MASSLESS FIELDS INFINITELY DEGENERATED WITH RESPECT TO THE HELICITY AND THEIR EFFECTS IN ASTROPHYSICS

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A tachyon field having a negative squared-mass $-m_t^2$ can be described in terms of massless fields degenerated infinitely with respect to the helicity. The degeneracy leads symmetry breakings of space-time. This picture for the tachyon does not contradict causality. The tachyon vector-field is quenched from the interactions with matter fields, and the effects can be represented by a phase factor. The accelerated expansion of the universe and the dark energies are interpreted in terms of the phase factor. An asymmetry between the distribution of particles and that of anti-particles in the universe is also derived from the phase. Membranes can be described by the tachyon wave packet.

1. Introduction

The Klein–Gordon equation for a tachyon having negative squared-mass $-m_t^2$ (m_t = a positive real number) is written as

$$\left[\partial^\mu \partial_\mu - m^2\right]\phi(r_\mu) = \left[\frac{\partial^2}{\partial \tau^2} - \Delta(x,y,z) - m^2\right]\phi(\tau, \vec{r}) = 0\,, \qquad (1)$$

where the metric tensors are taken as $\eta_{\tau\tau} = 1$, $\eta_{ii} = -1$ for $i = x, y, z$, and $\eta_{\mu\nu} = 0$ for $\mu \neq \nu$, and $\tau = ct$ (c = light velocity), $m = m_t c/\hbar$, and $\Delta(x,y,z) = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$. It is known that in relativistic transformations the tachyon must have a velocity exceeding the light velocity c, and then the existence of such a particle breaks causality. The tachyon has, of course, been not yet observed. Here an idea that the introduction of the tachyon does not contradict causality is proposed. In the idea the tachyon

will be described by massless fields infinitely degenerated with respect to the helicity. A vector field playing the role similar to a gauge field is included in the massless fields. It will be shown that the tachyon vector-field has a close connection with problems in astrophysics such as the dark energy and the accelerated expansion of the universe. The membrane in Lorentz space will also be constructed in terms of the wave packet of the tachyons.

2. Tachyon as infinitely degenerate massless-fields

Eq. (1) can be divided into two equations with two dimensions [1,2], *e.g.*

$$\left[\frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial z^2}\right] \phi_z(\tau, z) = 0 \tag{2}$$

and

$$\left[-\Delta(x, y) - m^2\right] A(x, y) = 0, \tag{3}$$

where the field ϕ is written by

$$\phi(\tau, \boldsymbol{r}) = \phi_z(\tau, z) A(x, y).$$

One for the two-dimensional $z\tau$ space is the relativistic equation of motions for a massless particle, of which solutions are written by the plane wave $e^{\pm i(p_0\tau - pz)/\hbar}$ with $p_0 = |p|$ and $-\infty < p < +\infty$, whereas the other for the two-dimensional xy space is a two-dimensional Schrödinger equation in a negative constant potential for the solutions with the zero-energy eigenvalue $E = 0$. The $+$ and $-$ symbols in the plane wave $e^{\pm i(p_0\tau - pz)/\hbar}$, respectively, correspond to the creation and the annihilation of the field in field theories. Note that the separation of the two directions perpendicular to the non-zero three-momentum $\boldsymbol{p}$ in the four-dimensional space can covariantly be carried out in terms of the four-momenta given by $p^+ = (|\boldsymbol{p}|, \boldsymbol{p})$ and $p^- = (|\boldsymbol{p}|, -\boldsymbol{p})$ such that $\varepsilon_{\mu\nu\lambda\sigma} p^{+\lambda} p^{-\sigma}/2|\boldsymbol{p}|^2$, where $\varepsilon_{\mu\nu\lambda\sigma}$ are the totally anti-symmetric tensors defined by $\varepsilon_{0123} = 1$. In the covariant expression $-\Delta(x, y)$ is written in terms of these tensors such as

$$\varepsilon_{\mu\nu\lambda\sigma}\partial^\nu p^{+\lambda} p^{-\sigma} \varepsilon^{\mu\nu'\lambda'\sigma'} \partial_{\nu'} p^+_{\lambda'} p^-_{\sigma'}/(2|\boldsymbol{p}|^2)^2.$$

Since the covariant expression is not convenient for making our discussion simple, the moving direction of the tachyon is fixed to the z direction in the following discussion. Note that in this choice the two axes $\tau \pm z$ are taken as two light-cone coordinates.

It has been shown that two-dimensional Schrödinger equations with the central potentials $V_a(\rho) = -a^2 g_a \rho^{2(a-1)}$ ($a \neq 0$ and $\rho = \sqrt{x^2 + y^2}$, *i.e.*, $x = \rho \cos\varphi$ and $y = \rho \sin\varphi$) have zero-energy eigenstates which are infinitely

degenerate [3–6]. Eq. (3) is nothing but the equation for $a = 1$. This fact indicates that the tachyon can be interpreted as fields with the infinite degeneracy. Let us briefly reconsider the argument for the two-dimensional zero-energy solutions with the infinite degeneracy. Putting the function $A(x,y)_n^{\pm} = f_n^{\pm}(x,y)e^{\pm imx}$ into Eq. (3), where $f_n^{\pm}(x,y)$ are polynomials of degree n $(n = 0, 1, 2, \cdots)$, the following equations for the polynomials are obtained;

$$\left[\Delta(x,y) \pm 2im\frac{\partial}{\partial x} \right] f_n^{\pm}(x,y) = 0 \,. \tag{4}$$

From the above equations it is seen that the relations $(f_n^-)^* = f_n^+$ hold for all values of a and n. General forms of the polynomials and the introduction of the angle to the x axis have been carried out [4, 5]. It has been shown that the functions A_n belong to the conjugate space of the nuclear space in the Gel'fand triplet $\mathcal{S}(\mathcal{R}) \subset L^2(\mathcal{R}) \subset \mathcal{S}(\mathcal{R})^{\times}$, where $\mathcal{S}(\mathcal{R})$, $L^2(\mathcal{R})$ and $\mathcal{S}(\mathcal{R})^{\times}$ are, respectively, the Schwartz space, a Lebesgue space and the conjugate space of $\mathcal{S}(\mathcal{R})$ [3–9]. It should be stressed that in the massless case for $m = 0$ Eq. (4) does not have the polynomial solutions, and in the positive squared-mass case for $+m^2$ the functions A_n have the factor $e^{\pm mx}$, instead of $e^{\pm imx}$, which diverges one of the limits of $x \to \pm\infty$ and then the solutions do not have any good nuclear space for the construction of Gel'fand triplet. That is to say, only in the negative squared-mass case the well-defined Gel'fand triplet can be found. Since all the functions have the factors $e^{\pm imx}$, it can be seen that the zero-energy states describe stationary flows in the xy plane [3–7]. In this picture the tachyon mass parameter m is understood to be the wave number of the stationary flows in the xy plane, and the stationary flows do not change under the relativistic transformations in the $z\tau$ space. In general, in the relativistic covariant expression in terms of the anti-symmetric tensors $\varepsilon^{\mu\nu\lambda\sigma}$, the tachyon fields behave as massless fields with the four-momentum p^+ or p^- under the four-dimensional relativistic transformations. It can be said that the tachyons are the massless fields which expand into the two-dimensional space perpendicular to the moving direction as the stationary flows with the zero-energy eigenvalue. The tachyon fields are no longer particles but represent the movements of the two-dimensional flows. Of course, they move with the light velocity as massless field, and therefore the existence of the tachyon fields does not contradict causality. It should be noticed that in this representation the tachyon fields definitely break the four dimensional space-time symmetry, although the original equation (1) trivially has it. The infinite degeneracy of the tachyon arises from this symmetry breaking.

3. Helicity representation of tachyon fields

The tachyon fields expressed by the above polynomials $f_n^{\pm}$, however, do not have any definite properties with respect to the rotations in the xy space. Let us study another representation of the infinite degeneracy of the zero-energy states in terms of the two-dimensional polar coordinate [7, 10]. Eq. (3) can be rewritten in the polar coordinate as

$$\left[-\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + \frac{1}{\rho^2}\left(\frac{\hat{L}_z}{\hbar}\right)^2 - m^2 \right] R(\rho,\varphi) = 0 \,, \tag{5}$$

where $\hat{L}_z = -i\hbar\frac{\partial}{\partial\varphi}$, $\rho^2 = x^2 + y^2$, and φ is the angle from the positive x axis. In general $\rho^2 = x^2 + y^2$ should be written as a four-dimensional scalar in terms of the anti-symmetric tensors $\varepsilon^{\mu\nu\lambda\sigma}$ and the four-momenta $p^{\pm}$ in the covariant expression. This equation has solutions with an infinite degeneracy with respect to the angular momentum $\hat{L}_z$. Putting the eigenfunctions of $\hat{L}_z$, i.e.

$$R_l = h_l(\rho)e^{il\phi}$$

into Eq. (5), the equation for h_l is given by

$$\left[-\frac{1}{\rho}\frac{d}{d\rho}(\rho\frac{d}{d\rho}) + \frac{l^2}{\rho^2} - m^2 \right] h_l(\rho) = 0 \,. \tag{6}$$

This is the equation for the Bessel function, and then the solution is given by $h_l(\rho) = J_l(q)$, where $q = m\rho$. We have a unique solution for each l. The solutions have the infinite degeneracy with respect to the eigenvalues of L_z. That is to say, the tachyon expressed by R_l represents a massless field with the helicity $\hbar l$. We see that the eigenfunctions R_l for $l = positve\ integers$ represent the boson fields and those for $l = positve\ integers + 1/2$ do the fermion fields. The coexistence of the boson and fermion fields suggests the introduction of the supersymmetry [7], but the theme will not be discussed here.

Note that, since these helicity eigenfunctions are not in any Hilbert space but in the conjugate space of Gel'fand triplet, the normalizations of the solutions cannot generally be introduced. This fact generates an important difference from the usual massless-fields like photons. That is to say, the infinite numbers of the tachyon fields are never observed as point-particles like photons, and they are observed as waves extending over the two-dimensional space perpendicular to the moving direction.

4. Tachyon tensor fields

Let us study the tensor representation of the tachyon fields. Before the study it should be noted that the multiplication of the dimensionless scalar

$$Q(\tau, z; p) = \frac{p^\mu r_\mu}{\hbar}$$

to R_l does not change the property of R_l as the solution of Eq. (1) for $p^\mu = p^{\pm\mu}$ because of the massless condition $p^2 = 0$. In general the functions

$$\chi_l = f(Q)R_l \tag{7}$$

are the solutions for the tachyon fields, where $f(Q)$ is an arbitrary differentiable-function with respect to Q. In general the solutions with $f(Q) \neq$ *constant* are no longer the plane waves in the $z\tau$ space, but they can be the solutions in the Gel'fand triplet. This fact means that tachyons have another infinite freedom with respect to the functions of Q, which is common to all massless fields in the Gel'fand triplet. Furthermore we see that $T_{l\mu\nu\,...} = \partial_\mu\partial_\nu \ldots \chi_l$ can be the solutions of Eq. (3) such that

$$\left[-\Delta(x, y) - m^2\right] T_{l\mu\nu\,...} = 0\,. \tag{8}$$

Taking into account that χ_0 is the scalar in the four-dimensional space-time, arbitrary tensors in the four-dimensional space-time can be constructed from the scalar χ_0 such that

$$T_{0\mu\nu\,...} = \partial_\mu\partial_\nu \ldots \chi_0\,. \tag{9}$$

The tensor fields

$$\phi_{\mu\nu\,...} = T_{0\mu\nu\,...}e^{ip^\mu r_\mu/\hbar}$$

satisfy Eq. (1), where the four-coordinate $r = (\tau, x, y, z)$ and the four-momentum $p = (|p|, 0, 0, p)$ are taken in the present model. Note that the factor $p^\mu r_\mu/|p|$ represents $x_- = \tau - z$ for $p = (|p|, 0, 0, |p|)$ and $x_+ = \tau + z$ for $p = (|p|, 0, 0, -|p|)$ in the light-cone coordinates. Using the recurrence formulae $dJ_l(q)/dq = lq^{-1}J_l(q) - J_{l+1}(q)$, we can see the relations between the helicity representation and the tensor one.

5. A kind of gauge of the tachyon field

It is seen that the field ϕ_μ describes a tachyon vector-field, and $T_{0\mu}$ represents the strength of the tachyon vector-field. Let us study the tachyon vector-field. Hereafter we put $T_\mu \equiv T_{0\mu}$. The anti-symmetric tensor representing the field strength can be introduced as same as the electromagnetic field strength such that

$$F_{\mu\nu} = \partial_\mu T_\nu - \partial_\nu T_\mu\,. \tag{10}$$

The tensor obviously fulfills the equation

$$\partial^\nu F_{\mu\nu} = 0 \Rightarrow \partial^\nu \partial_\mu T_\nu - \partial^2 T_\mu = 0 \tag{11}$$

that corresponds to the equation in the case for the absence of source. It can be said that T_μ is a kind of vector potentials. Now we can write the equation of motion of a matter field having spin $\frac{1}{2}$ with the vector potential as

$$\bar{\psi}(i\gamma^\mu(\partial_\mu - ig_c T_\mu) - M)\psi, \tag{12}$$

where g_c and M are, respectively, the coupling constant and the mass of the matter field. In this model the transformation similar to the gauge transformations of ψ and T_μ can be introduced such that

$$\psi \to \psi' = e^{ig_c \chi^T} \psi, \qquad T_\mu \to T'_\mu = T_\mu + \partial_\mu \chi^T, \tag{13}$$

and the tachyon ϕ_μ is transformed as

$$\phi_\mu \to \phi'_\mu = \left(T_\mu + \partial_\mu \chi^T\right) e^{ip^\mu r_\mu/\hbar}. \tag{14}$$

Here χ^T is an arbitrary function given in (7). Under these transformations the interaction does not change the form such that

$$\bar{\psi}' \left(i\gamma^\mu \left(\partial_\mu - ig_c T'_\mu\right) - M\right) \psi'. \tag{15}$$

If $\chi^T = -\chi_0$ is taken, T'_μ obviously vanishes, namely, $T'_\mu = 0$. In this case the matter field seems to be free from the vector potential induced by the tachyon vector-field. Actually the tachyon vector-field is quenched such that $\phi'_\mu = 0$. In the matter field, however, the effect of the interaction with the tachyon remains as the phase factor expressed by the function

$$\tilde{F} = e^{-ig_c \chi_0}. \tag{16}$$

If the gauge coupling constant g_c is common for all matter fields, the phase factor is also common for all the matter fields. Except the case for $f(Q) = constant$ the phase depends not only on ρ but also on τ and z. It should be stressed that this phase factor appears only in the extended space, that is, in the Gel'fand triplet.

The high tensor fields $\phi_{\mu\nu\ldots}$ have the property such that

$$\partial_\mu \phi_{\mu\nu\ldots} = m^2 \phi_{\nu\ldots}.$$

This means that in the vector couplings with matter fields like (12) all the tensor fields reduce to the vector one such that

$$\partial_{\nu_1} \ldots \partial_{\nu_n} \phi_{\mu\nu_1 \ldots \nu_n}/m^{2n} = \phi_\mu.$$

In the gauge theory the effects induced by all the tensor fields are described by the same factor with (16).

6. Dark fields in the universe

Now we know that the infinite numbers of tachyons can exist and they are no longer observed as particles. This means that we cannot observe them in any experiments where new particles are searched. But they can carry the large part of the total energy of this universe, because they have the infinite degeneracy. As seen in the last section, in the gauge interaction they decouple with any matter fields. This means that we cannot produce them in the interactions of particles observed in gauge interactions, That is to say, the tachyons are dark fields from the standpoint of the present gauge theories. Their existence can, of course, be seen in the gravitational interaction, because they possibly carry the large part of the energy in the universe, which has already been observed as the large missing energy. Do we have any other way to observe them? We can see their trace in the phases of all matter fields.

Let us study the effect of the phase factor. All the matter fields have the same phase factor $\tilde{F}$ such that

$$\tilde{\psi} = \tilde{F}\psi\,. \qquad (17)$$

The quantity corresponding to the four-momentum of the matter field

$$\tilde{p}_\mu \equiv i\frac{1}{2}\hbar\left[\tilde{\psi}^*\partial_\mu\tilde{\psi} - \partial_\mu\tilde{\psi}^*\tilde{\psi}\right]\Big/\left|\tilde{\psi}\right|^2$$

is evaluated as

$$\tilde{p}_\mu = i\tfrac{1}{2}\hbar\left[\psi^*\partial_\mu\psi - \partial_\mu\psi^*\psi\right]/|\psi|^2 + \tfrac{1}{2}\hbar g_c\left[\partial_\mu(\chi_0 + \chi_0^*)\right]\,. \qquad (18)$$

Note that in the case of the free field ψ that is described by a plane wave $e^{ip_{\psi\mu}r^\mu}$ the first term just gives the four-momentum $p_{\psi\mu}$ of the free motion. It is seen that the momentum $\tilde{p}_\mu$ for $\tilde{\psi}$ is different from that for ψ by the second term $\hbar g_c[\partial_\mu(\chi_0 + \chi_0^*)]/2 = \hbar g_c\mathrm{Re}[\partial_\mu\chi_0]$, which is brought by the phase factor $\tilde{F}$. The second term can be understood as the difference from the momentum of the field being free from the interaction with the tachyon vector-field. Here let us study a simple case for $f(Q) = Q$. It is important that the factor $Q = p^\mu r_\mu/\hbar$ in χ_0 brings the term that is proportional to time $g_c|p|ct\partial_i R_0$ for $i = x, y$. Since R_0 has no φ-dependence, the derivatives give non-zero effect only for the direction of ρ, that is evaluated as

$$g_c|p|ct\partial_\rho J_0(q) = -g_c|p|ctmJ_1(q)\,. \qquad (19)$$

This result tells us that the field $\tilde{\psi}$ moves under a force without time-dependence, that is to say, the motion of the field is the motion accelerated by the time-independent force. In order to recover the uniformity of the

space-time it is natural to consider that the distribution of the tachyons are uniform and have no specific direction for the momentum. Then the average with respect to the distribution of the tachyon momentum p_μ eliminates the term $p_z z$ depending on the direction of the tachyon momentum included in the second term, and the remaining term is given as

$$-g_c |\bar{p}| c t m J_1(q), \tag{20}$$

where $|\bar{p}|$ stands for the average of the magnitude of the tachyon momentum. The universal acceleration for the radial direction reminds us the accelerated expansion of the universe [14, 15]. For the short time after the birth of the universe many numbers of tachyons were produced as well as other particles. During the long time of the expansion they reached the stable distribution, and have been swallowed up as the phase factor by the matter fields. From the relation $q = m\rho$ it can be seen that, if the tachyon mass parameter $m = m_t c/\hbar$ is small enough to fulfill the condition that $q = m\rho$ is less than the first zero-point of J_1 for $q \neq 0$ all over the present universe, all the matter fields suffer the positive acceleration for $g_c < 0$ or the negative one for $g_c > 0$. In such a case the acceleration does not disturb the internal movement of each galaxy, because the sizes of the galaxies are very small in comparison with that of the universe, and therefore $J_1(q)$ can be taken as a constant in each galaxy. The effect of the phase factor can be an answer to the accelerated expansion of the present universe.

Let us estimate the magnitude of the effect. For the present time scale corresponding to the age of the universe the effects are observed as positive effects all over the universe [15]. This means that g_c must be negative for the matter fields like protons and neutrons, and $q = m\rho$ must be smaller than the first zero-point of J_1 but $q = 0$ so as to satisfy the condition $J_1(q) > 0$. By using the age of the universe ($\sim 1.4 \times 10^{10}$ years) the order of the radius of the universe is estimated as $10^{26}(m)$. From the condition that the first zero-point of J_1 is less than 4, that is, $q = m\rho < 4$ for $\rho \sim 10^{26}$, the order of the tachyon mass m_t is estimated as

$$m_t \leq 10^{-68}(\text{kg}). \tag{21}$$

This value being nearly $10^{-38} m_e$ (m_e = electron mass) is very small in comparison with masses of other particles known at present. It should, however, be noted that m_t is not the mass but the parameter of the potential for deriving the zero-energy solutions.

Let us study a little more the magnitude of the momentum. In order to interpret the acceleration experimentally observed [14, 15] the order of Eq. (20) should be the same as those of the momenta of the matter fields on the present time scale of the universe. Since the magnitude of

ctm for *t = the age of the universe* and also that of $J_1(m\rho)$ for $\rho <$ *the size of the universe* are of the order $O(1)$, the order of the momentum of Eq. (20) is estimated as

$$-g_c|\bar{p}| \sim M\gamma c\beta\,,$$

where the Lorentz factors are given by $\gamma = 1/\sqrt{1 - \beta^2}$ and $\beta = v/c$ for the velocity of the matter v. The order of the dimensionless coupling constant g_c is obtained as

$$|g_c| \sim M\gamma c\beta/|\bar{p}|\,. \tag{22}$$

Considering that the average of the tachyon momentum $|\bar{p}|$ will be not very small and also not very large in comparison with those of the matter fields, we can say that the coupling constant g_c has a reasonable magnitude.

Tachyons must be created also in the growth of each galaxy. Such tachyons give another effect to each galaxy. It can be said that the effect on the momentum is very small, since $q = m\rho$ for the galaxy scale is very small in comparison with that for the universe size and $J_1(q)$ can be taken to be nearly 0 for such small values of q.

This effect also increases or decreases the energy of the matter field as well as the three-momentum, which is evaluated in terms of the 0-component of $\tilde{p}_\mu$. The effect obtained as $-g_c|\bar{p}|cJ_0(q)$ is positive for $g_c < 0$ as well as that on the momentum. By using the above estimations of Eqs (21) and (22), the order of the energy correction is seen to be the same order with that for the matter. The effect of the tachyons produced in the growth of each galaxy, however, cannot be neglected in the energy corrections, because $J_0(q)$ has the largest value at $q = 0$. This means that the dark energies can have two origins, that is to say, one arises from the birth of the universe and the other from the birth of each galaxy. The former spreads all over the universe, while the latter is trapped in the galaxy. It will be observed in gravitational effects. It can be said that the tachyons can be the candidates for the dark fields in the universe and also in galaxies.

7. Remarks on few interesting problems

In the gauge theory the coupling constant g_c for anti-particles changes the sign from that for particles. This means that the effects by the phase factor are opposite between particles and antiparticles. In the universe where the particles are accelerated the anti-particles are braked. In the early universe the expansion speeds of the anti-particles became slow in comparison with those of the particles. This effect can cause the asymmetry between the distribution of the particles and that of the anti-particles in the present universe. It is possible that the anti-particles which have lost their velocities through the interaction with the tachyon fields are gathered by the gravitational force and construct another world being different from our world.

Such an asymmetry is possibly observed in each galaxy. The production rates of the anti-particles are small in galaxies mainly constituted by the particles, but their expansion is suppressed and they are left behind near the center of the galaxy. Those anti-particles left behind possibly make a large group by the gravitational interaction. We shall observe such a large group at the center of the galaxy, sometimes as a black hole.

The zero-energy solutions contain the $l = \pm 2$ states written by $R_{\pm 2}$. The massless tensor fields are written by the zero-energy solutions such that [2]

$$h_{\mu\nu} = g\partial_\mu\partial_\nu\chi_0 \,, \tag{23}$$

where g is a constant. The tensor field satisfies the equation

$$\partial^2 h_{\mu\nu} - \partial^\sigma(\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\mu\sigma}) + \eta^{\mu\nu}h_{\mu\nu} = 0 \tag{24}$$

that is the linearized equation for the small fluctuation of gravitational field $g_{\mu\nu}$ around the Minkowski metric $\eta_{\mu\nu}$, which is given by

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \,.$$

The gravitational interaction can be induced through the interactions with the tachyons. Note, however, that all of $h_{\mu\nu}$ will possibly be quenched by the choice of the appropriate phase factor.

It can be said that the tachyons presented here bring some interesting standpoints for the problems in astrophysics.

Finally, it is mentioned that the argument presented here can be extended to the case for the two-dimensional potentials $V_a(\rho)$ in terms of the conformal transformations [1,3–7]. Actually the cases for $a = 1/2$ ($V_a(\rho) \propto \rho^{-1}$; Coulomb type potential) and $a = 3/2$ ($V_a(\rho) \propto \rho$; confinement type potential) were discussed in the previous works [1,7]. It can also be said that the case of $a = 2$ representing the parabolic potential ($V_a(\rho) \propto \rho^2$) will have a certain relation with string theory. Actually the two-dimensional membrane can be described as follows:

Consider the wave packet in terms of the tachyon such that

$$\Phi_l(x,y,z,t) = \int\limits_{-\infty}^{\infty} d(p/\hbar) f(p - p_z)\chi_l(x,y)e^{\pm ip(z-\tau)/\hbar} \,. \tag{25}$$

Note that in this integration both states having the energy factors $e^{-i|p|\tau/\hbar}$ and $e^{i|p|\tau/\hbar}$ are included. If $f(p - p_z) = e^{-\alpha^2(p-p_z)^2/\hbar^2}/\sqrt{\pi\alpha}$ is taken, the result is given as

$$\Phi_l(x,y,z,t) = \chi_l(x,y)e^{-(z\mp\tau)^2/4\alpha^2}e^{\pm ip_z(z-\tau)/\hbar} \,. \tag{26}$$

It is seen that this wave packet is a membrane spreading in the xy plane, of which peak in the z direction is at $z = \pm\tau$. It can be seen that the wave packet does not change the structure at all in the time development, that is, not only the stationary structure in the xy plane but the thickness for the z direction as well. The wave packet, of course, moves with the light velocity c. The thickness goes to zero as α goes to zero. We see that the stable membrane moving with the light velocity can be described by the tachyons. It is noted that the membrane with $l \neq 0$ has a vortex at the origin of the two-dimensional space. The vortex may possibly be interpreted as a string interacting with the membrane. On the other hand, membranes without any vortices can be described in terms of the zero-energy solutions given in Eq. (4). The membrane of the tachyons may be related to that of string theory, and possibly observed as some astrophysical walls.

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