ABOUT THE MORRIS–THORNE WORMHOLE AND VACUUM SOLUTIONS IN THE CONFORMAL POINCARÉ-GAUGE THEORY OF GRAVITATION

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The Morris–Thorne wormhole and vacuum equations in the conformal Poincaré-gauge theory of gravitation are considered. It is shown that wormholes cannot be realized as configurations of a "usual" matter. It is obtained also that dynamic vacuum solutions for spherical symmetric case coincide with the corresponding GR solutions.

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1. Introduction

It is known, that in framework of the Einstein's GR there is a number of difficulties. One of them is the impossibility of construction of the Morris–Thorne Wormhole (MTW). The vacuum MTW [1] has an event horizon on throat, and non vacuum solutions describing them inevitably contain an "exotic" matter (with negative radial pressure).

As one of possible ways of exception of above mentioned problems the transition to the conformal Poincaré-gauge theory of gravitation in the Einstein gauge and in the torsionless limit [2] can be considered. This theory is based on equations ($C_{\alpha\beta\mu\nu}$ is the Weyl tensor, $\varkappa = 8\pi G/c^4$, f — an arbitrary parameter)

$$R^i_k - \tfrac{1}{2}\delta^i_k R + \varkappa f C^{ij}{}_{kl} R^l_j \;=\; \varkappa T^i_k \,, \tag{1a}$$

$$f C^{ij}{}_{kl;i} \;=\; 0\,. \tag{1b}$$

The asymptotical solutions of system (1) near to singular points for the perfect fluid configurations have been investigated in work [3]. In [2] it has been shown that for a static spherical symmetric line element

$$dS^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + e^{\mu(r)}\left(d\theta^2 + \sin^2\theta d\Omega^2\right), \tag{2}$$

in vacuum case the solutions of equations (1) coincide with corresponding GR solutions. Here we shall consider the system (1) for the MTW with a matter and its dynamic vacuum equations.

2. MTW in the conformal Poincaré-gauge theory of gravitation

According to definition [1] the MTW is a topological handle in space-time which connects the distant regions of Universe. A line element for MTW corresponds to (2) for $\lambda = 0$. The functions $e^{\nu(r)}$ and $e^{\mu(r)}$ have symmetric positive minima at $r = 0$ (the MTW throat). These functions are also monotonous to the left and to the right of the throat.

For a line element (2) the equations (1) have a form (a prime denoted differentiation with respect to r) [4]

$$\varkappa P_{\mathrm{r}} = \frac{x'^2}{x^2} - \frac{1}{x^2} + \frac{x'y}{x} + \frac{2A^3}{x^3}\left(-\frac{x''}{x} - \frac{x'^2}{x^2} + \frac{1}{x^2} + \frac{1}{2}y' + \frac{1}{4}y^2 + \frac{x'y}{2x}\right), \tag{3a}$$

$$-\varkappa\varepsilon = \frac{2x''}{x^2} - \frac{x'^2}{x^2} - \frac{1}{x^2} + \frac{2A^3}{x^3}\left(\frac{x''}{x} - \frac{x'^2}{x^2} + \frac{1}{x^2} + \frac{1}{2}y' + \frac{1}{4}y^2 + \frac{x'y}{2x}\right), \tag{3b}$$

$$\varkappa P_{\perp} = \frac{x''}{x^2} + \frac{1}{2}y' + \frac{1}{4}y^2 + \frac{x'y}{2x} + \frac{2A^3}{x^3}\left(\frac{x'^2}{x^2} - \frac{1}{x^2} - \frac{1}{2}y' - \frac{1}{4}y^2\right), \tag{3c}$$

$$y' = \frac{2x''}{x} - \frac{2x'^2}{x^2} - \frac{1}{2}y^2 + \frac{2}{x^2} + \frac{x'y}{x} - \frac{6r_{\mathrm{g}}}{x^3}, \tag{3d}$$

here $\varepsilon = -T_0^0$; $P_{\mathrm{r}} = T_1^1$; $P_{\perp} = T_2^2 = T_3^3$ — the energy density, the radial pressure and the tangent pressure respectively; $x = e^{\mu/2}$, $y = \nu'$, $A^3 = \frac{1}{2}\varkappa f r_{\mathrm{g}}$, r_{g} is a gravitation radius of object.

We shall consider the equation (3d). Let us present a function $\boldsymbol{y}$ as

$$y = \frac{2x'}{x} + \xi(x). \tag{4}$$

Substituting the expression (4) in (3d), we obtain

$$x' = \frac{-\frac{\xi^2}{2} + \frac{2}{x^2} - \frac{6r_{\mathrm{g}}}{x^3}}{\frac{d\xi}{dx} + \frac{\xi}{x}}. \tag{5}$$

According to definition

$$y(x_{\mathrm{t}}) = x'(x_{\mathrm{t}}) = 0 \tag{6}$$

($x_{\rm t}$ — the value $x(r)$ on throat). From (4) follows the condition $\xi(x_{\rm t}) = 0$. As a result from (5) we find

$$x_{\rm t} = 3r_{\rm g}\,. \tag{7}$$

Substituting y' from (3d) in expression (3a) we obtain

$$\varkappa P_{\rm r} = \frac{x'^2}{x^2} - \frac{1}{x^2} + \frac{x'y}{x} + \frac{2A^3}{x^3}\left(\frac{x'y}{2x} - 2\frac{x'^2}{x^2} + \frac{2}{x^2} - \frac{3r_{\rm g}}{x^3}\right). \tag{8}$$

Using the conditions (6) and (7) in (8) we find a radial pressure on throat

$$\varkappa P_{\rm r} = -\frac{1}{9r_{\rm g}^2} + \frac{2A^3}{(3r_{\rm g})^5}\,. \tag{9}$$

In [4] it was shown that for this problem takes place a requirement $f < 0$. As the parameters $\varkappa$ and $r_{\rm g}$ are positive then $A < 0$. Hence $P_{\rm r}(x_{\rm t}) < 0$. Thus by analogy with GR here is inevitable presence of "exotic" matter on throat.

3. The vacuum spherical-symmetric equations

Let us consider now the vacuum equations (1) for line element

$$dS^2 = -e^{-v}dt + 2adtdr + e^{\lambda}dr^2 + e^{\mu}d\Omega^2\,, \tag{10}$$

where v, a, μ, λ are functions of t and r.

The system (3a)–(3c) can be present as

$$G_1^1 + 2z\left(G_2^2 - G_0^0\right) \;=\; 0\,, \tag{11a}$$

$$G_0^0 + 2z\left(G_2^2 - G_1^1\right) \;=\; 0\,, \tag{11b}$$

$$z\left(G_0^0 + G_1^1\right) + (1-2z)G_2^2 \;=\; 0\,, \tag{11c}$$

(here $z = (A/x)^3$, $G_k^i = R_k^i - \frac{1}{2}R\delta_k^i$).
Obviously, the solutions of equations (11) can be different from the vacuum GR solutions $G_k^i = 0$ only for

$$z \;=\; \tfrac{1}{4}\,, \tag{12a}$$

or

$$z \;=\; -\tfrac{1}{2}\,. \tag{12b}$$

For condition (12a) we have

$$G_1^1 = G_0^0 = -G_2^2\,. \tag{13}$$

From (12a) it follows that $x = 4^{1/3}$. Then it is easy to examine that in this case the relations (for line element (10)) take place .

$$G_0^0 = G_1^1 = -\frac{1}{x^2}\,, \tag{14a}$$

$$R_2^2 = 0\,. \tag{14b}$$

However

$$G_0^0 + G_1^1 + G_2^2 = -R\,. \tag{15}$$

It is obvious that conditions (13), (14) contradict equality (15). Let us consider now the case corresponding (12b). According to (11) we have

$$G_0^0 = G_1^1\,, \tag{16a}$$

$$G_2^2 = 0\,. \tag{16b}$$

From (12b) it follows that $x = -2^{1/3}A$. Then we have a contradiction between conditions (14), (16) and equality (15). Thus, the vacuum solutions within conformal Poincaré-gauge theory of gravitation coincide with GR vacuum solutions. An analogical conclusion was obtained in work [2] for static case.

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