# WHAT CAN THE POLARIZATION PUZZLE IN $B \rightarrow \Phi K^{*}$ TELL US ABOUT THE FOURTH QUARK GENERATION AND VECTOR QUARK? 

M.R. Ahmady<br>Physics Department, Mount Allison University<br>Sackville, New Brunswick E4L 1E6, Canada

F. Behzadi, F. Falahati, S.M. Zebarjad ${ }^{\dagger}$

Physics Department, Shiraz University, Shiraz 71454, Iran
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We investigate the effect of an additional generation of ordinary quarks and vector quarks on longitudinal and transverse amplitudes associated with the exclusive $B^{0} \rightarrow \Phi K^{*}$ decays. We perform $\chi^{2}$ fits to the experimental data with respect to these two model parameters. Even though distinct minima $\left(\chi_{0}^{2}\right)$ are observed but $\chi_{0}^{2} /$ d.o.f. values much larger than one indicates that such a constrained extension of the standard model cannot resolve the polarization puzzle in the $B^{0} \rightarrow \Phi K^{*}$ decay mode.

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## 1. Introduction

The polarization puzzle in $B \rightarrow \Phi K^{*}$ decays could be interpreted as a sign of new physics beyond the standard model (SM). In this two-body decay of $B$-mesons the final-state particles are both vector mesons and, therefore, we have three distinct decay amplitudes which are classified according to their helicities. The conservation of angular momentum requires that the helicity of the produced vector mesons be both either longitudinal (00) or negative $(--)$ or positive $(++)$. Within the SM, where only left-handed quarks participate in charged weak currents, one expects much greater longitudinal polarization amplitude, $H_{00}$, for two vector mesons $\Phi$ and $\bar{K}^{0 *}$ than transverse ones, $H_{--}$and $H_{++}$, in which one and two helicity flips are needed, respectively. In fact, this can roughly translate into the following

[^0]relation for helicity amplitudes: $\bar{H}_{00}: \bar{H}_{--}: \bar{H}_{++} \sim \mathcal{O}(1): \mathcal{O}\left(1 / m_{b}\right)$ : $\mathcal{O}\left(1 / m_{b}{ }^{2}\right)$. However, the experimental data gathered from BABAR and BELLE show a different picture: $\bar{H}_{00}: \bar{H}_{--}: \bar{H}_{++} \sim \mathcal{O}\left(1 / m_{b}\right): \mathcal{O}(1)$ : $\mathcal{O}\left(1 / m_{b}{ }^{2}\right)$, which is in contradiction with naive SM predictions [1-4]. There have been attempts to resolve this discrepancy both within and beyond the SM. The contribution of penguin annihilation [5], rescattering [6-11], and enhanced penguin contributions due to the dipole operator $[12,13]$ are examples of the former approach. Reference [14] discusses the testing of the first two mechanisms with the future measurements of $U$-spin related charmless $B$ decays to two vector mesons. New physics (NP) approach, on the other hand, can take two main routes, either assume new four-Fermi operators which do not exist in the SM low-energy effective Hamiltonian [15] or alternatively, work with the same set of operators but assume extra contributions to the Wilson coefficients. In this article, we follow the latter approach.

Two possible extensions of SM are extra fourth generation (SM4) of quarks $\left(t^{\prime}, b^{\prime}\right)[16-21,23]$ and Vector Quark Model (VQM). There are various arguments in support of SM4, among them, the flavor democracy in the three generations of the SM [24]. In this scenario, the masses of the first three fermion families, as well as inter-generational mixing are generated by small braking of flavor democracy $[25,26]$. The fourth family quarks are nearly degenerate and their common mass scale is constrained by the experimental value of $\rho$ and $S$ parameters. Considering the latest data $\rho=1.0002_{-0.0004}^{+0.0007}$ [29], the mass of the fourth quark $m_{t^{\prime}}$ lies between 300 GeV and 700 GeV . As such, these exotic quarks, if exist, could be produced in good numbers at LHC via gluon fusion mechanism. The indirect effects of the fourth generation scenario have already shown to close the gap between the data and theoretical predictions on CP violation in penguin-dominated nonleptonic $B$ decays [21]. On the other hand, the Vector Quark Model (VQM) is an extension of the SM with an extra generation of iso-singlet quarks [22]. Unlike the three generations of ordinary quarks in the SM, both the left- and the right-handed components of the quarks of this additional generation are invariant under $\mathrm{SU}(2)_{\mathrm{L}}$ gauge group. Therefore, the flavor changing weak interactions of these exotic quarks proceeds only through mixing with ordinary quarks and this results in the non-unitarity of the extended $4 \times 4$ quark mixing matrix and thus non-vanishing flavor changing neutral currents ( FCNC ) at the tree level.

In this paper, we study the effect of an additional quark generation (SM4) and Vector Quark Model (VQM) on the helicity amplitudes of $B \rightarrow \Phi K^{*}$ decay. Our motivation is to examine these NP scenarios which have only left handed operators to explain the discrepancies between the SM and experimental data. Using $\chi^{2}$ fits, the best estimates for the models parameters in SM4 and VQM are obtained. However, the large value of $\chi^{2} /$ d.o.f. indicates
that these scenarios in their simplest forms cannot explain the polarization puzzle. One interpretation of these results, among others, could be that using the left handed operators alone cannot solve the puzzle.

This paper is organized as follows. In Section 2, we give a brief review of helicity amplitude calculation within the SM. Section 3 and 4 are devoted to deriving these amplitudes within the extended SM with an extra generation of quarks (SM4) and Vector Quark Model (VQM). Then, we perform $\chi^{2}$ fits to the available experimental data and conclude with an analysis of our results.

## 2. $\bar{B}^{0} \rightarrow \Phi \bar{K}^{0 *}$ polarization amplitudes in the SM

The effective Hamiltonian for hadronic $b \rightarrow s$ transitions can be written as:

$$
\begin{align*}
\mathcal{H}_{\mathrm{eff}}= & G_{\mathrm{F}} / \sqrt{2}\left[V_{u b} V_{u s}^{*}\left(c_{1} O_{1}^{u}+c_{2} O_{2}^{u}\right)+V_{c b} V_{c s}^{*}\left(c_{1} O_{1}^{c}+c_{2} O_{2}^{c}\right)\right. \\
& \left.-V_{t b} V_{t s}^{*}\left(\sum_{i=3}^{10} c_{i} O_{i}\right)+c_{g} O_{g}\right]+ \text { h.c. } \tag{1}
\end{align*}
$$

where 4-quark and 2-quark operators have the following definitions:

- Current-current operators:

$$
\begin{align*}
O_{1}^{u} & =(\bar{u} b)_{V-A}(\bar{s} u)_{V-A}, & & O_{2}^{u} \\
O_{1}^{c} & =\left(\bar{c} \bar{u}_{\alpha} b_{\beta}\right)_{V-A}\left(\bar{s}_{\beta} u_{\alpha}\right)_{V-A}(\bar{s} c)_{V-A}, & & O_{2}^{c} \tag{2}
\end{align*}=\left(\bar{c}_{\alpha} b_{\beta}\right)_{V-A}\left(\bar{s}_{\beta} c_{\alpha}\right)_{V-A} .
$$

- QCD-penguin operators:

$$
\begin{equation*}
O_{3(5)}=(\bar{s} b)_{V-A} \sum_{q}(\bar{q} q)_{V \mp A}, \quad O_{4(6)}=\left(\bar{s}_{\alpha} b_{\beta}\right)_{V-A} \sum_{q}\left(\bar{q}_{\beta} q_{\alpha}\right)_{V \mp A} \tag{3}
\end{equation*}
$$

- Electroweak penguin operators:

$$
\begin{align*}
O_{7(9)} & =\frac{3}{2}(\bar{s} b)_{V-A} \sum_{q} e_{q}(\bar{q} q)_{V \pm A} \\
O_{8(10)} & =\frac{3}{2}\left(\bar{s}_{\alpha} b_{\beta}\right)_{V-A} \sum_{q} e_{q}\left(\bar{q}_{\beta} q_{\alpha}\right)_{V \pm A} \tag{4}
\end{align*}
$$

- Choromomagnetic dipole operator:

$$
\begin{equation*}
O_{8 \mathrm{~g}}=g_{s} /\left(8 \pi^{2}\right) m_{b} \bar{s} \sigma^{\mu \nu}\left(1+\gamma_{5}\right) T^{a} b G_{\mu \nu}^{a} \tag{5}
\end{equation*}
$$

where $(V \pm A)=\gamma^{\mu}\left(1 \pm \gamma_{5}\right), e$ and $g$ are QED and QCD coupling constants, respectively, $T^{a}$ 's are $\mathrm{SU}(3)$ color matrices, $G_{\mu \nu}$ is the gluon field strength and $q \in\{u, d, s, c, b\}$.

The complete calculation of $\bar{B}^{0} \rightarrow \Phi \bar{K}^{* 0}$ in SM can be found in Refs [15, 27], which is briefly explained in the remainder of this section. Using the QCD factorization (QCDF) approach with the $\Phi$ meson factorized, the amplitude ${ }^{1}$ of the above decay, which is penguin dominated, has the following expression:

$$
\begin{equation*}
\bar{A}\left(\bar{B}^{0} \rightarrow \Phi \bar{K}^{0 *}\right)=G_{\mathrm{F}} / \sqrt{2}\left(-V_{t b} V_{t s}^{*}\right) a_{\mathrm{SM}}^{h} X^{\left(\bar{B}^{0} \bar{K}^{0 *}, \Phi\right)} \tag{6}
\end{equation*}
$$

where

$$
\begin{align*}
X^{\left(\bar{B}^{0} \bar{K}^{0 *}, \Phi\right)}= & \left\langle\Phi\left(q, \varepsilon_{1}\right)\right|(\bar{s} s)_{V-A}|0\rangle\left\langle\bar{K}^{*}\left(p^{\prime}, \varepsilon_{2}\right)\right|(\bar{s} b)_{V-A}\left|\bar{B}^{0}(p)\right\rangle \\
= & i f_{\Phi} m_{\Phi}\left[\frac{-2 i}{m_{\bar{B}^{0}}+m_{\bar{K}^{0 *}}} \varepsilon_{\mu \nu \alpha \beta} \varepsilon_{1}^{* \mu} \varepsilon_{2}^{* \nu} p^{\alpha} p^{\prime \beta} V\left(q^{2}\right)\right] \\
& -i f_{\Phi} m_{\Phi}\left[\left(m_{\bar{B}^{0}}+m_{\bar{K}^{0 *}}\right) \varepsilon_{1}^{*} \cdot \varepsilon_{2}^{*} A_{1}\left(q^{2}\right)\right. \\
& \left.-\left(\varepsilon_{1}^{*} \cdot p\right)\left(\varepsilon_{2}^{*} \cdot p\right) \frac{2 A_{2}\left(q^{2}\right)}{m_{\bar{B}^{0}}+m_{\bar{K}^{0 *}}}\right] \tag{7}
\end{align*}
$$

in which the decay constant $f_{\Phi}$ and hadronic form factors are defined by the following relations:

$$
\begin{align*}
\left\langle\Phi\left(q, \varepsilon_{1}\right)\right| V^{\mu}|0\rangle= & f_{\Phi} m_{\Phi} \varepsilon_{1}^{* \mu} \\
\left\langle\bar{K}^{* 0}\left(p^{\prime}, \varepsilon_{2}\right)\right| V^{\mu}\left|\bar{B}^{0}(p)\right\rangle= & \frac{2}{m_{\bar{B}^{0}}+m_{\bar{K}^{0 *}}} \varepsilon^{\mu \nu \alpha \beta} \varepsilon_{2 \nu}^{*} p_{\alpha} p_{\beta}^{\prime} V\left(q^{2}\right) \\
\left\langle\bar{K}^{* 0}\left(p^{\prime}, \varepsilon_{2}\right)\right| A^{\mu}\left|\bar{B}^{0}(p)\right\rangle= & i\left[\left(m_{\bar{B}^{0}}+m_{\bar{K}^{0 *}}\right) \varepsilon_{2}^{* \mu} A_{1}\left(q^{2}\right)\right. \\
& \left.-\left(\varepsilon_{2}^{*} \cdot p\right)\left(p+p^{\prime}\right)^{\mu} \frac{A_{2}\left(q^{2}\right)}{m_{\bar{B}^{0}}+m_{\bar{K}^{0 *}}}\right] \\
& -2 i m_{\bar{K}^{0 *}} \frac{\varepsilon_{2}^{*} \cdot p}{q^{2}} q^{\mu}\left[A_{3}\left(q^{2}\right)-A_{0}\left(q^{2}\right)\right] \tag{8}
\end{align*}
$$

In the above expressions, $m_{\bar{B}^{0}}$ and $m_{\bar{K}^{0 *}}$ are the masses of $\bar{B}^{0}$ and $\bar{K}^{0 *}$, respectively, $q=p-p^{\prime}, A_{3}(0)=A_{0}(0)$, and

$$
\begin{equation*}
A_{3}\left(q^{2}\right)=\frac{\left(m_{\bar{B}^{0}}+m_{\bar{K}^{0 *}}\right)}{2 m_{\bar{K}^{0 *}}} A_{1}\left(q^{2}\right)-\frac{\left(m_{\bar{B}^{0}}-m_{\bar{K}^{0 *}}\right)}{2 m_{\bar{K}^{0 *}}} A_{2}\left(q^{2}\right) \tag{9}
\end{equation*}
$$

[^1]The multiplicative factor $a_{\mathrm{SM}}^{h}$ in Eq. (6) is a combination of the Wilson coefficients in $\mathcal{H}_{\text {eff }}$ (Eq. (1)) with the superscript $h$ denoting the polarization states of $\Phi$ and $K^{0 *}$ mesons; $h=0$ is for the helicity 00 state and $h= \pm$ for helicity $\pm \pm$ states:

$$
\begin{equation*}
a_{\mathrm{SM}}^{h}=a_{3}^{h}+a_{4}^{h}+a_{5}^{h}-\frac{1}{2}\left(a_{7}^{h}+a_{9}^{h}+a_{10}^{h}\right) . \tag{10}
\end{equation*}
$$

Within the QCDF approach, the $h$-dependence of $a_{i}$ 's are due to nonfactorizable effects which appear at $\mathcal{O}\left(\alpha_{S}\right)$ and in fact, in the naive factorization (NF) approximation we have $a_{\mathrm{SM}}^{0}=a_{\mathrm{SM}}^{+}=a_{\mathrm{SM}}^{-}=a_{\mathrm{SM}}$. Using EqS (6) to (9), it is then straightforward to calculate the decay width of $\bar{B}^{0} \rightarrow \Phi \bar{K}^{\star 0}$ in terms of the helicity amplitudes:

$$
\begin{equation*}
\Gamma\left(\bar{B}^{0} \rightarrow \Phi \bar{K}^{\star 0}\right)=\frac{p_{\mathrm{c}}}{8 \pi m_{\bar{B}^{0}}}\left(\left|\bar{H}_{00}\right|^{2}+\left|\bar{H}_{++}\right|^{2}+\left|\bar{H}_{--}\right|^{2}\right) \tag{11}
\end{equation*}
$$

where $p_{\mathrm{c}}$ is the center of mass momentum of the $\Phi$ and $K^{*}$ mesons in the $\bar{B}$ rest frame and the amplitudes $\bar{H}_{00}, \bar{H}_{++}$and $\bar{H}_{--}$are given as:

$$
\begin{align*}
\bar{H}_{00}= & G_{\mathrm{F}} / \sqrt{2} C^{0}\left(i f_{\Phi} m_{\Phi}\right)\left(m_{\bar{B}^{0}}+m_{\bar{K}^{0 *}}\right)\left[a A_{1}\left(m_{\Phi}^{2}\right)-b A_{2}\left(m_{\Phi}^{2}\right)\right] \\
\bar{H}_{ \pm \pm}= & -G_{\mathrm{F}} / \sqrt{2} C^{ \pm}\left(i f_{\Phi} m_{\Phi}\right)\left[\left(m_{\bar{B}^{0}}+m_{\bar{K}^{0 *}}\right) A_{1}\left(m_{\Phi}^{2}\right)\right. \\
& \left.\mp \frac{2 m_{\bar{B}^{0}} p_{\mathrm{c}}}{m_{\bar{B}^{0}}+m_{\bar{K}^{0 *}}} V\left(m_{\Phi}^{2}\right)\right] \tag{12}
\end{align*}
$$

with

$$
\begin{aligned}
a & \equiv \frac{m_{\bar{B}^{0}}^{2}-m_{\Phi}^{2}-m_{\bar{K}^{0 *}}^{2}}{2 m_{\Phi} m_{\bar{K}^{0 *}}}, \\
b & \equiv \frac{2 m_{\bar{B}^{0}}^{2} p_{c}^{2}}{m_{\Phi} m_{\bar{K}^{0 *}}\left(m_{\bar{B}^{0}}+m_{\bar{K}^{0 *}}\right)^{2}}, \\
C^{h} & =V_{t b} V_{t s}^{*} a_{\mathrm{SM}}^{h} .
\end{aligned}
$$

We note that, unlike $\bar{H}_{ \pm \pm}, \bar{H}_{00}$ only receives contributions from those terms which are symmetric under $\varepsilon_{1} \leftrightarrow \varepsilon_{2}$. The experimental data on this decay mode is usually given in terms of transversity amplitudes, which are related to the helicity amplitudes via the following relations:

$$
\begin{align*}
& \bar{A}_{0}=\bar{H}_{00} \\
& \bar{A}_{\|}=\left(\bar{H}_{++}+\bar{H}_{--}\right) / \sqrt{2} \\
& \bar{A}_{\perp}=-\left(\bar{H}_{++}-\bar{H}_{--}\right) / \sqrt{2} \tag{13}
\end{align*}
$$

$\|$ and $\perp$ subscripts refer to the polarizations of $\Phi$ and $\bar{K}^{0 *}$ both being transverse (in the $\bar{B}^{0}$ rest frame) and either parallel or perpendicular to each other. One can then rewrite the decay rate (Eq. (11)) in terms of these amplitudes:

$$
\begin{equation*}
\Gamma\left(\bar{B}^{0} \rightarrow \phi \bar{K}^{* 0}\right)=\frac{p_{c}}{8 \pi m_{\bar{B}^{0}}^{2}}\left(\left|\bar{A}_{0}\right|^{2}+\left|\bar{A}_{\|}\right|^{2}+\left|\overline{A_{\perp}}\right|^{2}\right) \tag{14}
\end{equation*}
$$

Table I shows the experimental data for the fractional decay rates defined as:

$$
\begin{equation*}
\bar{R}_{i}=\frac{\Gamma\left(\bar{B}^{0} \rightarrow \phi \bar{K}^{* 0}\right)_{i}}{\Gamma\left(\bar{B}^{0} \rightarrow \phi \bar{K}^{* 0}\right)}=\frac{\left|\bar{A}_{i}\right|^{2}}{\left|\bar{A}_{0}\right|^{2}+\left|\bar{A}_{\|}\right|^{2}+\left|\bar{A}_{\perp}\right|^{2}}, \quad i=0, \|, \perp \tag{15}
\end{equation*}
$$

and the arguments of $\bar{A}_{\|}$and $\overline{A_{\perp}}$, keeping in mind that $\bar{R}_{\|}=1-\bar{R}_{0}-\bar{R}_{\perp}$.
TABLE I
The BABAR and BELLE data for $\bar{B}^{0} \rightarrow \Phi \bar{K}^{* 0}$ and its CP conjugate $B^{0} \rightarrow \Phi K^{* 0}$ decay amplitudes [34].

| Observable | BABAR | BELLE |
| :--- | ---: | ---: |
| $\bar{R}_{0}$ | $0.49 \pm 0.07$ | $0.59 \pm 0.10$ |
| $\bar{R}_{\perp}$ | $0.20 \pm 0.07$ | $0.26 \pm 0.09$ |
| $\arg \left(\bar{A}_{\\|}\right)$ | $-2.61 \pm 0.31$ | $-2.05 \pm 0.31$ |
| $\arg \left(\bar{A}_{\perp}\right)$ | $0.31 \pm 0.36$ | $0.81 \pm 0.32$ |
| $R_{0}$ | $0.55 \pm 0.08$ | $0.41 \pm 0.10$ |
| $R_{\perp}$ | $0.24 \pm 0.08$ | $0.24 \pm 0.10$ |
| $\arg \left(A_{\\|}\right)$ | $-2.07 \pm 0.31$ | $-2.29 \pm 0.37$ |
| $\arg \left(A_{\perp}\right)$ | $1.03 \pm 0.36$ | $0.74 \pm 0.33$ |

We observe that according to the data $\bar{R}_{T}=\bar{R}_{\|}+\bar{R}_{\perp} \sim \bar{R}_{0}$. However, SM estimates via Eqs (12) and (13) produce a much bigger longitudinal fraction, ranging from $\bar{R}_{0} \sim 0.92$ to 0.87 depending on the nonperturbative parameters of the QCDF and leading to $\mathcal{O}\left(1 / m_{b}^{2}\right)$ suppression of $\bar{R}_{T} / \bar{R}_{0}$. Since, to the leading order, there is no CP odd phase present in $b \rightarrow s$ transition within the SM , one would expect similar predictions for the CP conjugate $B^{0} \rightarrow \Phi K^{* 0}$ decay. The experimental data for this decay mode is shown in the bottom half of Table I. In the next section, we investigate how an additional generation of heavy quarks, SM4, can affect this discrepancy between the SM and experimental data.

## 3. Effects of an additional quark generation

The presence of an extra generation of ordinary quarks results in additional contributions, $c_{i}^{\text {SM4 }}$, to the Wilson coefficients of the operators in Eq. (1) which are dependent on $x=m_{t^{\prime}}^{2} / m_{W}^{2}$. The unitary quark mixing matrix is now $4 \times 4$ which can be written in terms of 6 mixing angles and 3 CP-violating phases. The relevant elements of this matrix for $b \rightarrow s$ transition satisfy the relation

$$
\begin{equation*}
V_{t b} V_{t s}^{*}=-V_{u b} V_{u s}^{*}-V_{c b} V_{c s}^{*}-V_{t^{\prime} b} V_{t^{\prime} s}^{*} . \tag{16}
\end{equation*}
$$

Consequently, one can write the new effective Hamiltonian as:

$$
\begin{align*}
\mathcal{H}_{\mathrm{eff}}= & G_{\mathrm{F}} / \sqrt{2}\left\{V_{u b} V_{u s}^{*}\left(c_{1} O_{1}^{u}+c_{2} O_{2}^{u}\right)+V_{c b} V_{c s}^{*}\left(c_{1} O_{1}^{c}+c_{2} O_{2}^{c}\right)\right. \\
& \left.-V_{t b} V_{t s}^{*}\left[\sum_{i=3}^{10} c_{i}^{\mathrm{new}} O_{i}+c_{g}^{\mathrm{new}} O_{g}\right]+\text { h.c. }\right\} \tag{17}
\end{align*}
$$

where $c_{i}^{\text {new }}$ and $c_{g}^{\text {new }}$ are:

$$
\begin{equation*}
c_{i}^{\text {new }}=c_{i}+\frac{V_{t^{\prime} b} V_{t^{\prime} s}^{*}}{V_{t b} V_{t s}^{*}} c_{i}^{\mathrm{SM} 4}, \quad i=3 \ldots 10, g \tag{18}
\end{equation*}
$$

These new Wilson coefficients modify Eq. (12) with the following substitution:

$$
\begin{equation*}
C^{h} \longrightarrow C_{\text {new }}^{h} \equiv V_{t b} V_{t s}^{*} a_{\text {new }}^{h} \tag{19}
\end{equation*}
$$

where $a_{\text {new }}^{h}$ is obtained from Eq. (10) with $a_{i}$ 's calculated via the substitution $c_{i} \rightarrow c_{i}^{\text {new }}$. Clearly in obtaining Eq. (19), one needs to run down $c_{i}^{\text {new }}$ to the scale $\mu=m_{b}$. The product of the new mixing elements in Eq. (16) is expressed in terms of two model parameters:

$$
\begin{equation*}
V_{t^{\prime} b} V_{t^{\prime} s}^{*}=u e^{i \varphi}, \tag{20}
\end{equation*}
$$

where $\varphi$ is one of the CP-violating phases. Thus, the coefficients $C_{\text {new }}^{h}$ are sensitive to all three model parameters: $x=m_{t^{\prime}}^{2} / m_{W}^{2}, u$ and $\varphi$. To find the optimal values for the model parameters, we perform a $\chi^{2}$ fit to the experimental data in Table I plus the branching ratio (10 input data for $\bar{B}^{0} \rightarrow \Phi \bar{K}^{0 *}$ and its CP conjugate process $\left.B^{0} \rightarrow \Phi K^{0 *}\right)$. In the numerical evaluation of the decay amplitudes within the QCDF formalism, three hadronic parameters appear in the calculation of the matrix element, $\rho$ and $\theta$ parametrize the logarithmic divergent integral involving the light-cone distribution amplitudes for the vector meson [30,31], and a mass scale $w_{B}$ in the $B$-mesons's wavefunction. Our fits are performed for $\rho=0$ and $\rho=1$,
where $\theta=0$ or $180^{\circ}$ are used in the latter case, and with $w_{B}$ taken to be 0.20 . Also, the phase ambiguity associated with the fact that the angular distribution analysis from which $\arg \left(\bar{A}_{\|}\right)$and $\arg \left(\bar{A}_{\perp}\right)$ are obtained is sensitive only to the interference terms like $\operatorname{Re}\left(\bar{A}_{\|} \bar{A}_{0}^{*}\right), \operatorname{Im}\left(\bar{A}_{\perp} \bar{A}_{0}^{*}\right)$ and $\operatorname{Im}\left(\bar{A}_{\perp} \bar{A}_{\|}^{*}\right)$ allows the alternate choice of $\left(-\arg \left(\bar{A}_{\|}\right), \pm \pi-\arg \left(\bar{A}_{\perp}\right)\right)$ for the data pair $\left(\arg \left(\bar{A}_{\|}\right), \arg \left(\bar{A}_{\perp}\right)\right)[2,4]$. That is, for example, the BABAR data on the arguments of the transvesity amplitudes in Table I can be read as either $\arg \left(\bar{A}_{\|}\right)=-2.61 \pm 0.31, \arg \left(\bar{A}_{\perp}\right)=0.31 \pm 0.36$ or alternatively as $\arg \left(\bar{A}_{\|}\right)=2.61 \pm 0.31, \arg \left(\bar{A}_{\perp}\right)=2.83 \pm 0.36$. We refer to the former choice as plus and the latter one as minus phase convention. We have shown the results for minus and plus phase convention for the case $\rho=1, \theta=0$ in Figs 1 and 2.


Fig. 1. The results of four generations for $\rho=1, \theta=0$ in the minus phase.
In this case, the value of the model parameters which result in a minimum $\chi^{2}$ are given in Table II. It is clear from this table the helicity amplitudes obtained from the SM4 are not able to match the observation. One conjecture is that perhaps the unitarity requirement, Eq. (16), is too restrictive and has to be relaxed if adding a new generation of quarks has any chance at explaining the experimental data on helicity amplitudes. This is, in fact, what we have if the extra generation of quarks are vector like. In the next section, we consider this possibility.


Fig. 2. The results of four generations for $\rho=1, \theta=0$ in the plus phase.

The values of helicity amplitudes for SM and SM4.

| Model | Phase | $\frac{\chi^{2}}{7}$ | $x$ | $u$ | $\phi$ | $\bar{R}_{0}$ | $\bar{R}_{\perp}$ | $R_{0}$ | $R_{\perp}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SM | - | - | - | - | - | 0.87 | 0.062 | 0.94 | 0.030 |
| SM4 | minus | 19.82 | 95 | 0.007 | -0.03 | 0.89 | 0.053 | 0.93 | 0.035 |
|  | plus | 18.23 | 92 | 0.060 | 3.29 | 0.99 | 0.003 | 0.84 | 0.080 |

## 4. Effects of vector like down quark (VLDQ)

Another simple extension of the SM where deviations from unitary of CKM matrix naturally arise by introducing extra quarks $[22,36]$ which are weak isosinglets but which mix with the SM quarks. Here, we consider only the case of one down-type vector quark. The nonunitarity of CKM $\operatorname{matrix}\left(V_{t b} V_{t s}^{*}+V_{u b} V_{u s}^{*}+V_{c b} V_{c s}^{*}=U_{s b}\right)$ leads to a new $b s z^{0}$ vertex which is proportional to $U^{s b}$ (FCNC at tree level). One can show the Wilson

Coefficients, $c_{3}, c_{7}, c_{9}$, and $c_{g}$ receive the following additional contributions at scale $\mu=M_{\mathrm{W}}$ [36]:

$$
\begin{align*}
& C_{3}^{\mathrm{VLDQ}}=\frac{1}{6} \\
& C_{7}^{\mathrm{VLDQ}}=\frac{2}{3} \sin ^{2} \theta_{W}, \\
& C_{9}^{\mathrm{VLDQ}}=-\frac{2}{3}\left(1-\sin ^{2} \theta_{W}\right), \\
& C_{g}^{\mathrm{VLDQ}}=-\frac{1}{3} . \tag{21}
\end{align*}
$$

Consequently, we can write the new effective Hamiltonian as:

$$
\begin{align*}
\mathcal{H}_{\mathrm{eff}}= & G_{\mathrm{F}} / \sqrt{2}\left\{V_{u b} V_{u s}^{*}\left(c_{1} O_{1}^{u}+c_{2} O_{2}^{u}\right)+V_{c b} V_{c s}^{*}\left(c_{1} O_{1}^{c}+c_{2} O_{2}^{c}\right)\right. \\
& \left.-V_{t b} V_{t s}^{*}\left[\sum_{i=3}^{10} c_{i}^{\mathrm{new}} O_{i}+c_{g}^{\mathrm{new}} O_{g}\right]+\text { h.c. }\right\} \tag{22}
\end{align*}
$$

where $c_{i}^{\text {new }}$ and $c_{g}^{\text {new }}$ are:

$$
\begin{equation*}
c_{i}^{\mathrm{new}}=c_{i}-\frac{U_{s b}}{V_{t b} V_{t s}^{*}} c_{i}^{\mathrm{VLDQ}}, \quad i=3 \ldots 10, g \tag{23}
\end{equation*}
$$

Using the Renornmalization Group Equations, we can run the above Wilson Coefficients down to scale $\mu=m_{b}[36]$ and finally obtain the helicity amplitudes in Eq. (12) by substituting:

$$
\begin{equation*}
C^{h} \longrightarrow C_{\text {new }}^{h} \equiv V_{t b} V_{t s}^{*} a_{\text {new }}^{h}, \tag{24}
\end{equation*}
$$

where $U_{s b}=\left|U_{s b}\right| e^{i \phi_{s b}} . \quad a_{\text {new }}^{h}$ is again obtained from Eq. (10) with $a_{i}$ 's calculated via the substitution $c_{i} \rightarrow c_{i}^{\text {new }}$. Using the experimental data on $\operatorname{Br}\left(B \longrightarrow X_{s} \ell^{+} \ell^{-}\right)[37,38]$, and assuming the dominance of the tree level contribution, one can extract the rough constraint $\left|U_{s b}\right| \leq 10^{-3}$ and therefore in this paper we take the parameters $\left|U_{s b}\right|$ and $\phi_{s b}$ as the following:

$$
\begin{equation*}
0 \leq\left|U_{s b}\right| \leq 10^{-3}, \quad 0 \leq \phi_{s b} \leq 2 \pi \tag{25}
\end{equation*}
$$

We have preformed a $\chi^{2}$ fit similar to the one in the previous section within the above restricted parameter space and the results are shown in Table III and Figs 3 and 4. We observe from Table III that, unlike the addition of ordinary quarks, the extra vector quark leads to a better fit to the data when minus sign convention is assumed. However, the resulted values of the helicity amplitudes are far from the experimental data.


Fig. 3. The results of VQM for $\rho=1, \theta=0$ in the minus phase.


Fig. 4. The results of VQM for $\rho=1, \theta=0$ in the plus phase.
TABLE III
The values of helicity amplitudes for SM and VLDQ.

| Model | Phase | $\frac{\chi^{2}}{8}$ | $U_{s b}$ | $\phi_{s b}$ | $\bar{R}_{0}$ | $\bar{R}_{\perp}$ | $R_{0}$ | $R_{\perp}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SM | - | - | - | - | 0.87 | 0.062 | 0.94 | 0.030 |
| VLDQ | minus | 17.33 | 0.001 | 3.11 | 0.89 | 0.053 | 0.93 | 0.035 |
|  | plus | 34.81 | 0.001 | 1.88 | 0.87 | 0.061 | 0.93 | 0.035 |

## 5. Results and discussion

Although, adding extra quarks, ordinary or vector like, to the SM can have strong effects on the SM observables in some processes [39-41], our $\chi^{2}$ analysis shows that such models cannot explain the experimental data for the helicity amplitudes of $B \rightarrow \Phi K^{*}$ decays within the constrained parameter
space. Assuming the smallness of the annihilation diagram in QCDF, one can conclude that considering the left handed operators alone are not enough to decrease the gap between SM and observed values and the presence of new operators in the effective Hamiltonian is perhaps necessary. This issue has not been directly proved previously.

## REFERENCES

[1] [BABAR Collaboration] B. Aubert et al., Phys. Rev. Lett. 91, 171802 (2003).
[2] [BABAR Collaboration] B. Aubert et al., Phys. Rev. D69, 031102 (2004); Phys Rev. Lett. 93, 231801 (2004).
[3] [BELLE Collaboration] K.F. Chen et al., Phys. Rev. Lett. 91, 201801 (2003).
[4] [BELLE Collaboration] K. Abe et al., hep-ex/0408141.
[5] A.L. Kagan, Phys. Lett. B601, 151 (2004) [hep-ph/0407076].
[6] P. Colangelo, F. De Fazio, T.N. Pham, Phys. Lett. B597, 291 (2004).
[7] M. Ladisa, V. Laporta, G. Nardulli, P. Santorelli, Phys. Rev. D70, 114025 (2004).
[8] H.Y. Cheng, C.K. Chua, A. Soni, Phys. Rev. D71, 014030 (2005).
[9] C.W. Bauer, D. Pirjol, I.Z. Rothstein, I.W. Stewart, Phys. Rev. D70, 054015 (2004).
[10] M. Ciuchini, E. Franco, G. Martinelli, L. Silvestrini, Nucl. Phys. B501, 271 (1997).
[11] M. Ciuchini, E. Franco, G. Martinelli, M. Pierini, L. Silvestrini, Phys. Lett. B515, 33 (2001).
[12] M. Beneke, J. Rohrer, D. Yang, Phys. Rev. Lett. 96, 141801 (2006).
[13] W.S. Hou, M. Nagashima, hep-ph/0408007.
[14] A. Datta, A.V. Gritsan, D. London, M. Nagashima, A. Szynkman, Phys. Rev. D76, 034015 (2007).
[15] P.K. Das, K.C. Yang, Phys. Rev. D71, 094002 (2005).
[16] V. Bashiry, F. Falahati, hep-ph/07073242.
[17] A. Arhrib, W.-S. Hou, Eur. Phys. J. C27, 555 (2003).
[18] A. Arhrib, W.-S. Hou, Phys. Rev. D64, 073016 (2001) [hep-ph/0012027].
[19] C.-S. Huang, W.-J. Huo, Y.-L. Wu, Phys. Rev. D64, 016009 (2001).
[20] V. Bashiry, K. Azizi, J. High Energy Phys. 0707, 064 (2007)
[hep-ph/0702044].
[21] W.-S. Hou, H.-N. Li, S. Mishima, M. Nagashima, Phys. Rev. Lett. 98, 131801 (2007) [hep-ph/0611107].
[22] M. Ahmady, M. Nagashima, A. Sugamoto, Phys. Rev. D64, 054011 (2001) [hep-ph/0105049].
[23] W.-S. Hou, M. Nagashima, A. Soddu, Phys. Rev. Lett. 95, 141601 (2005) [hep-ph/0503072].
[24] S. Sultansoy, hep-ph/0004271.
[25] M.S. Chanowitz, M.A. Furlan, I. Hinchliffe, Nucl. Phys. B153, 402 (1998).
[26] A. Datta, S. Rayachaudhiri, Phys. Rev. D49, 4762 (1994).
[27] H.Y. Cheng, K.C. Yang, Phys. Lett. B511, 40 (2001).
[28] A.J. Buras, Rev. Mod. Phys. 52, 199 (1980).
[29] [Particle Data Group] W.-M. Yao et al., J. Phys. G 33, 1 (2006).
[30] G. Savard et al., Phys. Rev. Lett. 95, 102501 (2005).
[31] M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, Nucl. Phys. B591, 313 (2000) [hep-ph/0006124].
[32] H.N. Li, hep-ph/0411305.
[33] A.J. Buras, hep-ph /980471.
[34] X.Q. Li, G.R. Lu, Y.D. Yand, Phys. Rev. D68, 114015 (2003).
[35] [BABAR Collaboration] B. Aubert et al., Phys. Rev. Lett. 93, 231804 (2004); A. Gritsan, hep-ex/0409059.
[36] C.-H.V. Chang, D. Chang, W.-Y. Keung, Phys. Rev. D61, 053007 (2000).
[37] [BABAR Collaboration] B. Aubert et al., Phys. Rev. Lett. 93, 081802 (2004).
[38] [BELLE Collaboration] M. Iwasaki et al., Phys. Rev. D72, 092005 (2005).
[39] S.M. Zebarjad, F. Falahati, H. Mehranfar, Phys. Rev. D79, 075006 (2009).
[40] V. Bashiry, S.M. Zebarjad, F. Falahati, K. Azizi, J. Phys. G 35, 065005 (2008).
[41] A. Ahmady, S.M. Zebarjad, F. Falahati, Charmless hadronic two-body $B_{s}$ decays within the vector quark model, in preparation.


[^0]:    ${ }^{\dagger}$ Corresponding author: zebarjad@physics.susc.ac.ir

[^1]:    ${ }^{1}$ The annihilation contribution which is power suppressed is neglected here [27].

