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WORMHOLES WITH VARYING EQUATION OF STATE PARAMETER

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We propose wormholes solutions by assuming space dependent equation of state parameter. Our models show that the existence of wormholes is supported by phantom energy. Here, the phantom energy is characterized by variable equation state parameter. We show that the averaged null energy condition (ANEC) violating phantom energy can be reduced as desired.

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1. Introduction

A Wormhole is a 'tunnel' through curved space-time, connecting two widely separated regions of our Universe or even of different universe. In a pioneer work, Morris and Thorne [1] observed, to hold a wormhole open, one has usually used an exotic matter, which violates the well known energy conditions. The exotic matter is a hypothetical form of matter that violates the weak or null energy conditions. In last few years, exotic matter has been becoming an active area of research in wormhole physics [2]. Since all known matters obey the null energy condition, $T_{\mu\nu}k^{\mu}k^{\nu} > 0$, where $T_{\mu\nu}$ is the energy stress tensor and k^{μ} any null vector, several authors [3] have considered scalar tensor theories to build wormhole-like space-time with the presence of ordinary matter in which scalar field may play the role of exotic matter. In an interesting paper, Vollick has shown how to produce exotic matter

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using scalar field [4]. Recent astrophysical observations indicated that the Universe at present is accelerating. There are different ways of evading this unexpected behavior. Most of these attempts focus on Alternative gravity theories or the supposition of existence of a hypothetical dark energy with a positive energy density and a negative pressure [5].

The matter with the property, energy density, $\rho > 0$ but pressure $p < -\rho < 0$ is known as Phantom Energy. The idea of phantom was proposed by Caldwell [6] to describe acceleration state of the Universe. As phantom energy violates the null energy condition what is needed to support traversable wormholes. So phantom energy may play a possible role for constructing wormhole-like space-time.

Several authors have recently discussed the physical properties and characteristics of traversable wormholes by taking Phantom Energy as source [7]. Recent observational analysis involving X-ray luminosity of galaxy clusters and SNe type Ia data suggests that we live in a flat Universe and its present acceleration stage is driven by a dark energy component whose equation of state may evolve in time [8]. Several authors have studied cosmological models assuming variable equation of state parameter [9]. Since in the literature of wormhole physics, this dark energy component is known as phantom energy, in this article, we propose wormhole solutions supported by phantom energy where equation of state parameter is a function of radial coordinate rather than a constant. The present work falls into two categories. In the first one, we provide phantom energy matter sources that produce wormhole-like geometry. In this category, we discuss two toy models. In the second one, we are trying to search phantom energy matter sources that produce some specified wormhole-like structures. In this category, we provide two specific toy models of wormholes. In all cases, we have established a matching of each interior wormhole metric with an exterior Schwarzschild metric.

The layout of the paper is as follows: In the second section, we shall present the model of our system. In Section 3, we shall provide four toy models of the wormholes. Section 4 is devoted to a brief summary and discussion.

2. The models and the basic equations

We consider the model, which is characterized by the exotic equation of state,

$$\frac{p}{\rho} = -w(r)\,, \tag{1}$$

where $w(r)$ is a positive function of radial coordinate.

A static spherically symmetric Lorentzian wormhole can be described by a manifold $R^2 X S^2$ endowed with the general metric in Schwarzschild co-ordinates (t, r, θ, ϕ) as

$$ds^2 = -e^{2f(r)}dt^2 + \frac{1}{\left[1 - \frac{b(r)}{r}\right]}dr^2 + r^2 d\Omega_2^2 \,, \tag{2}$$

where, $r\epsilon(-\infty, +\infty)$.

To describe a wormhole, the redshift function $f(r)$ should be finite and the shape function obeys the following properties

$$b(r_0) = r_0 \,, \tag{3}$$

where r_0 is the throat of the wormhole.

$$b'(r_0) < 1 \,, \tag{4}$$

$$b(r) < r \,, \qquad r > r_0 \,. \tag{5}$$

Also the space-time is asymptotically flat *i.e.* $\frac{b(r)}{r} \to 0$ as $\mid r \mid \to \infty$.

According to Morris and Thorne [1], we assume $f =$ constant, to make the problem simpler. This assumption implies that a traveller feels a zero tidal force. This supposition would help for an advanced engineer to construct a traversable passage.

Using the Einstein field equations $G_{\mu\nu} = 8\pi T_{\mu\nu}$, in orthonormal reference frame (with $c = G = 1$) , we obtain the following stress energy scenario,

$$\rho(r) = \frac{b'}{8\pi r^2} \,, \tag{6}$$

$$p(r) = \frac{1}{8\pi}\left[-\frac{b}{r^3}\right] , \tag{7}$$

$$p_{\mathrm{tr}}(r) = \frac{1}{8\pi}\left(1 - \frac{b}{r}\right)\left[\frac{(-b'r + b)}{2r^2(r-b)}\right] , \tag{8}$$

where $\rho(r)$ is the energy density, $p(r)$ is the radial pressure and $p_{\mathrm{tr}}(r)$ is the transverse pressure.

Using the conservation of stress energy tensor $T^{\mu\nu}_{;\nu} = 0$, one can obtain the following equation

$$p' + \frac{2}{r}p - \frac{2}{r}p_{\mathrm{tr}} = 0 \,. \tag{9}$$

Now from equation (1), by using (6) and (7), one gets,

$$\frac{p}{\rho} = -w(r) = -\frac{b}{rb'} \,. \tag{10}$$

3. Toy models of wormholes

Now, we will discuss several toy models of wormholes:

Specialization 1: $w(r) = w(\text{constant})$.

Now consider the special case, $w(r) = w(\text{constant})$, then equation (10) yields,

$$b = b_0 r^{1/w} \tag{11}$$

[b_0 is an integration constant].

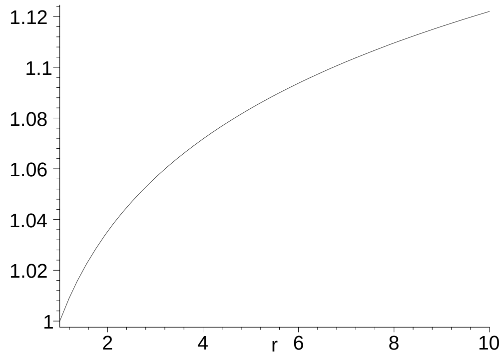

Fig. 1. Diagram of the shape function of the wormhole.

Since the space-time is asymptotically flat *i.e.* $\frac{b(r)}{r} \to 0$ as $| r | \to \infty$, then the equation (11) is consistent only when $w > 1$.

The throat of the wormhole occurs at

$$r = r_0 = b_0^{w/(w-1)} . \tag{12}$$

Now we match the interior wormhole metric to the exterior Schwarzschild metric. To match the interior to the exterior, we impose the continuity of the metric coefficients, $g_{\mu\nu}$, across a surface, S, *i.e.* $g_{\mu\nu\,(\text{int})}|_S = g_{\mu\nu\,(\text{ext})}|_S$.

[This condition is not sufficient to different space-times. However, for space-times with a good deal of symmetry (here, spherical symmetry), one can use directly the field equations to match [10].]

The wormhole metric is continuous from the throat, $r = r_0$ to a finite distance $r = a$. Now we impose the continuity of g_{tt} and g_{rr},

$$\begin{aligned} g_{tt\,(\text{int})}|_S &= g_{tt\,(\text{ext})}|_S \,, \\ g_{rr\,(\text{int})}|_S &= g_{rr\,(\text{ext})}|_S \end{aligned}$$

at $r = a$ [*i.e.* on the surface S] since $g_{\theta\theta}$ and $g_{\phi\phi}$ are already continuous. The continuity of the metric then gives generally

$$e^{2f}{}_{\text{int}}(a) = e^{2f}{}_{\text{ext}}(a) \qquad \text{and} \qquad g_{rr\,(\text{int})}(a) = g_{rr\,(\text{ext})}(a) \,.$$

Hence one can find

$$e^{2f} = \left(1 - \frac{2GM}{a}\right) \tag{13}$$

and $1 - \frac{b(a)}{a} = \left(1 - \frac{2GM}{a}\right)$ *i.e.* $b(a) = 2GM$. This implies $b_0 a^{1/w} = 2GM$. Hence,

$$a = \left(\frac{2GM}{b_0}\right)^w \tag{14}$$

i.e. matching occurs at $a = \left(\frac{2GM}{b_0}\right)^w$.

The interior metric $r_0 < r \leq a$ is given by

$$ds^2 = -\left[1 - b_0 a^{(1-w)/w}\right] dt^2 + \frac{dr^2}{\left[1 - b_0 r^{(1-w)/w}\right]} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) . \tag{15}$$

The exterior metric $a \leq r < \infty$ is given by

$$ds^2 = -\left[1 - \frac{b_0 a^{1/w}}{r}\right] dt^2 + \frac{dr^2}{\left[1 - \frac{b_0 a^{1/w}}{r}\right]} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) . \tag{16}$$

Here, one can see that the null energy condition is violated, $p + \rho < 0$ and consequently all the other energy conditions. Now we will check whether the wormhole geometry is, in principle, suffered by arbitrary small amount averaged null energy condition (ANEC) violating phantom energy. The ANEC violating matter can be quantified by the integrals $I = \oint \rho dV$, $I = \oint (p_i + \rho) dV$. In the model, we have assumed that the ANEC violating matter is related only to p (radial pressure), not to the transverse components [as one can see from field equations (6)–(8), $p_{\rm tr} = \left(\frac{-1+w(r)}{2}\right)\rho$ and $w(r) > 1$].

According to Visser *et al.* [11], the information about the 'total amount' of ANEC violating matter in the space-time is given by the integral,

$$I = \oint (p + \rho) dV = 2 \int_{r_0}^{\infty} (p + \rho) 4\pi r^2 dr \tag{17}$$

[$dV = r^2 \sin\theta dr d\theta d\phi$, factor two comes from including both wormhole mouths].

From the field equations, one can get,

$$p + \rho = \frac{1}{8\pi r}\left(1 - \frac{b}{r}\right)\left[\ln \frac{1}{\left(1 - \frac{b}{r}\right)}\right]' . \tag{18}$$

Hence,

$$I = \left[(r-b) \ln \left(\frac{r}{r-b} \right) \right]_{r_0}^{\infty} - \int_{r_0}^{\infty} \left[(1-b') \ln \left(\frac{r}{r-b} \right) \right] dr \,. \tag{19}$$

For the first expression, we see that, at the throat r_0, $b(r_0) = r_0$, the boundary term at r_0 vanishes. Now we consider the boundary term at infinity.

Let us denote as χ the contribution into this term from infinity. Then,

$$\chi = \lim_{r\to\infty} (r-b) \ln \left[\frac{r}{r-b} \right] \,. \tag{20}$$

This can be rewritten as

$$\chi = \lim_{r\to\infty} r \left(1 - \frac{b}{r} \right) \ln \left[\frac{r}{r-b} \right] \,. \tag{21}$$

Now, as in this limit the quantity $\frac{b}{r}$ is small, we may expand the logarithm as $\ln\left(1-\frac{b}{r}\right) = -\frac{b}{r} + ...$, where only the main term is retained. Neglecting here the term $\frac{b}{r}$ in parentheses, one obtains $\chi = b(\infty)$. Here, $b(\infty) = \infty$. Hence, in this case the total amount of ANEC violating matter is infinitely large. This case is not physically interesting.

Specialization 2: $w(r) = Ar^n$

In this case, we consider $w(r) = Ar^n$, where n and A are two positive constants. For this consideration, equation (10) gives,

$$b = \exp\left(B - \frac{1}{Anr^n} \right) \,, \tag{22}$$

where B is an integration constant.

We assume, throat of the wormhole occurs at $r = r_0$, then $b(r_0) = r_0$ implies

$$B = \frac{1}{Anr_0^n} + \ln r_0 \,. \tag{23}$$

So, the shape function takes the following form as

$$b = \exp\left(\frac{1}{Anr_0^n} + \ln r_0 - \frac{1}{nAr^n} \right) \,. \tag{24}$$

Since, $n > 0$, $w > 1$, for all $r > r_0 > [\frac{1}{A}]^{1/n}$. So the assumption $w(r) = Ar^n$ is justified to explore the phantom energy with r dependent equation of state. Here the space time is asymptotically flat *i.e.* $\frac{b(r)}{r} \to 0$ as $|\, r \,| \to \infty$.

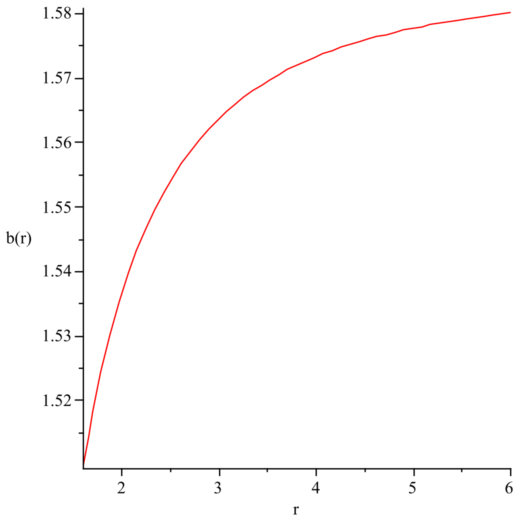

Fig. 2. Diagram of the shape function of the wormhole.

From the graph (Fig. 3), one can also note that when $r > r_0$, $G(r) < 0$ *i.e.* $b(r) - r < 0$. This implies $\frac{b(r)}{r} < 1$ when $r > r_0$. Also, from the graph, we see that G is a decreasing function of r for $r \geq r_0$ and hence $G'(r) < 0$ for $r \geq r_0$. In other words, $b'(r_0) < 1$ *i.e.* flare-out condition has been satisfied. Thus obtained shape function would represent a wormhole structure.

Now we can match this interior wormhole metric with exterior Schwarzschild metric at a, where

$$a = \left[\frac{1}{An\left(\frac{1}{Anr_0^n} + \ln \frac{r_0}{2GM} \right)} \right]^{1/n}.$$

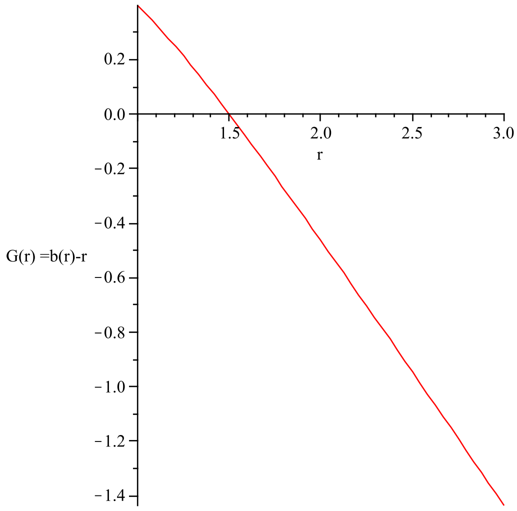

Fig. 3. Throat occurs where $G(r)$ cuts r axis *i.e.* at $r = 1.5$ (choosing suitably the parameters as $r_0 = 1.5$, $A = 4$ and $n = 2$).

Here the interior metric $r_0 < r \leq a$ is given by

$$
\begin{aligned}
ds^2 &= -\left[1 - \frac{\exp\left(\frac{1}{Anr_0^n} + \ln r_0 - \frac{1}{nAa^n}\right)}{a}\right] dt^2 \\
&\quad + \frac{dr^2}{\left[1 - \frac{\exp\left(\frac{1}{Anr_0^n} + \ln r_0 - \frac{1}{nAa^n}\right)}{r}\right]} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\,. \qquad (25)
\end{aligned}
$$

The exterior metric $a \leq r < \infty$ is given by

$$
\begin{aligned}
ds^2 &= -\left[1 - \frac{\exp\left(\frac{1}{Anr_0^n} + \ln r_0 - \frac{1}{nAa^n}\right)}{r}\right] dt^2 \\
&\quad + \frac{dr^2}{\left[1 - \frac{\exp\left(\frac{1}{Anr_0^n} + \ln r_0 - \frac{1}{nAa^n}\right)}{r}\right]} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\,. \qquad (26)
\end{aligned}
$$

In this case, we are interested in measuring the total amount of ANEC violating matter. We consider the wormhole field deviates from the throat out to a radius a. Thus the total amount of ANEC violating matter is matching the interior solution to an exterior space-time at a. Then the volume integral takes the value,

$$\begin{aligned} I &= [b(a) - a]\left[\ln\left(1 - \frac{b(a)}{a}\right)\right] + (a - r_0) \\ &\quad -(a\ln a - r_0 \ln r_0) - [b(a) - a]\left[\ln\left(\frac{a - b(a)}{e}\right)\right] \\ &\quad +[b(a)\ln a - b(r_0)\ln r_0] - \ln\frac{a}{r_0} - \frac{1}{An^2}\left(\frac{1}{a^n} - \frac{1}{r_0^n}\right) + \ldots . \end{aligned} \tag{27}$$

This implies that the total amount of ANEC violating matter depends on several parameters, namely, a, n, A, r_0. If we kept the parameters n, r_0, A fixed, then the parameter a plays a significant role in reducing the total amount of ANEC violating matter. Thus total amount of ANEC violating matter can be made small by taking suitable position, where interior wormhole metric will match the exterior Schwarzschild metric. This proves that it is possible to construct a wormhole with small amount of phantom energy characterized by variable equation of state parameter.

Specialization 3: Specific shape function: $b(r) = D\left(1 - \frac{A}{r}\right)\left(1 - \frac{B}{r}\right)$**.**
Consider the specific form of the shape function as

$$b(r) = D\left(1 - \frac{A}{r}\right)\left(1 - \frac{B}{r}\right), \tag{28}$$

where A, B and $D(> 0)$ are arbitrary constants.

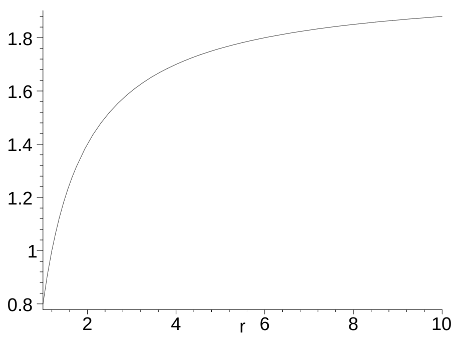

Fig. 4. Diagram of the shape function of the wormhole.

For this case, the equation of state parameter function takes the form

$$w(r) = \frac{[(r - A)(r - B)]}{[(A + B)r - 2AB]}. \tag{29}$$

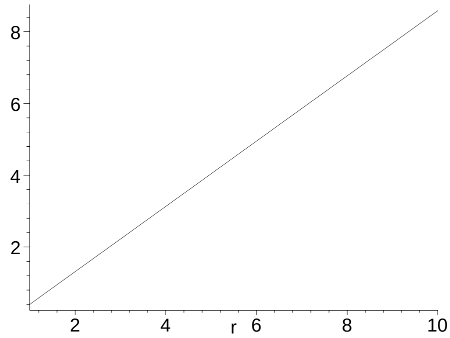

Fig. 5. Diagram of the equation of state parameter. Here r cannot be taken arbitrarily large. The figure is limited by values $\leq a$, where interior wormhole metric will match with exterior Schwarzschild metric.

We will now verify whether the particular choice of the shape function would represent the wormhole structure. One can easily see that $\frac{b(r)}{r} \to 0$ as $\mid r \mid \to \infty$.

Throat of the wormhole occurs at $r = r_0$, where r_0 satisfies the following equation $b(r_0) = r_0$ *i.e.* $r_0^3 - Dr_0^2 + (A+B)Dr_0 - ABD = 0$.

The solution of this equation is

$$r_0 = S + T + \frac{D}{3}, \tag{30}$$

where

$$\begin{aligned} S &= \left[R + \sqrt{Q^3 + R^2}\right]^{1/3} \quad \text{and} \quad T = \left[R - \sqrt{Q^3 + R^2}\right]^{1/3}, \\ Q &= \frac{3D(A+B) - D^2}{9}, \qquad R = \frac{27ABD + 2D - 9D^2(A+B)}{54}. \end{aligned}$$

Since r_0 is a root of the above equation, then by standard theorem of algebra, either $g(r) \equiv b(r) - r < 0$ for $r > r_0$ and $g(r) > 0$ for $r < r_0$ or $g(r) > 0$ for $r > r_0$ and $g(r) < 0$ for $r < r_0$. Let us take the first possibility and one can note that for $r > r_0$, $g(r) < 0$, in other words, $b(r) < r$. But when $r < r_0$, $g(r) > 0$, this means, $b(r) > r$, which violates the wormhole structure given in Eq. (2).

Now we are matching our interior wormhole metric with the exterior Schwarzschild metric at a where

$$a = \frac{AD + BD + \sqrt{(AD+BD)^2 - 4ABD(D - 2GM)}}{2(D - 2GM)}.$$

Here the interior metric $r_0 < r \leq a$ is given by

$$\begin{aligned} ds^2 &= -\left[1 - \frac{D}{a}\left(1 - \frac{A}{a}\right)\left(1 - \frac{B}{a}\right)\right] dt^2 \\ &+ \frac{dr^2}{\left[1 - \frac{D}{r}\left(1 - \frac{A}{r}\right)\left(1 - \frac{B}{r}\right)\right]} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\,. \end{aligned} \tag{31}$$

The exterior metric $a \leq r < \infty$ is given by

$$\begin{aligned} ds^2 &= -\left[1 - \frac{D}{r}\left(1 - \frac{A}{a}\right)\left(1 - \frac{B}{a}\right)\right] dt^2 \\ &+ \frac{dr^2}{\left[1 - \frac{D}{r}\left(1 - \frac{A}{a}\right)\left(1 - \frac{B}{a}\right)\right]} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\,. \end{aligned} \tag{32}$$

In this case, the total amount of ANEC violating matter in space-time with a cutoff of the stress energy at a is given by

$$\begin{aligned} I &= D + [a - b(a)]\left[1 + \ln\left(1 - \frac{b(a)}{a}\right)\right] + (a - r_0) - D\ln\frac{a}{r_0} \\ &- D(A+B)\left(\frac{1}{a} - \frac{1}{r_0}\right) - 2ABD\left(\frac{1}{a^2} - \frac{1}{r_0^2}\right)\,. \end{aligned} \tag{33}$$

This implies that the total amount of ANEC violating matter depends on several parameters, namely, a, A, B, D, r_0. If we kept the parameters A, B, D, r_0 fixed, then the parameter a plays a significant role in reducing the total amount of ANEC violating matter. Thus total amount of ANEC violating matter can be made small by taking a suitable position, where interior a wormhole metric will match the exterior Schwarzschild metric. This proves that it is possible to construct a wormhole with small amount of phantom energy characterized by variable equation of state parameter.

According to Morris and Thorne [1], the r co-ordinate is ill-behaved near the throat, but proper radial distance

$$l(r) = \pm\int_{r_0^+}^{r} \frac{dr}{\sqrt{1 - \frac{b(r)}{r}}} \tag{34}$$

must be well behaved everywhere *i.e.* we must require that $l(r)$ is finite throughout the space-time.

For our model, (taking $B = 0$), one can determine the proper distance through the wormhole as

$$l(r) = \sqrt{r^2 - Dr + DA} + \frac{D}{2}\ln\left[\frac{2\sqrt{r^2 - Dr + DA} + 2r - D}{2r_0 - D}\right]\,. \tag{35}$$

The radial proper distance is measured from r_0 to any $r > r_0$. Note that on the throat $r = r_0$, $l = 0$.

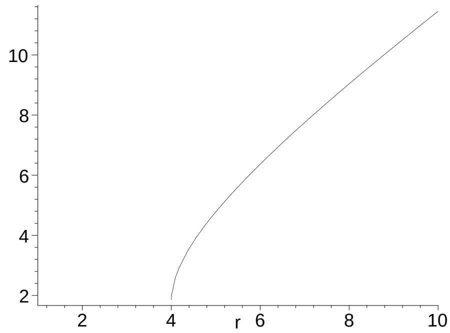

Fig. 6. Diagram of the radial proper distance ($D = 2,\ A = -4,\ r_0 = 4$).

Specialization 4: Specific shape function: $b(r) = A \tanh Cr$.

Now we make the specific choice for the shape function as

$$b(r) = A \tanh Cr\,, \tag{36}$$

where A (> 0) and C are arbitrary constants.

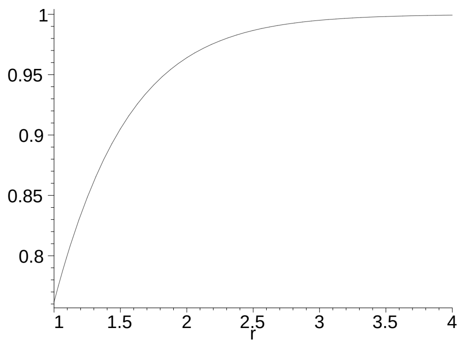

Fig. 7. Diagram of the shape function of the wormhole.

Using the equation (10), one gets

$$w(r) = \frac{C}{2r} \sinh 2Cr\,. \tag{37}$$

It is easy to verify that the above particular choice of the shape function would represent the wormhole structure. Here, $\frac{b(r)}{r} \to 0$ as $\mid r \mid \to \infty$ and throat occurs at $r = r_0$ for which $b(r_0) = r_0$ *i.e.* $A \tanh Cr_0 = r_0$.

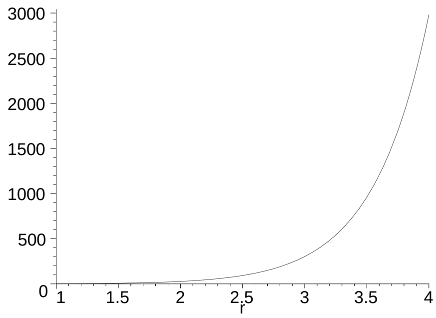

Fig. 8. Diagram of the equation of state parameter. Here r cannot be taken arbitrarily large. The figure is limited by values $\leq a$, where interior wormhole metric will match the exterior Schwarzschild metric.

[If one chooses $A = 2$ and $C = 1$, the graph of the function $F(r) = b(r) - r$ indicates the point r_0 where $F(r)$ cuts the 'r' axis. From the graph, one can also note that when $r > r_0$, $F(r) < 0$ *i.e.* $b(r) - r < 0$. This implies $\frac{b(r)}{r} < 1$ when $r > r_0$.]

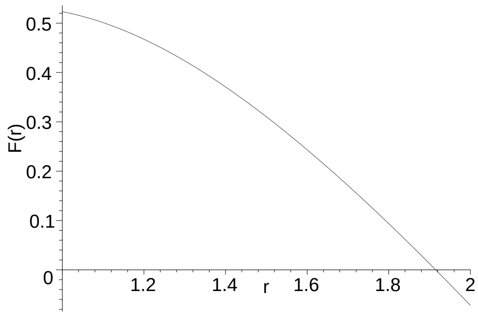

Fig. 9. Throat occurs where $F(r)$ cuts r axis.

Now matching this interior metric of wormhole with the exterior Schwarzschild metric at a, where $a = \frac{1}{2C} \ln \frac{A+2GM}{A-2GM}$, one gets, the interior metric $r_0 < r \leq a$ as

$$ds^2 = -\left[1 - \frac{A \tanh Ca}{a}\right] dt^2 + \frac{dr^2}{[1 - \frac{A \tanh Cr}{r}]} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\,. \quad (38)$$

Here the exterior metric $a \leq r < \infty$ is given by

$$ds^2 = -\left[1 - \frac{A \tanh Ca}{r}\right] dt^2 + \frac{dr^2}{[1 - \frac{A \tanh Ca}{r}]} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\,. \quad (39)$$

In this case, the amount of ANEC violating matter in the space-time with a cutoff of the stress energy at a is given by

$$\begin{aligned} I &= A + [a - A\tanh Ca]\ln a + (a - r_0) \\ &-A\Big[C(a-r_0) - \frac{C^3}{9}(a^3 - r_0^3) + \frac{2C^5}{75}(a^5 - r_0^5) \\ &- \ldots \frac{(-1)^{n-1}B_{\mathrm{n}}2^{2n}(2^{2n}-1)}{(2n-1)(2n!)}C^{2n-1}(a^{2n-1} - r_0^{2n-1}) + \ldots\Big] \\ &+(a - A\tanh Ca)\ln[(a - A\tanh Ca) - 1], \end{aligned}$$

where, B_{n} is the Bernoulli number.

Also, in this case, if we treat the parameters A, C, r_0 as fixed constants, then total amount of ANEC violating matter can be reduced to small quantity by taking a suitable position, where interior wormhole metric will match the exterior Schwarzschild metric. In other words, this type of wormhole can be constructed with small quantity of ANEC violating phantom energy material.

4. Concluding remarks

Our aim in this paper is to provide a prescription for obtaining a wormhole where stress energy tensor is characterized by phantom energy with variable equation of state parameter. We have provided several toy models according to this new proposal. In the first two models we have considered phantom energy sources that give birth to wormhole-like structure, whereas in the last two models, we have considered specific forms of the shape functions of wormhole and tried to search for matter sources (phantom like) that generate the above wormhole structures.

As mentioned above, to be a wormhole solution, the condition $b'(r_0) < 1$ is to be imposed. Now for the first case, $b'(r_0) = \frac{1}{w} < 1$, since $w > 1$, corresponding to solution (11). For the second case, $b'(r_0) < 1$ implies $r_0 > (\frac{1}{A})^{\frac{1}{n}}$, corresponding to solution (22) and for the last cases, one has to assume $r_0 > (A+B) \pm \sqrt{(A+B)^2 - 3AB}$ and $r_0 > \frac{1}{C}\cosh^{-1}\sqrt{AC}$, corresponding to the solutions (28) and (30), respectively. We have established a matching each of four interior wormhole metrics with the exterior Schwarzschild metric.

Except model 1, all the other models reveal the fact that one may construct wormholes with small amounts of phantom energy as desired which is characterized by a variable equation of state parameter.

The effective mass inside the radius r is defined by $M(r) = \frac{b(r)}{2}$ and the limit, $\lim_{r\to\infty} M(r) = M$, if exists, represents the asymptotic wormhole mass seen by an distant observer. In the first case, this limit does not exist whereas for the last three cases, one can see that M exists and is equal to $\frac{1}{2}\exp\left(\frac{1}{Anr_0^n}+\ln r_0\right)$ for the second case and to $\frac{D}{2}$ and $\frac{A}{2}$ for the last cases. This implies that a distant observer could not see any difference of gravitational nature between the wormhole and a compact mass 'M'.

The assumption for the redshift function to be constant implies the tidal gravitational force experienced by a traveller to be zero. Thus one of the traversibility condition is satisfied, in other words, our wormholes are traversable. Hence our wormholes containing small amount of exotic matter in spite of they are traversable for human beings.

The specializations 2–4 are taken from intuition and simplicity from the mathematical point of view. Since the proposed phantom energy is characterized by variable EoS parameter, so there are thousands numbers of choices that may be considered. Before selecting the specializations 2–4, we are dealing with several choices but we find these are the physically acceptable wormhole models (*i.e.* satisfying all the characteristics of wormhole as well as supported by small amount of ANEC violating matter). We hope scientists would be motivated by our approach and in future, will try to find sophisticated way for constructing wormholes.

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