Vol. 40 (2009) *ACTA PHYSICA POLONICA B* No 1

INFLUENCE OF Z' BOSON ON TOP QUARK SPIN CORRELATIONS AT THE LHC

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(Received August 14, 2008; revised version received November 6, 2008)

We study top–antitop pair production and top spin correlations in a model with an electrically neutral massive gauge boson, Z', at the Large Hadron Collider. In addition to the Standard Model processes, the Z' contributes to the top–antitop pair production process in the s-channel. Choosing a kinematical region of top invariant mass around the Z' resonance pole, we find sizable deviations of the top–antitop pair production cross-section and the top spin correlations from those of the Standard Model.

PACS numbers: 14.65.Ha, 14.70.Pw

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1. Introduction

The Standard Model (SM) based on the gauge group SU(3)$\times$ SU(2)$_{\rm L}\times$ U(1)$_{\rm Y}$ is phenomenologically quite successful in the description of phenomena around the electroweak scale. However, it is widely believed that new physics beyond the SM takes place above certain energy scale. In a class of new physics models, the SM gauge group is embedded in a larger gauge group. Such a model often predicts an electrically neutral massive gauge boson, referred as Z'. There are many example models such as the left-right symmetric model [1] and Grand Unified Theories based on the gauge groups SO(10) [2] and E$_6$ [3] (for a review, see for example [4–6]).

At the hadron collider a Z' boson could be observed as a resonance in the Drell–Yan process. Current direct search of Z' bosons for several models has been performed by the CDF Collaboration at the Tevatron in the e^+e^- decay channel with the use of the di-electron invariant mass and angular distributions. No evidence of a signal has been found and 95% CL lower limits of the Z' mass are set to be in the range from 650 to 900 GeV [7]. Recently studies about measurement of Z' bosons at the Large Hadron Collider (LHC) has been performed [8–10].

If a Z' boson mass lies around TeV scale, it could be discovered at the LHC. Once a Z'-like resonance is observed, the next task is to precisely measure its properties, such as mass, spin, couplings to the SM particles *etc.* The spin of the Z' boson affects angular distributions and spin configurations for outgoing particles produced by Z' decays. A good tool to study the spin configuration is a top–antitop quark pair. Since the top quark with mass in the range of 175 GeV [11] decays electroweakly before hadronization takes place [12], a spin polarization of the top–antitop quark pair is directly transferred to its decay products. Therefore, there are significant angular correlations between the top quark spin axis and the direction of motion of the decay products.

The spin correlations for the hadronic top–antitop pair production process have been extensively studied in the quantum chromodynamics (QCD) [13–15]. It is found that there is a spin asymmetry between the produced top–antitop pairs, namely, the number of produced top–antitop quark pairs with both spin up or spin down (like pair) is different from the number of pairs with the opposite spin combinations (unlike pair). If the top quark is coupled to new physics beyond the SM, it could alter the top–antitop spin correlations. Therefore, the top–antitop spin correlations can provide useful information to test not only the SM but also a possible new physics at the hadron collider. The LHC has a big advantage to study the top spin correlations, since it will produce almost 10 millions of top quarks a year. In Refs. [16] and [17], effects of the Kaluza–Klein gravitons on the top spin correlations in the brane world models at the LHC were studied and sizable deviations of the top spin correlations from the SM one were found[1].

The purpose of this paper is to investigate effects of the Z' boson on the top–antitop pair production and its spin correlations. In addition to the SM processes, the Z' boson gives rise to a new contribution for the top–antitop pair production process in the s-channel and alters the top–antitop pair production cross-section and the top spin correlations from the SM ones. Choosing a kinematical region of top invariant mass around the Z' resonance pole, we find their sizable deviations from those of the SM.

This paper is organized as follows. In the next section, we briefly review the top spin correlations. In Section 3, we present a model with the Z' boson. In Section 4, we derive the invariant amplitudes for the polarized top–antitop pair production processes mediated by the Z' boson in the s-channel. We show the results of our numerical analysis in Section 5. Section 6 is devoted to conclusions. Appendices ensemble formulas we use in the calculation.

2. Spin correlation

At hadron collider, the top–antitop quark pair is produced through the processes of quark–antiquark pair annihilation and gluon fusion:

$$i \to t + \bar{t}\,, \qquad i = q\bar{q}, gg\,. \tag{2.1}$$

The former is the dominant process at the Tevatron, while the latter is dominant at the LHC. The produced top–antitop pairs decay before hadronization takes place. The main decay modes in the SM involve leptonic and hadronic modes:

$$t \to bW^+ \to bl^+\nu_l, bu\bar{d}, bc\bar{s}\,, \tag{2.2}$$

where $l = e, \mu, \tau$. The differential decay rates to a decay product $f = b$, l^+, ν_l, *etc.* at the top quark rest frame can be parameterized as

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_f} = \frac{1}{2}(1 + \kappa_f \cos\theta_f)\,, \tag{2.3}$$

where Γ is the partial decay width of the respective decay channel and θ_f is the angle between the top quark polarization and the direction of motion of the decay product f. The coefficient κ_f called the top spin analyzing power is a constant between -1 and 1. The ability to distinguish the polarization of

[1] There are also several studies of effects of new physics on the top spin correlations at e^+e^- collider [18] and photon collider [19].

the top quark evidently increases with κ_f. The most powerful spin analyzer is a charged lepton, for which $\kappa_{l^+} = +1$ at tree level [20]. Other values of κ_f are $\kappa_b = -0.41$ for the b-quark and $\kappa_{\nu_l} = -0.31$ for the ν_l, respectively. In hadronic decay modes, the role of the charged lepton is replaced by the $\bar{d}$ or $\bar{s}$ quark.

Now we see how top spin correlations appear in the chain of processes of $i \to t\bar{t}$ and decay of the top quarks. The total matrix element squared for the top–antitop pair production (2.1) and their decay channels (2.2) is given by

$$|\mathcal{M}|^2 \propto \mathrm{Tr}[\rho R^i \bar{\rho}] = \rho_{\alpha'\alpha} R^i_{\alpha\beta,\alpha'\beta'} \bar{\rho}_{\beta'\beta} \tag{2.4}$$

in the narrow-width approximation for the top quark. Here the subscripts denote the top and antitop spin indices, and R^i denotes the density matrix corresponding to the production of the on-shell top–antitop quark pair through the process i in (2.1):

$$R^i_{\alpha\beta,\alpha'\beta'} = \sum_{\text{initial spin}} \mathcal{M}(i \to t_\alpha \bar{t}_\beta) \mathcal{M}^*(i \to t_{\alpha'} \bar{t}_{\beta'})\,, \tag{2.5}$$

where $\mathcal{M}(i \to t_\alpha \bar{t}_\beta)$ is the amplitude for the top–antitop pair production. The matrices ρ and $\bar{\rho}$ are the density matrices corresponding to the decays of polarized top and antitop quarks into some final states at the top and antitop rest frame, respectively. In the leptonic decay modes, the matrices ρ, which lead to (2.3), can be obtained as (see, for instance, [21])

$$\rho_{\alpha'\alpha} = \mathcal{M}(t_\alpha \to bl^+\nu_l) \mathcal{M}^*(t_{\alpha'} \to bl^+\nu_l) = \frac{\Gamma}{2}(1 + \kappa_f \vec{\sigma} \cdot \vec{q}_f)_{\alpha'\alpha}\,, \tag{2.6}$$

where q_f is the unit vector of the direction of motion of the decay product f. The density matrix for the polarized antitop quark is obtained by replacing $\kappa_f \to -\kappa_f$ in (2.6) if there is no CP violation. In the SM, there is no CP violation in the top quark decay at the leading order. In the model presented in the next section, there is no contribution to break CP symmetry at the leading order, and thus this relation holds.

A way to analyze the top–antitop spin correlations is to see the angular correlations of two charged leptons l^+l^- produced by the top–antitop quark leptonic decays. In the following, we consider only the leptonic decay channels. Using (2.4)–(2.6) and integrating over the azimuthal angles of the charged leptons, we obtain the following double distribution [13, 14]

$$\frac{1}{\sigma}\frac{d^2\sigma}{d\cos\theta_{l^+} d\cos\theta_{l^-}} = \frac{1}{4}\left(1 + B_1 \cos\theta_{l^+} + B_2 \cos\theta_{l^-} - C \cos\theta_{l^+} \cos\theta_{l^-}\right). \tag{2.7}$$

Here σ denotes the cross-section for the process of the leptonic decay modes, and $\theta_{l^+}(\theta_{l^-})$ denotes the angle between the top (antitop) spin axis and the direction of motion of the antilepton (lepton) at the top (antitop) rest frame. In what follows, we use the helicity spin basis which is almost optimal one to analyze the top spin correlation at the LHC[2]. In this basis, the top (antitop) spin axis is regarded as the direction of motion of the top (antitop) in the top–antitop center-of-mass system. The coefficients B_1 and B_2 are associated with a polarization of the top and antitop quarks, and C encodes the top spin correlations, whose explicit expression is given by

$$C = \mathcal{A}\kappa_{l^+}\kappa_{l^-}\,, \qquad \kappa_{l^+} = \kappa_{l^-} = 1\,, \tag{2.8}$$

where the coefficient $\mathcal{A}$ represents the spin asymmetry between the produced top–antitop pairs with like and unlike spin pairs defined as

$$\mathcal{A} = \frac{\sigma(t_\uparrow\bar{t}_\uparrow) + \sigma(t_\downarrow\bar{t}_\downarrow) - \sigma(t_\uparrow\bar{t}_\downarrow) - \sigma(t_\downarrow\bar{t}_\uparrow)}{\sigma(t_\uparrow\bar{t}_\uparrow) + \sigma(t_\downarrow\bar{t}_\downarrow) + \sigma(t_\uparrow\bar{t}_\downarrow) + \sigma(t_\downarrow\bar{t}_\uparrow)}\,. \tag{2.9}$$

Here $\sigma(t_\alpha\bar{t}_\beta)$ is the cross-section of the top–antitop pair production at parton level with denoted spin indices.

In the SM, at the lowest order of $\alpha_{\rm s}$, the spin asymmetry is found to be $\mathcal{A} = +0.319$ for the LHC[3]. At the LHC in the ATLAS experiment, the spin asymmetry of the top–antitop pairs will be measured with a precision of several percent, after one LHC year at low luminosity ($10\,\mathrm{fb}^{-1}$) [24].

3. A simple model with Z' boson

As a simple example which includes a Z' boson, we consider a model based on the gauge group SU(3)$\times$ SU(2)$\times$ U(1)$_1\times$ U(1)$_2$ [4–6]. In order to realize the gauge symmetry breaking U(1)$_1\times$ U(1)$_2$ $\rightarrow$ U(1)$_{\rm Y}$, where U(1)$_{\rm Y}$ is the SM hypercharge group, we introduce a scalar field Φ in the representation $(\mathbf{1},\mathbf{1},+1,-1)$. The covariant derivative for the scalar field is given by

$$\mathcal{D}^{\Phi}_\mu = \partial_\mu - ig_1B_{1\mu} + ig_2B_{2\mu}\,, \tag{3.1}$$

where g_i and $B_{i\mu}$ $(i=1,2)$ is the gauge coupling constant and the gauge boson of U(1)$_i$. Once Φ develops a vacuum expectation value ($\langle\Phi\rangle = v_\Phi/\sqrt{2}$),

[2] Recently another spin basis was constructed, which has a larger spin correlation than the helicity basis at the LHC [22].

[3] The parton distribution function set of CTEQ6L [23] has been used in our calculations. The resultant spin asymmetry somewhat depends on the parton distribution functions used.

the gauge symmetry is broken down to $\mathrm{U}(1)_1\times\ \mathrm{U}(1)_2 \to \mathrm{U}(1)_\mathrm{Y}$. Associated with this gauge symmetry breaking, the mass eigenstates of two gauge bosons are described as

$$B_\mu = B_{1\mu}\cos\phi + B_{2\mu}\sin\phi\,, \tag{3.2}$$
$$Z_{2\mu} = -B_{1\mu}\sin\phi + B_{2\mu}\cos\phi\,, \tag{3.3}$$

where ϕ is the mixing angle defined as $\tan\phi = g_1/g_2$, B_μ is the massless $\mathrm{U}(1)_\mathrm{Y}$ gauge boson and $Z_{2\mu}$ is the Z' boson with mass

$$M_{Z_2}^2 = \left(\frac{g'}{\sin\phi\cos\phi}\right)^2 v_\Phi^2\,. \tag{3.4}$$

Here, the SM $\mathrm{U}(1)_\mathrm{Y}$ gauge coupling constant is defined through the relation $1/g'^2 = 1/g_1^2 + 1/g_2^2$.

For a fermion with a charge (Y_{1f}, Y_{2f}) under $\mathrm{U}(1)_1\times\ \mathrm{U}(1)_2$, the interaction term with B_μ and $Z_{2\mu}$ is given by

$$\mathcal{L}_\mathrm{int} = \bar\psi_f\gamma^\mu\psi_f\left[g'Y_fB_\mu + g'\left(-Y_{1f}\tan\phi + Y_{2f}\cot\phi\right)Z_{2\mu}\right]\,, \tag{3.5}$$

where $Y_f = Y_{1f}+Y_{2f}$. In our following analysis, we assume for simplicity that under the gauge group, the quarks and leptons in each generation have quantum numbers, $q = (\mathbf{3},\mathbf{2},1/6,0)$, $u_\mathrm{R} = (\mathbf{3},\mathbf{1},2/3,0)$, $d_\mathrm{R} = (\mathbf{3},\mathbf{1},-1/3,0)$, $L = (\mathbf{1},\mathbf{2},-1/2,0)$, $e_\mathrm{R} = (\mathbf{1},\mathbf{1},-1,0)$ and $\nu_\mathrm{R} = (\mathbf{1},\mathbf{1},0,0)$.

After the spontaneous breaking of the electroweak symmetry $\mathrm{SU}(2)\times$ $\mathrm{U}(1)_\mathrm{Y}$ to the electromagnetic subgroup $\mathrm{U}(1)_\mathrm{EM}$, the neutral current interactions for the SM leptons and quarks are written as[4]

$$\mathcal{L}_\mathrm{NC} = J^\mu_\mathrm{EM}A_\mu + J^\mu_{Z_1}Z_{1\mu} + J^\mu_{Z_2}Z_{2\mu}\,. \tag{3.6}$$

Here J_EM and J_{Z_1} are the SM electromagnetic and neutral currents

$$J^\mu_\mathrm{EM} = e\sum_f Q^f\bar\psi_f\gamma^\mu\psi_f\,, \qquad J^\mu_Z = \sum_f\bar\psi_f\gamma^\mu(g^f_{\mathrm{L},1}P_\mathrm{L} + g^f_{\mathrm{R},1}P_\mathrm{R})\psi_f\,, \tag{3.7}$$

where $e = g\sin\theta_\mathrm{W} = g'\cos\theta_\mathrm{W}$, Q^f is the electric charge of fermion f and $g^f_{\mathrm{L},1}$ and $g^f_{\mathrm{R},1}$ are the chiral couplings given by (see also Appendix B)

$$g^f_{\mathrm{L},1} = \frac{e}{\cos\theta_\mathrm{W}\sin\theta_\mathrm{W}}(T^f_{3\mathrm{L}} - Q^f\sin^2\theta_\mathrm{W})\,, \qquad g^f_{\mathrm{R},1} = -eQ^f\tan\theta_\mathrm{W}\,, \tag{3.8}$$

[4] Although the mixing between the SM Z and Z' bosons, in general, emerges through the electroweak symmetry breaking, here we assume a negligibly small mixing, which is required by the current precision measurements. This situation is achievable in some models even when the Z' boson mass is small (see, for a concrete example, [25]).

with the third component of weak isospin for the left chiral component of ψ_f, $T^f_{3\mathrm{L}}$ ($T^u_{3\mathrm{L}} = +1/2$ for up-type and $T^d_{3\mathrm{L}} = -1/2$ for down-type fermions). The current J_{Z_2} is given as

$$\begin{aligned} J^\mu_{Z_2} &= g' \sum_f (-Y_{1f} \tan\phi + Y_{2f} \cot\phi) \bar{\psi}_f \gamma^\mu \psi_f \\ &= \sum_f \bar{\psi}_f \gamma^\mu (g^f_{\mathrm{L},2} P_\mathrm{L} + g^f_{\mathrm{R},2} P_\mathrm{R}) \psi_f \,, \end{aligned} \tag{3.9}$$

where the chiral couplings $g^f_{\mathrm{L},2}$ and $g^f_{\mathrm{R},2}$ are explicitly given in Appendix B. In the following analysis, we fix the mixing angle ϕ as $\tan\phi = 1$, for simplicity.

4. Scattering amplitude

In this section we calculate the squared invariant amplitudes for $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$ processes. First we calculate $q\bar{q} \to t\bar{t}$ process. In this process, top quark pairs are produced via the s-channel photon, Z boson and Z' boson exchanges. An amplitude for quark annihilation process is given by

$$\mathcal{M}(q\bar{q} \to t\bar{t}) = \mathcal{M}_{\mathrm{QCD}} + \mathcal{M}_{\mathrm{NC}} \,. \tag{4.1}$$

Here $\mathcal{M}_{\mathrm{QCD}}$ denotes the QCD process and $\mathcal{M}_{\mathrm{NC}}$ is the contribution of the neutral current. Since there is no interference between the QCD process and the neutral current process, the squared amplitude is simply written as

$$|\mathcal{M}|^2 = |\mathcal{M}_{\mathrm{QCD}}|^2 + |\mathcal{M}_{\mathrm{NC}}|^2 \,. \tag{4.2}$$

The helicity amplitude of the QCD process, $\mathcal{M}_{\mathrm{QCD}}$, is given by (for one flavor initial state)

$$|\mathcal{M}(q\bar{q} \to t_\uparrow \bar{t}_\uparrow)|^2 = |\mathcal{M}(q\bar{q} \to t_\downarrow \bar{t}_\downarrow)|^2 = \frac{g_s^4}{9}(1-\beta_\mathrm{t}^2)\sin^2\theta \,, \tag{4.3}$$

$$|\mathcal{M}(q\bar{q} \to t_\uparrow \bar{t}_\downarrow)|^2 = |\mathcal{M}(q\bar{q} \to t_\downarrow \bar{t}_\uparrow)|^2 = \frac{g_s^4}{9}(1+\cos^2\theta) \,, \tag{4.4}$$

where $q(\bar{q})$ denotes an initial (anti)quark, g_s is the strong coupling constant, $\beta_\mathrm{t} = \sqrt{1 - 4m_\mathrm{t}^2/s}$, m_t is the top quark mass, θ is the scattering angle between incoming quark and outgoing top quark, and $\sqrt{s}$ is energy of colliding partons. The helicity amplitude of the neutral current process, $\mathcal{M}_{\mathrm{NC}}(q\bar{q} \to t\bar{t})$, is described by

$$|\mathcal{M}_{\mathrm{NC}}(q\bar{q} \to t_\delta \bar{t}_\gamma)|^2 = \left(\frac{1}{2}\right)^2 \sum_{\alpha,\beta} |\mathcal{M}_{\mathrm{NC}}(\alpha,\beta;\delta,\gamma)|^2 \,, \tag{4.5}$$

where $\mathcal{M}_{\rm NC}(\alpha,\beta;\delta,\gamma)$ is the decomposition of the helicity amplitude with respect to the initial spin and $\alpha(\delta)$ and $\beta(\gamma)$ denote initial (final) spin states for quark and antiquark, respectively. The form is described by

$$\mathcal{M}_{\rm NC}(+,-;\pm,\pm) = \mp s\sqrt{1-\beta_{\rm t}^2}\sin\theta$$
$$\times\left[\frac{(eQ^f)(eQ^t)}{s}+\sum_{i=1}^{2}\frac{g^f_{\rm R,i}}{2}\frac{g^t_{{\rm L},i}+g^t_{\rm R,i}}{s-M^2_{Z_i}+iM_{Z_i}\mathit{\Gamma}_{Z_i}}\right], \tag{4.6}$$
$$\mathcal{M}_{\rm NC}(-,+;\pm,\pm) = \mp s\sqrt{1-\beta_{\rm t}^2}\sin\theta$$
$$\times\left[\frac{(eQ^f)(eQ^t)}{s}+\sum_{i=1}^{2}\frac{g^f_{{\rm L},i}}{2}\frac{g^t_{{\rm L},i}+g^t_{\rm R,i}}{s-M^2_{Z_i}+iM_{Z_i}\mathit{\Gamma}_{Z_i}}\right], \tag{4.7}$$
$$\mathcal{M}_{\rm NC}(+,-;+,-) = -s(1+\cos\theta)$$
$$\times\left[\frac{(eQ^f)(eQ^t)}{s}+\sum_{i=1}^{2}\frac{g^f_{\rm R,i}}{2}\frac{(1-\beta_{\rm t})g^t_{{\rm L},i}+(1+\beta_{\rm t})g^t_{{\rm R},i}}{s-M^2_{Z_i}+iM_{Z_i}\mathit{\Gamma}_{Z_i}}\right], \tag{4.8}$$
$$\mathcal{M}_{\rm NC}(+,-;-,+) = s(1-\cos\theta)$$
$$\times\left[\frac{(eQ^f)(eQ^t)}{s}+\sum_{i=1}^{2}\frac{g^f_{{\rm R},i}}{2}\frac{(1+\beta_{\rm t})g^t_{{\rm L},i}+(1-\beta_{\rm t})g^t_{{\rm R},i}}{s-M^2_{Z_i}+iM_{Z_i}\mathit{\Gamma}_{Z_i}}\right], \tag{4.9}$$
$$\mathcal{M}_{\rm NC}(-,+;+,-) = s(1-\cos\theta)$$
$$\times\left[\frac{(eQ^f)(eQ^t)}{s}+\sum_{i=1}^{2}\frac{g^f_{{\rm L},i}}{2}\frac{(1-\beta_{\rm t})g^t_{{\rm L},i}+(1+\beta_{\rm t})g^t_{{\rm R},i}}{s-M^2_{Z_i}+iM_{Z_i}\mathit{\Gamma}_{Z_i}}\right], \tag{4.10}$$
$$\mathcal{M}_{\rm NC}(-,+;-,+) = -s(1+\cos\theta)$$
$$\times\left[\frac{(eQ^f)(eQ^t)}{s}+\sum_{i=1}^{2}\frac{g^f_{{\rm L},i}}{2}\frac{(1+\beta_{\rm t})g^t_{{\rm L},i}+(1-\beta_{\rm t})g^t_{{\rm R},i}}{s-M^2_{Z_i}+iM_{Z_i}\mathit{\Gamma}_{Z_i}}\right], \tag{4.11}$$

with the decay widths of Z_1 and Z_2 bosons given by

$$\mathit{\Gamma}_{Z_i} = \mathit{\Gamma}(Z_i\to f\bar{f}) = \frac{M_{Z_i}}{96\pi}\sum_f \beta_i^f$$
$$\times\left\{\left(3+\left(\beta_i^f\right)^2\right)\left(\left(g^f_{{\rm L},i}\right)^2+\left(g^f_{{\rm R},i}\right)^2\right)+6\left(1-\left(\beta_i^f\right)^2\right)g^f_{{\rm L},i}g^f_{{\rm R},i}\right\}, \tag{4.12}$$

where M_{Z_i} is the mass of the gauge boson Z_i, $\beta_i^f=\sqrt{1-4m_f^2/M^2_{Z_i}}$, m_f is the mass of the fermion f. The explicit form of $\mathit{\Gamma}_{Z_i}$ for each decay mode is given in Appendix C. The sum in Eq. (4.12) does not include the right-handed neutrinos since their masses are naturally around $v_{\rm R}$.

For the squared amplitude for the QCD process with the gg initial state, we have

$$\begin{aligned} |\mathcal{M}(gg \to t_\uparrow \bar{t}_\uparrow)|^2 &= |\mathcal{M}(gg \to t_\downarrow \bar{t}_\downarrow)|^2 \\ &= \frac{g_s^4}{96}\mathcal{Y}(\beta_\mathrm{t}, \cos\theta)(1-\beta_\mathrm{t}^2)(1+\beta_\mathrm{t}^2+\beta_\mathrm{t}^2\sin^4\theta)\,, \end{aligned} \tag{4.13}$$

$$\begin{aligned} |\mathcal{M}(gg \to t_\uparrow \bar{t}_\downarrow)|^2 &= |\mathcal{M}gg \to t_\downarrow \bar{t}_\uparrow)|^2 \\ &= \frac{g_s^4\beta_\mathrm{t}^2}{96}\mathcal{Y}(\beta_\mathrm{t}, \cos\theta)\sin^2\theta(1+\cos^2\theta)\,. \end{aligned} \tag{4.14}$$

Here $\mathcal{Y}(\beta_\mathrm{t}, \theta)$ is defined by

$$\mathcal{Y}(\beta_\mathrm{t}, \cos\theta) = \frac{7+9\beta_\mathrm{t}^2\cos^2\theta}{(1-\beta_\mathrm{t}^2\cos^2\theta)^2}\,. \tag{4.15}$$

Using above formulas, we calculate the double distribution (2.7) including the Z' contributions. Explicit calculation tells us that the transverse polarization is vanishing, *i.e.* $B_1 = B_2 = 0$ while the spin asymmetry $\mathcal{A}$ is altered from the SM one.

5. Numerical results

Here we show various numerical results and demonstrate interesting properties of measurable quantities in our model. In our analysis we use the parton distribution function of CTEQ6L [23] with the factorization scale $Q = m_\mathrm{t} = 175\,\mathrm{GeV}$ and $\alpha_\mathrm{s}(Q) = 0.1074$. We choose $M_{Z_2} = 900\,\mathrm{GeV}$ to be consistent with the current experimental results [7]. In the whole analysis, the center of mass energy of the colliding protons, E_CMS, is taken to be 14 TeV at the LHC.

Fig. 1 shows the cross-sections of the top–antitop pair production through $u\bar{u} \to t\bar{t}$ and $d\bar{d} \to t\bar{t}$ at the parton level as a function of parton center-of-mass energy $\sqrt{s} = M_{t\bar{t}} = \sqrt{(p_t + p_{\bar{t}})^2}$, where $p_t(p_{\bar{t}})$ is a momentum of (anti)top quark. The figure exhibits the large peak corresponding to the resonant production of Z' boson.

The dependence of the cross-section on the top–antitop invariant mass $M_{t\bar{t}}$ is given by

$$\begin{aligned} \frac{d\sigma_\mathrm{tot}(pp \to t\bar{t})}{d\sqrt{s}} &= \sum_{a,b}\int_{-1}^{1} d\cos\theta \int_{\frac{s}{E_\mathrm{CMS}^2}}^{1} dx_1 \frac{2\sqrt{s}}{x_1 E_\mathrm{CMS}^2} f_a(x_1, Q^2) \\ &\times f_b\left(\frac{s}{x_1 E_\mathrm{CMS}^2}, Q^2\right)\frac{d\sigma(t\bar{t})}{d\cos\theta}\,. \end{aligned} \tag{5.1}$$

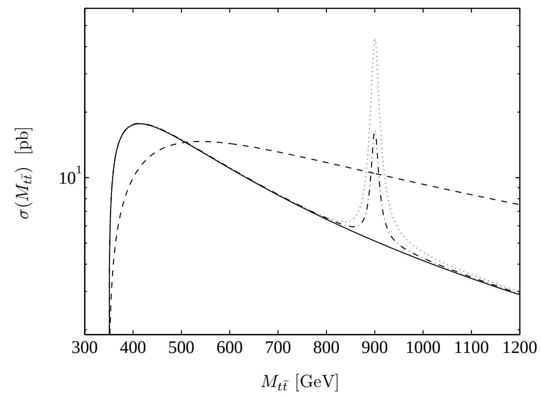

Fig. 1. The dependence of the cross-section of the top–antitop quark pair production by quark pair annihilation and gluon fusion on the center-of-mass energy of colliding partons $M_{t\bar{t}}$. The solid and dashed lines correspond to the results of the quark annihilation and gluon fusion for the SM, respectively. The dotted and dash-dotted lines correspond to the results of $u\bar{u} \to t\bar{t}$ and $d\bar{d} \to t\bar{t}$, respectively.

Fig. 2 depicts the result of the differential cross-section (5.1). Here, the decomposition of the total cross-section into the like $(t_\uparrow \bar{t}_\uparrow + t_\downarrow \bar{t}_\downarrow)$ and unlike $(t_\uparrow \bar{t}_\downarrow + t_\downarrow \bar{t}_\uparrow)$ top–antitop spin pairs is also shown. The deviation from the SM around the resonant pole is large only for unlike top–antitop spin pairs, because of the helicity conserving interactions between the Z' boson and the SM fermions. Note that the resonance of Z' will be firstly measured in the leptonic process $pp \to \mu^+\mu^-$ since the background is quite small.

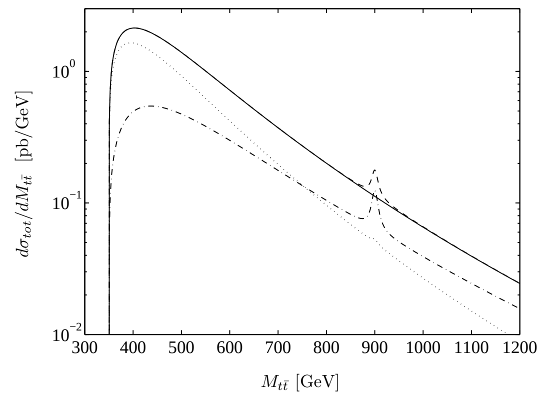

Fig. 2. Differential cross-section (5.1) as a function of the top–antitop invariant mass $M_{t\bar{t}}$. The solid and dashed lines correspond to the results of the SM and our model. The breakdown of the differential cross-sections into the like (dotted) and the unlike (dash-dotted) top–antitop spin pair productions are also depicted.

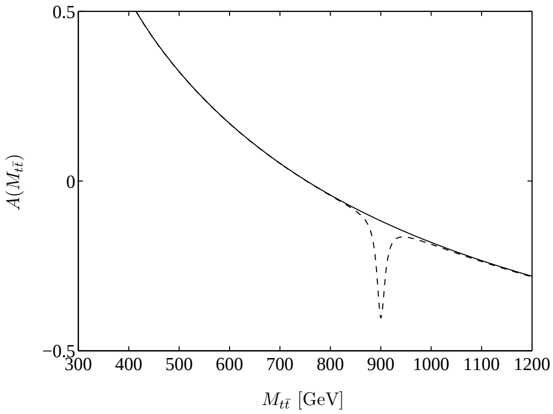

Fig. 3. Spin asymmetry $\mathcal{A}$ as a function of the top–antitop invariant mass $M_{t\bar{t}}$. The solid line corresponds to the SM, while the dashed line corresponds to our model.

The information of the pole will be then confirmed in the top–antitop quark production. Furthermore, it should be useful to observe large deviation of the top spin correlation as we will discuss below.

Now we show the result for the spin asymmetry $\mathcal{A}$ in Fig. 3 as a function of the center-of-mass energy of colliding partons. As expected, deviation from the SM is enhanced around the Z' boson resonance pole. In Fig. 4, we show the spin asymmetry $\mathcal{A}$ as a function of the Z' boson mass M_{Z_2}, after integration with respect to $M_{t\bar{t}}$ in the range $2m_{\rm t} \leq M_{t\bar{t}} \leq E_{\rm CMS}$.

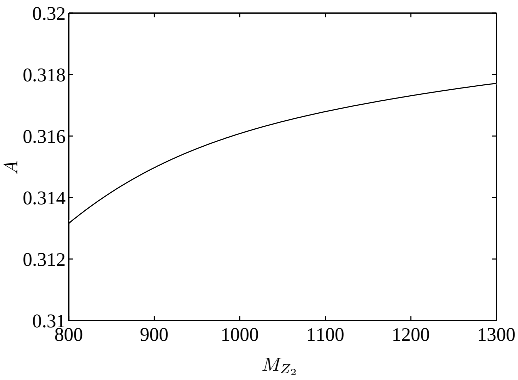

Fig. 4. Spin asymmetry $\mathcal{A}$ as a function of M_{Z_2}.

For $M_{Z_2} \gtrsim 900\,\text{GeV}$, deviation from the SM is less than a few percent. In order to enhance the deviation of the spin asymmetry from the SM one, we impose the selection criteria $M_{Z_2} - 50\,\text{GeV} < M_{t\bar{t}} < M_{Z_2} + 50$ GeV for the range of the integration. Table I presents values of the spin asymmetry for three chosen masses $M_{Z_2} = 900,\ 1100$ and $1300\,\mathrm{GeV}$. We see sizable deviations from the SM ones.

TABLE I

Spin asymmetry $\mathcal{A}$ and $t\bar{t}$ total cross-section for the top–antitop events without the constraint on the invariant mass (second and third column) and with the invariant mass cut in the range $M_{Z_2} - 50\,\mathrm{GeV} < M_{t\bar{t}} < M_{Z_2} + 50\,\mathrm{GeV}$ (from fourth to seventh column) for various values of M_{Z_2}. The values in the fifth and seventh column correspond to the SM, while in the other columns correspond to our Z' model. The last line shows the SM results.

M_{Z_2} [GeV]	$\mathcal{A}$	σ [pb]	$\mathcal{A}^{(\mathrm{cut})}$	$\mathcal{A}^{(\mathrm{cut})}_{\mathrm{SM}}$	$\sigma^{(\mathrm{cut})}$ [pb]	$\sigma^{(\mathrm{cut})}_{\mathrm{SM}}$ [pb]
900	0.315	491	-0.199	-0.114	12.8	11.4
1100	0.317	490	-0.331	-0.232	4.65	3.99
1300	0.317	490	-0.430	-0.319	1.91	1.57
SM	0.319	489	—	—	—	—

The corresponding values of the total cross-section[5] imply statistically sufficient number of events in the selected kinematical range (under assumption of integral luminosity $\mathcal{L} = 10\,\mathrm{fb}^{-1}$ for one LHC year at low luminosity and $\mathcal{L} = 100\,\mathrm{fb}^{-1}$ for one LHC year at high luminosity).

6. Conclusions

We studied top–antitop pair production and its spin correlation in the Z' model at the LHC. In this model, in addition to the SM processes, there is a new contribution to the top–antitop pair production process mediated by an electrically neutral gauge boson Z' in the s-channel. We calculated the squared invariant amplitudes for the top–antitop pair production including the new contribution from the Z' boson and showed numerical results for the top pair production cross-section and the top spin correlations. We found that after integration with respect to $M_{t\bar{t}}$ for the full range $2m_{\mathrm{t}} \leq M_{t\bar{t}} \leq E_{\mathrm{CMS}}$, the deviation of the spin asymmetry $\mathcal{A}$ from the SM result is very small, below the estimated sensitivity of the ATLAS experiment [24]. When we imposed the selection criteria $M_{Z_2} - 50\,\mathrm{GeV} < M_{t\bar{t}} < M_{Z_2} + 50\,\mathrm{GeV}$ for the range of the integration, the deviation of the spin asymmetry from the SM one is remarkably enhanced (around 50% of the SM value), even if the total cross-sections are almost the same. By using the same analysis

[5] Cross sections in Table I are computed in the leading order. In the SM, NLO contributions significantly increase the total cross-section, nevertheless the spin asymmetry is insensitive to NLO contributions [15].

performed in [24], we can estimate the sensitivity for our model. In Ref. [24], it is shown that the spin asymmetry of the top–antitop pairs in the SM will be measured with a precision of 6% after one LHC year at low luminosity, 10 fb^{-1}. In our model, for instance, for the case of $M_{Z_2} = 900\,\mathrm{GeV}$ and $\sigma^{(\mathrm{cut})} = 12.8\,\mathrm{pb}$, we will reach the same accuracy of the measurement of the spin correlation with that of the SM when the integrated luminosity is $270\,\mathrm{fb}^{-1}$ (= 2.7 years of high luminosity LHC run). Note that it is very rough estimation since the sensitivity of the ATLAS experiment on the spin correlation published in [24] was estimated selecting low energetic top quarks with $M_{t\bar{t}} < 550\,\mathrm{GeV}$. In order to estimate the sensitivity more accurately with a high $M_{t\bar{t}}$ region for our case, we need Monte Carlo simulations including the detector response. It must be the subject of next study. Finally we compare our results with our previous analysis for Kaluza–Klein productions in the large extra-dimensions [16] and the Randall–Sundrum model [17]. All these models affect to reduce the spin asymmetry $\mathcal{A}$ from the SM results, when new physics effects are sizable. This is because the interactions between new particles and top quarks conserve chirality and as a result the cross-section of the top–antitop quark pair production for the unlike spin pair is enhanced from the SM one. In our model with Z' boson, the situation is the same and the spin asymmetry is decreased. Although the spin asymmetry is decreased in these models, they can be distinguishable since they have different pole structure, which is specified via the cross-section of the top–antitop pair production. There is a single pole in the model with Z' boson while no pole exists in the large extra-dimensions and multiple poles for the Kaluza–Klein gravitons exist in the Randall–Sundrum model. Actually information of the pole in our model is important to observe a large deviation of the spin correlation around the pole.

In summary, the top spin correlations is a powerful tool to reveal the property of the Z' boson at the LHC.

Note added: Recently top quark pair productions and their spin correlations through possible new physics s-channel resonances have been investigated in a model independent way with the use of Monte Carlo simulation [26]. In their analysis, a Z' gauge boson couples with the same strength to fermions as the SM Z boson. Then the peak of the Z' resonance pole in the cross-section against $M_{t\bar{t}}$ is larger than ours for the same Z' boson masses. Therefore, the absolute value of the peak corresponding the Z' boson resonance in the top spin correlation against $M_{t\bar{t}}$ is larger than ours and consequently their spin asymmetry is much smaller than ours. We have checked that our results are consistent with ones in [26] when we choose the same setting as in [26].

The work of M.A. is supported by the Science Research Center Program of the Korea Science and Engineering Foundation through the Center for Quantum Spacetime (CQUeST) of the Sogang University with grant number R11-2005-021. M.A. would also like to thank to the Czech Technical University in Prague and theory division, KEK, for their hospitality during his visit. The work of N.O. is supported in part by the Scientific Grants from the Ministry of Education, and the Science and Culture of Japan (No. 18740170) and the Academy of Finland Finnish–Japanese Core Programme grant 112420. N.O. would also like to thank the High Energy Physics Division of the Department of Physical Sciences, the University of Helsinki, for their hospitality during his visit. The work of K.S. is supported by the Research Program MSM6840770029 and by the project of the International Cooperation ATLAS-CERN of the Ministry of Education, Youth and Sports of the Czech Republic.

Appendix A

General formula

Consider the following Lagrangian

$$\mathcal{L}^f_{\rm int} = eJ^\mu Z_\mu = \bar{\psi}_f \gamma^\mu (g^f_{\rm L} P_{\rm L} + g^f_{\rm R} P_{\rm R}) \psi_f Z_\mu \,, \tag{A.1}$$

where Z_μ denotes a massive gauge boson whose mass is M. A polarized invariant amplitude for the process $f(\alpha)\bar{f}(\beta) \to F(\delta)\bar{F}(\gamma)$ is given by

$$\mathcal{M}(\alpha,\beta;\gamma,\delta) = \frac{1}{p^2 - M^2 + iM\Gamma_Z} J^\mu_{\rm in}(\alpha,\beta) J_{{\rm out}\,\mu}(\gamma,\delta)\,, \tag{A.2}$$

where $\alpha, \beta(\gamma, \delta)$ denote initial(final) spin states for quark and antiquark, respectively, and Γ is the decay width of Z boson. The currents of gauge bosons for initial (massless) and final (massive) states are given by

$$J^\mu_{\rm in}(+,-) = -\sqrt{s} g^f_{\rm R}(0,1,i,0)\,, \qquad J^\mu_{\rm in}(-,+) = -\sqrt{s} g^f_{\rm L}(0,1,-i,0)\,, \tag{A.3}$$

and

$$\begin{aligned}
J^\mu_{\rm out}(+,+) &= \omega_+\omega_- \left\{ g^{\rm F}_{\rm L}(1,-\sin\theta,0,-\cos\theta) - g^{\rm F}_{\rm R}(1,\sin\theta,0,\cos\theta) \right\}, && \text{(A.4)}\\
J^\mu_{\rm out}(-,-) &= \omega_+\omega_- \left\{ g^{\rm F}_{\rm L}(1,\sin\theta,0,\cos\theta) - g^{\rm F}_{\rm R}(1,-\sin\theta,0,-\cos\theta) \right\}, && \text{(A.5)}\\
J^\mu_{\rm out}(+,-) &= \omega_-^2 g^{\rm F}_{\rm L}(0,-\cos\theta,i,\sin\theta) - \omega_+^2 g^{\rm F}_{\rm R}(0,\cos\theta,-i,-\sin\theta)\,, && \text{(A.6)}\\
J^\mu_{\rm out}(-,+) &= \omega_+^2 g^{\rm F}_{\rm L}(0,-\cos\theta,-i,\sin\theta) - \omega_-^2 g^{\rm F}_{\rm R}(0,\cos\theta,i,-\sin\theta)\,, && \text{(A.7)}
\end{aligned}$$

where $\omega^2_\pm = \sqrt{s}/2\,(1 \pm \beta_{\rm F})$, $\beta_{\rm F} = \sqrt{1 - (4m^2_{\rm F})/s}$ and $f(F)$ denotes a flavor of initial (final) state of fermions.

Appendix B

Couplings

The couplings for the SM Z boson:

$$\begin{aligned}
g^{\nu}_{\mathrm{L},1} &= \frac{e}{\cos\theta_{\mathrm{W}}\sin\theta_{\mathrm{W}}}\tfrac{1}{2}\,, \\
g^{\nu}_{\mathrm{R},1} &= 0\,, && \text{(B.1)} \\
g^{l}_{\mathrm{L},1} &= \frac{e}{\cos\theta_{\mathrm{W}}\sin\theta_{\mathrm{W}}}\left(-\tfrac{1}{2}-\sin^2\theta_{\mathrm{W}}(-1)\right)\,, \\
g^{l}_{\mathrm{R},1} &= -e(-1)\tan\theta_{\mathrm{W}}\,, && \text{(B.2)} \\
g^{u}_{\mathrm{L},1} &= \frac{e}{\cos\theta_{\mathrm{W}}\sin\theta_{\mathrm{W}}}\left(\tfrac{1}{2}-\sin^2\theta_{\mathrm{W}}\tfrac{2}{3}\right)\,, \\
g^{u}_{\mathrm{R},1} &= -e\tfrac{2}{3}\tan\theta_{\mathrm{W}}\,, && \text{(B.3)} \\
g^{d}_{\mathrm{L},1} &= \frac{e}{\cos\theta_{\mathrm{W}}\sin\theta_{\mathrm{W}}}\left(-\tfrac{1}{2}-\sin^2\theta_{\mathrm{W}}\left(-\tfrac{1}{3}\right)\right)\,, \\
g^{d}_{\mathrm{R},1} &= -e\left(-\tfrac{1}{3}\right)\tan\theta_{\mathrm{W}}\,, && \text{(B.4)}
\end{aligned}$$

The couplings for Z' boson:

$$g^{\nu}_{\mathrm{L},2} = \frac{e}{\cos\theta_{\mathrm{W}}}\tfrac{1}{2}\tan\phi\,, \qquad g^{\nu}_{\mathrm{R},2} = 0\,, \tag{B.5}$$

$$g^{l}_{\mathrm{L},2} = \frac{e}{\cos\theta_{\mathrm{W}}}\tfrac{1}{2}\tan\phi\,, \qquad g^{l}_{\mathrm{R},2} = \tfrac{e}{\cos\theta_{\mathrm{W}}}\tan\phi\,, \tag{B.6}$$

$$g^{u}_{\mathrm{L},2} = \frac{e}{\cos\theta_{\mathrm{W}}}\left(-\tfrac{1}{6}\right)\tan\phi\,, \qquad g^{u}_{\mathrm{R},2} = \frac{e}{\cos\theta_{\mathrm{W}}}\left(-\tfrac{2}{3}\right)\tan\phi\,, \tag{B.7}$$

$$g^{d}_{\mathrm{L},2} = \frac{e}{\cos\theta_{\mathrm{W}}}\left(-\tfrac{1}{6}\right)\tan\phi\,, \qquad g^{d}_{\mathrm{R},2} = \frac{e}{\cos\theta_{\mathrm{W}}}\tfrac{1}{3}\tan\phi\,. \tag{B.8}$$

Appendix C

Decay width

The decay widths of $Z(Z_1)$ and $Z'(Z_2)$ bosons:

$$\varGamma(Z_i\to\nu\bar\nu) = \frac{M_{Z_i}}{24\pi}\left(\left(g^{\nu}_{\mathrm{L},i}\right)^2+(g^{\nu}_{\mathrm{R},i})^2\right)\,, \tag{C.1}$$

$$\varGamma(Z_i\to l\bar l) = \frac{M_{Z_i}}{24\pi}\left(\left(g^{l}_{\mathrm{L},i}\right)^2+\left(g^{l}_{\mathrm{R},i}\right)^2\right)\,, \tag{C.2}$$

$$\varGamma(Z_i\to u\bar u) = \frac{M_{Z_i}}{24\pi}3\left(\left(g^{u}_{\mathrm{L},i}\right)^2+(g^{u}_{\mathrm{R},i})^2\right)\,, \tag{C.3}$$

$$\varGamma(Z_i\to d\bar d) = \frac{M_{Z_i}}{24\pi}3\left(\left(g^{d}_{\mathrm{L},i}\right)^2+(g^{d}_{\mathrm{R},i})^2\right)\,, \tag{C.4}$$

$$\begin{aligned}\Gamma\left(Z_i\to t\bar{t}\right) &= \frac{M_{Z_i}}{24\pi}3\beta_{\rm t}\\ &\times \left[\left(1-\frac{m_{\rm t}^2}{M_{Z_i}^2}\right)\left(\left(g_{{\rm L},2}^t\right)^2+\left(g_{{\rm R},2}^t\right)^2\right)+6\frac{m_{\rm t}^2}{M_{Z_i}^2}g_{{\rm L},2}^t g_{{\rm R},2}^t\right]\delta_{i2}\,,\end{aligned} \tag{C.5}$$

where $\beta_{\rm t}=\sqrt{1-4m_{\rm t}^2/M_{Z_2}^2}$.

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