large muon and taon masses we may safely restrict to electron lepton number conservation alone[2].

[1] Equivalent values are: $a_3\, m_s = -134,\, -222,\, -310\,\mathrm{MeV}$.

[2] Neutrino oscillation phenomenon indicates conservation of the total lepton number only. Therefore, (10c) could be replaced with $x_e + Y_{\nu_e} + Y_{\nu_\mu} + Y_{\nu_\tau} = Y_L \equiv Y_{\mathrm{Le}}$, where we have put $Y_L\mu = Y_L\tau = 0$. As neutrinos have very small masses all three terms are identical and finally we get $x_e + 3Y_{\nu_e} = Y_{\mathrm{Le}}$. Only difference is "degeneracy factor": $g = 3$ instead of $g = 1$ in Eq. (6) for neutrinos.

Lepton number conservation can be introduced into Eq. (11) in the following manner. We rewrite (10c) using chemical potentials:

$$\mu_e{}^3 + \tfrac{1}{2}\,\mu_{\nu_e}{}^3 = 3\pi^2\, n_B\, Y_{\rm Le}\,. \tag{16}$$

Eq. (13a) minus (13c) gives:

$$\mu_K = \mu_e - \mu_{\nu_e}\,. \tag{17}$$

Eq. (17) and (16) are used to derive μ_e and μ_{ν_e} as a functions of μ_K, n_B and $Y_{\rm Le}$. Now, minimalized function is:

$$\begin{aligned}\tilde{\varepsilon}(Y_{\rm Le}, n_B, x, \theta, \mu_K) \;&=\; \varepsilon_{F_n} + \varepsilon_{F_p} + \varepsilon_{\rm kaon} + \varepsilon_{F_{\nu_e}}(\mu_K,\, Y_{\rm Le}) \\ &\quad - \mu_K\Big[n_p - n_K - n_e(\mu_K,\, Y_{\rm Le})\Big]\,.\end{aligned} \tag{18}$$

Expression above is *explicite* very complex due to presence of the third order radicals resulting from (16).

Two parameter family of solutions is presented in Figs. 1–5. Fig. 1 illustrate lepton number conservation. Without kaons electrons are required by electric charge conservation. If condensate is present, negative charge is provided by preferred due to strong interactions kaons, and neutrinos begins to provide required lepton number amount. Proton fraction strongly depends on trapped lepton number until kaon condensation threshold. This behavior depends somewhat on $\Sigma_{\rm KN}$, *cf.* Fig. 2.

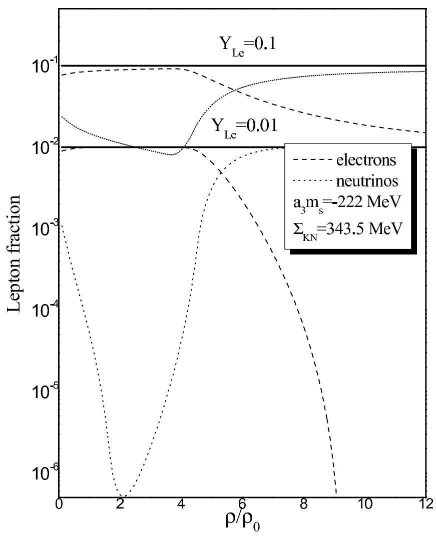

Fig. 1. Leptons fraction *versus* baryon density. Model described by Eq. (18) with no nucleon interactions.

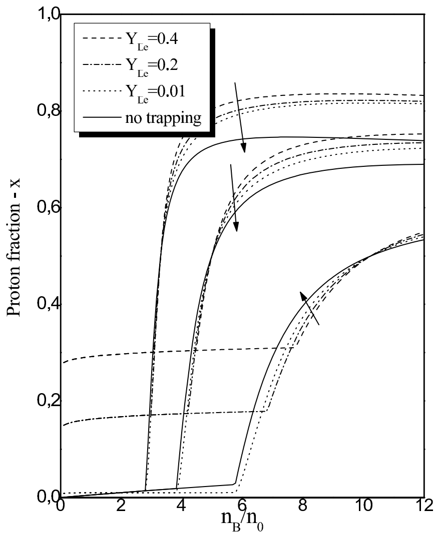

Fig. 2. Proton fraction for three values of the Σ_{KN}. Arrows indicate direction of the deleptonization effects. Model as in Fig. 1.

The most important effect of deleptonization is decrease in kaon condensation threshold, *cf.* Fig. 3. Direction of the effect does not depend on Σ_{KN}. Therefore, as kaons tends to "soften" EOS, deleptonization cause decrease in maximum neutron star mass. If PNS is born in stable state with large Y_{Le}

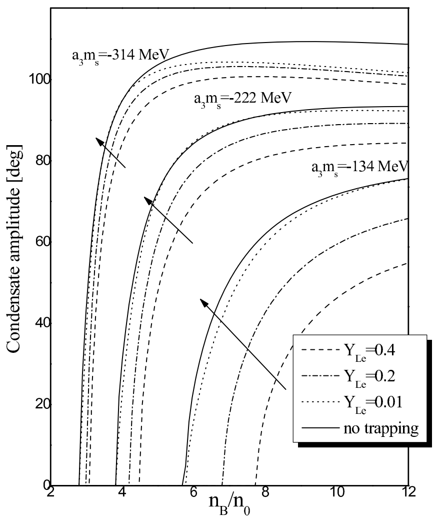

Fig. 3. Deleptonization effects on the amplitude of the kaon condensate θ. Threshold for condensation with trapped neutrinos is always higher and proportional to the lepton number density Y_{Le}. Model as in Fig. 1.

then deleptonization may cause delayed collapse to a black hole if $Y_{\rm Le}$ reach small values long time (*i.e.* tens of seconds [5]) after core-collapse[3].

Model with $Y_{\rm Le} \to 0$ is clearly different from model without neutrino trapping (Fig. 4). Initially, for large values of $Y_{\rm Le}$ model works well, but transition to the free streaming regime requires solving transport equations rather than quasistatic evolution.

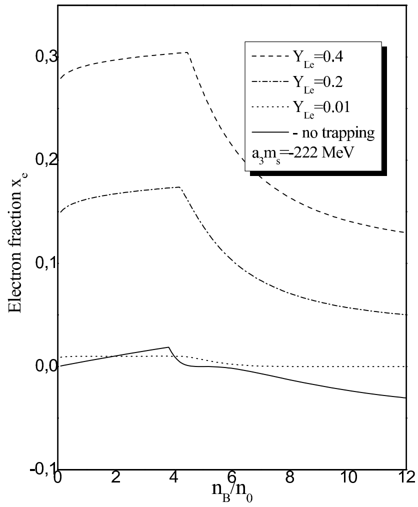

Fig. 4. Electron fraction with and without neutrino trapping. Model as in Fig. 1.

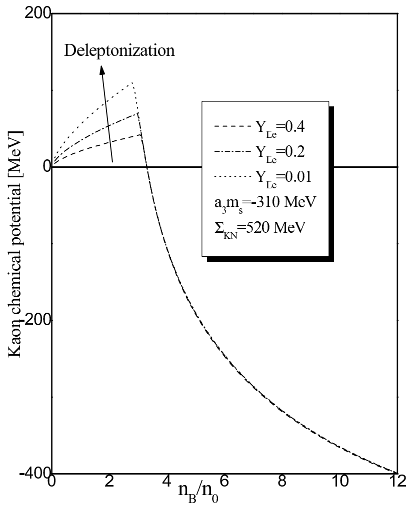

Fig. 5. Kaon chemical potential as a function of nuclear density. Model as in Fig. 1.

[3] Neutron star has not been found in the remnant of the supernova 1987A [22]. Delayed collapse is the most probable explanation.

5. Symmetry energy effects

High density energy symmetry behavior is very important for neutron stars and core-collapse supernovae (Lattimer, Yamada in [1]) but is a source of the great uncertainty [17–20]. Two qualitative behavior has been found:

- always growing (from mean field theories)
- decreasing at high densities (variational methods).

Mean field theory results can be approximated with [15]:

$$V_2 = au\,, \qquad V_2 = a\sqrt{u}\,, \qquad V_2 = a\frac{2\,u^2}{1+u}\,, \tag{19}$$

where $a = 17\,\mathrm{MeV}$.

Minimalized energy is:

$$\begin{aligned}\tilde{\varepsilon}(Y_{\mathrm{Le}}, n_B, x, \theta, \mu_K) \;=\; & \varepsilon_{F_n} + \varepsilon_{F_p} + \varepsilon_{\mathrm{kaon}} + \varepsilon_{F_{\nu_e}}(\mu_K, Y_{\mathrm{Le}}) \\ & -\mu_K\left[n_p - n_K - n_e(\mu_K, Y_{\mathrm{Le}}\right] + \varepsilon_{\mathrm{int}}\,. \end{aligned} \tag{20}$$

Fig. 6 compares mean field and variational results. In the latter, symmetry energy not only decreases, but can reach negative values and pure proton or pure neutron states are preferred.

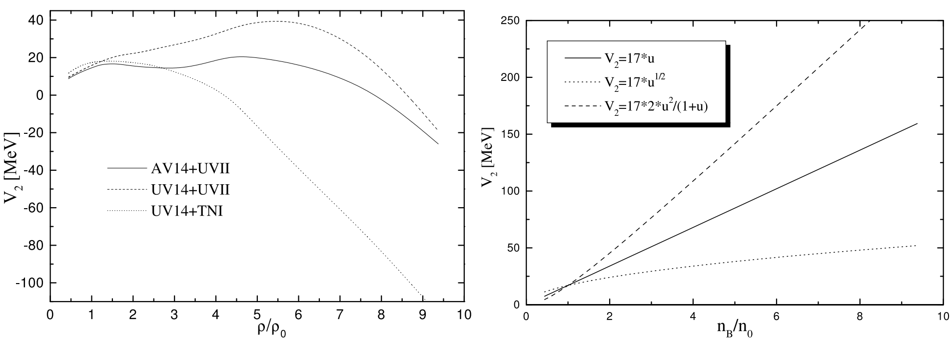

Fig. 6. Symmetry energy in variational models (left) and mean field models (right). Results are presented for UV14+UVII and linear cases (solid lines).

Symmetry energy strongly influences on matter properties both below and above condensation threshold (Figs. 7–9).

General tendency to decrease condensation threshold is however unaffected, and amplitude is still growing with density (Fig. 10).

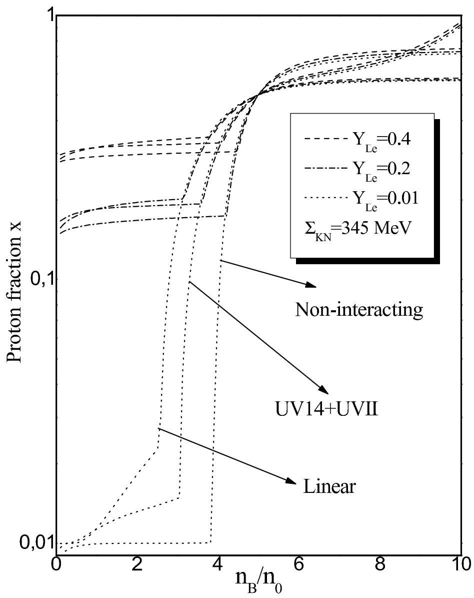

Fig. 7. Deleptonization effects on proton fraction for three symmetry energy models from Fig. 6 for $a_3m_s = -222$ MeV. It is clear that uncertainty due to symmetry energy leads to larger effects than deleptonization itself.

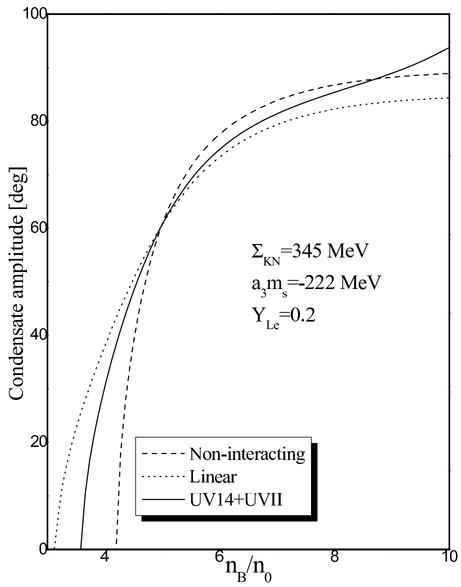

Fig. 8. Kaon condensate amplitude for $Y_{\rm Le} = 0.2$ in various symmetry energy models from Fig. 6.

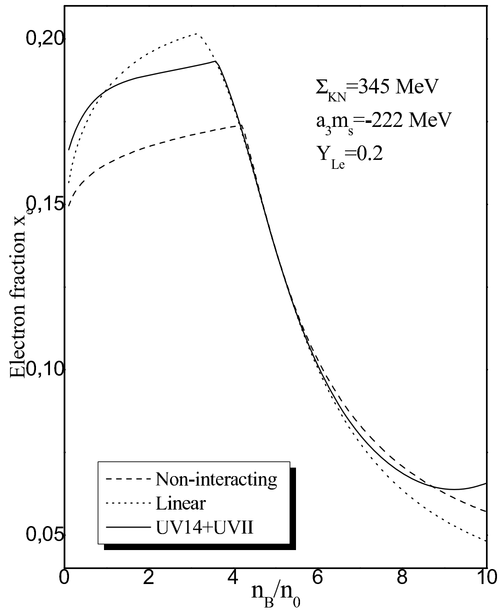

Fig. 9. The same as in Fig. 8 for the electron fraction.

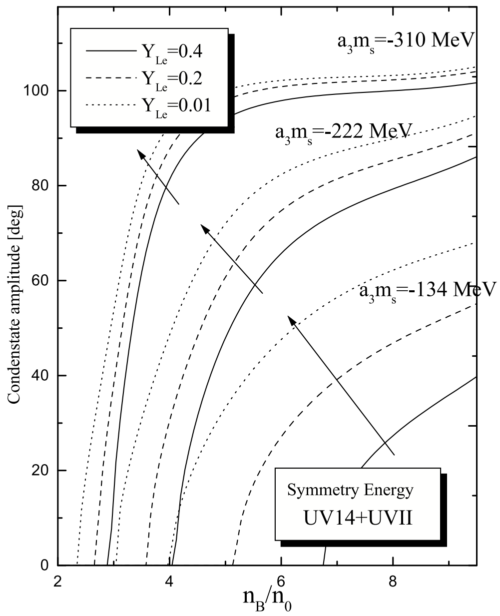

Fig. 10. Deleptonization effect on kaon condensate with symmetry energy are qualitatively similar to the case without symmetry energy, *cf.* Fig. 3.

6. Conclusions

Decrease of the kaon condensation threshold during deleptonization has been shown to be universal in the considered class of models. Main source of the uncertainty is the high density behavior of the symmetry energy and value of the kaon–nucleon interaction parameter $\Sigma_{\rm KN}$. Decrease in the condensation threshold may cause newborn PNS to be unstable as trapped lepton number is carried away from the neutrinospheres [9].

Increasing condensate volume finally may cause collapse to a black hole and immediate disappearance of the neutrino flux (effect observable for a next Galactic supernova) as neutrino spheres are swallowed under event horizon [21].

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