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HIGH PRECISION CALCULATIONS IN THE MSSM*

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We review recent developments and results for performing high precision calculations in the Minimal Supersymmetric Extension of the Standard Model (MSSM). As an example, the effects of the three-loop order radiative corrections on the unification of the gauge couplings will be discussed in some details.

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1. Introduction

One of the main purposes of the upcoming experiments to be conducted at the Large Hadron Collider (LHC) and the International Linear Collider (ILC) is to reveal the nature of electroweak symmetry breaking. The mechanism of spontaneous symmetry breaking in the Standard Model (SM) is probably incomplete, and a theory with a higher symmetry is necessary to solve the hierarchy problem. Supersymmetry (SUSY) provides a natural and highly predictive solution. The most studied SUSY extension of the SM is the Minimal Supersymmetric Standard Model (MSSM), which, in particular, agrees with all precision data at least as well as the SM.

Another compelling argument in favour of SUSY is the particle content of the MSSM, that leads in a natural way to the unification of the three gauge couplings at a high energy scale $\mu \simeq 10^{16}$ GeV, in agreement with predictions of Grand Unification Theories (GUT). It is often argued (for reviews see *e.g.* Refs [1]) that, from the precise knowledge of the low-energy supersymmetric parameters one can shed light on the origin and mechanism of supersymmetry breaking and even on physics at much higher energies, like the GUT scale. The extrapolation of the supersymmetric parameters measured at the TeV energy scale to the GUT-scale raises inevitably the question of

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uncertainties involved. Currently, there are four publicly available spectrum generating codes [2] based on two-loop order MSSM Renormalization Group Equations (RGEs) [3] subjected to two types of boundary conditions. One set of constraints accounts for the weak-scale matching between the MSSM and SM parameters to one-loop order [4]. The second one allows for the SUSY breaking at the high scale according to specific models like minimal supergravity, gauge mediation and anomaly mediation. The approximations within the codes differ by high-order corrections and by the treatment of the low-energy threshold corrections. The typical spread of the results is within few percents, which does not always meet the experimental accuracy. Recent analyses [5] have proven that the three-loop order effects on the running of the strong coupling constant α_s and the bottom quark mass m_b may exceed those induced by the current experimental accuracy [6, 7].

2. Prediction of $\alpha_\mathrm{s}(\mu_\mathrm{GUT})$

The aim of this study is to compute α_s at the high-energy scale $\mu \simeq \mathcal{O}(\mu_\mathrm{GUT})$, starting from the strong coupling constant at the mass of the Z boson M_Z. We denote this parameter $\alpha_\mathrm{s}^{\overline{\mathrm{MS}},(5)}(M_Z)$ to specify that the underlying theory is QCD with five active flavours and $\overline{\mathrm{MS}}$ is the renormalization scheme. The value of $\alpha_\mathrm{s}(\mu_\mathrm{GUT})$ is the strong coupling constant in the MSSM renormalized in the $\overline{\mathrm{DR}}$-scheme, that we denote as $\alpha_\mathrm{s}^{\overline{\mathrm{DR}},(\mathrm{full})}(\mu_\mathrm{GUT})$. For the evaluation of $\alpha_\mathrm{s}^{\overline{\mathrm{DR}},(\mathrm{full})}$ from $\alpha_\mathrm{s}^{\overline{\mathrm{MS}},(n_f)}$ we follow the "common scale approach", which requires a unique scale for the matching between QCD and MSSM. More precisely, for mass independent renormalization schemes like $\overline{\mathrm{MS}}$ or $\overline{\mathrm{DR}}$, the decoupling of heavy particles has to be performed explicitly. In practice, this means that the heavy degrees of freedom are integrated out from the full theory. For SUSY models with roughly degenerate mass spectrum at the scale $\tilde{M}$, one can consider the MSSM as the full theory valid from the GUT scale μ_GUT down to $\tilde{M}$, which we assume to be around 1 TeV. Below the scale $\tilde{M}$ the QCD is the effective theory describing the low-energy physics.

For the running analysis of the strong coupling constant, we can distinguish four individual steps that we detail below.

1. Running of $\alpha_\mathrm{s}^{\overline{\mathrm{MS}},(n_f)}$ from $\mu = M_Z$ to $\mu = \mu_\mathrm{dec}$.
   The energy dependence of the strong coupling constant is governed by the RGEs. In QCD with n_f quark flavours, the β function is known through four loops both in the $\overline{\mathrm{MS}}$ [8] and the $\overline{\mathrm{DR}}$-scheme [9].

2. Conversion $\alpha_\mathrm{s}^{\overline{\mathrm{MS}},(n_f)}(\mu_\mathrm{dec}) \to \alpha_\mathrm{s}^{\overline{\mathrm{DR}},(n_f)}(\mu_\mathrm{dec})$.
   For the three-loop running analysis we are focusing on, one needs to

evaluate the dependence of α_s values in the $\overline{\rm DR}$ scheme from those evaluated in $\overline{\rm MS}$ scheme to two-loop accuracy [9]. This requires the introduction of the so-called evanescent coupling constants due to the application of $\overline{\rm DR}$ to non supersymmetric theories (QCD in this case). In particular, it describes the coupling of the 2ε-dimensional components (so-called ε-scalars) of the gluon to a quark. It is an unphysical parameter that must decouple from any prediction for physical observables. We also used this property as a consistency check for our method.

3. Matching of $\alpha_s^{\overline{\rm DR},(n_f)}$ and $\alpha_s^{\overline{\rm DR},({\rm full})}$ at $\mu_{\rm dec}$.
   Integrating out all SUSY particles at the common scale of SUSY mass spectrum, one directly obtains the SM as the effective theory, valid at low energies. The transition between the two theories can be done at an arbitrary decoupling scale μ:

$$\alpha_s^{\overline{\rm DR},(n_f)}(\mu) \;=\; \zeta_s^{(n_f)}\,\alpha_s^{\overline{\rm DR},({\rm full})}(\mu)\,, \tag{1}$$

   ζ_s depends logarithmically on the scale μ, which is why one generally chooses $\mu \sim \tilde{M}$. In Eq. (1), $n_f = 6$ means that only the SUSY particles are integrated out, while for $n_f = 5$ at the same time the top quark is integrated out. In a supersymmetric theory, SUSY requires that the evanescent couplings are equal to the corresponding gauge couplings. As a consequence, if we assume that QCD is obtained by integrating out the heavy degrees of freedom (squarks and gluinos) from SUSY–QCD, the evanescent couplings are uniquely determined by matching conditions between the two theories, similar with those given in Eq. (1).

4. Running of $\alpha_s^{\overline{\rm DR},({\rm full})}$ from $\mu = \mu_{\rm dec}$ to $\mu = \mu_{\rm GUT}$.
   The energy dependence of the strong coupling constant is in this case governed by the MSSM RGEs. In SUSY–QCD, the β function has been evaluated in the $\overline{\rm DR}$-scheme through three loops [10].

Assembling the above mentioned steps, we can predict the value of $\alpha_s(\mu_{\rm GUT})$ with up to three-loop accuracy.

2.1. Numerical results

The decoupling scale is not a physical parameter and cannot be predicted by the theory. It is usually chosen to be of the order of the heavy particle mass in order to circumvent the appearance of large logarithms. At fixed order perturbation theory, it is expected that the relations between the running parameters evaluated at high-energy scales and their low-energy values become less sensitive to the choice of $\mu_{\rm dec}$, once higher order radiative corrections are considered. The dependence on the precise value of the decoupling scale is interpreted as a measure of the unknown higher order corrections. We discuss the scale dependence of $\alpha_{\rm s}(\mu_{\rm GUT})$ in Fig. 1.

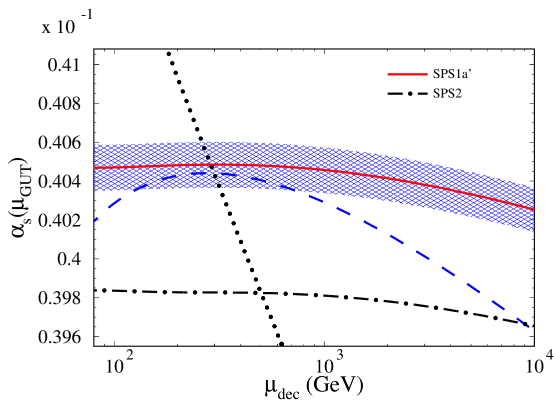

Fig. 1. $\alpha_{\rm s}(\mu_{\rm GUT})$ as a function of $\mu_{\rm dec}$.

For the SM parameters we used $\alpha_{\rm s}(M_Z) = 0.1189$ [6], where $M_Z = 91.1876$ GeV [11], $m_b(\mu_b) = 4.164$ GeV [7], with $\mu_b = m_b(\mu_b)$, and $M_t = 172.4$ GeV [12]. For the SQCD parameters, we implemented their values for the SPS1a$'$ scenario [1]: $m_{\tilde{g}} = 607.1$ GeV, $m_{\tilde{t}_1} = 366.5$ GeV, $m_{\tilde{t}_2} = 585.5$ GeV, $m_{\tilde{b}_1} = 506.3$ GeV, $m_{\tilde{b}_2} = 545.7$ GeV, $A_t^{\overline{\rm DR}}(1~{\rm TeV}) = -565.1$ GeV, $A_b^{\overline{\rm DR}}(1~{\rm TeV}) = -943.4$ GeV, $\mu = 396.0$ GeV, and $\tan\beta = 10.0$.

The dotted, dashed and solid lines denote the one-, two-, and three-loop running, where the corresponding exact results for the decoupling coefficients have been implemented. One can see the improved stability of the three-loop results w.r.t. the decoupling-scale variation. The uncertainty induced by the current experimental accuracy on $\alpha_{\rm s}(M_Z)$, $\delta\alpha_{\rm s} = 0.001$ [6], is indicated by the hatched band.

In order to get an idea of the effects induced by the SUSY mass parameters on $\alpha_{\rm s}(\mu_{\rm GUT})$, we show through the dash-dotted line the three-loop results, if the SUSY parameters corresponding to the Snowmass Point SPS2 [13] are adopted. Their explicit values are: $m_{\tilde{g}} = 784.4$ GeV, $m_{\tilde{t}_1} = 1003.9$ GeV, $m_{\tilde{t}_2} = 1307.4$ GeV, $m_{\tilde{b}_1} = 1296.6$ GeV, $m_{\tilde{b}_2} = 1520.1$ GeV, and $\tan\beta = 10.0$. The curves induced by the other benchmark points SPSi, with $i = 3, 4, \ldots, 9$ would lie between the two curves displayed here. One clearly notices the great impact of the SUSY-mass pattern on the predicted value of the strong coupling at high energies.

In conclusion, the three-loop effects are phenomenologically significant for the prediction of $\alpha_s^{\overline{\rm DR}}(\mu_{\rm GUT})$. The approach outlined here accounts for the effects induced by the individual mass parameters, that turn out to exceed the experimental uncertainty by more than a factor five.

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