NUMERICAL EVALUATION OF ONE-LOOP SCATTERING AMPLITUDES FOR HARD-PHOTON EMISSION IN BHABHA SCATTERING*

Stefano Actis

Paul Scherrer Institut, 5232 Villigen PSI, Switzerland

Pierpaolo Mastrolia

Theory Group, Physics Department, CERN, 1211 Geneva 23, Switzerland

Giovanni Ossola

Physics Department, New York City College of Technology
300 Jay Street, Brooklyn NY 11201, USA

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After a general introduction about the latest developments in the approach to multi-loop NLO calculations, we present a numerical evaluation of the one-loop QED corrections to the hard-bremsstrahlung emission in Bhabha scattering. The calculation is performed by employing the reduction method developed by Ossola, Papadopoulos and Pittau (OPP).

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1. A brief general introduction

Motivated by the upcoming LHC experiment, in the last few years we observed tremendous progress in the calculation of one-loop multi-leg processes. This involves several new results for cross-sections of interest for the LHC, but also the developments of many new techniques and codes [1].

Important results have been obtained with traditional methods [2–4]. Meanwhile, unitarity-based approaches led to the development of advanced tools [5,6] that proved to be very effective for processes such as $W+3$ jets [7]

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or multi-gluon [8] production. This last process was also the target of independent implementations of d-dimensional unitarity [9] and recursive relations for tensor integrals [10].

Three years ago, a new method was proposed for the numerical evaluation of one-loop amplitudes, now known as OPP reduction [11, 12], in which the reduction is performed at the integrand level [13]. With respect to other existing methods, it presents several advantages: the information required in order to perform the reduction is minimal, simply the numerical value of the numerator of the integrand for a set of values of the integration momentum is needed. Moreover, the method does not require any computer algebra and it incorporates a solid way to compute rational terms.

The next frontier is the systematic automatization of the calculations. In this process, important issues such as stability, precision and versatility should be addressed.

In the variety of alternative options available to performing one-loop multi-leg calculations, it is still too soon to decide which methods will prove themselves efficient and versatile enough to have an impact in future calculations [14]. During the last year, the progress in the automated generation of the numerators [15] and in the generation of dipole terms [16], coupled with the OPP reduction engine, led to the construction of a very powerful tool for multi-leg processes, that already proved its enormous potential [17, 18].

2. NLO QED corrections to photon emission in Bhabha scattering

Bhabha scattering is employed at e^+e^- colliders as a luminosity candle. At high-energy colliders such as LEP, the luminosity was measured by considering Bhabha scattering events at small scattering angles, while at meson factories operating at lower center-of-mass energies, such as DAΦNE or CESR, the luminosity is determined by analyzing large-angle Bhabha scattering events.

It is, therefore, important to reduce the theoretical uncertainty which affects the cross-section for this process, in order to achieve a better determination of the luminosity for the colliders [19].

Next-to-leading (NLO) corrections to $e^-e^+ \to e^-e^+$ in the full Standard Model were calculated long ago [20]. In order to proceed to the next order in the perturbative expansion (NNLO), there are three different sets of calculations that should be completed: two-loop corrections to the process $e^-e^+ \to e^-e^+$ [21, 22]; one-loop corrections including a single hard photon in addition to the outgoing electron–positron pair; tree-level corrections with two hard photons or a hard electron–positron pair in addition to the e^-e^+ couple in the final state of Bhabha scattering.

Very recently, we completed the calculation of the one-loop QED corrections to the process $e^-e^+ \to e^-e^+\gamma$ [23], the last missing contribution towards the completion of the full NNLO correction to Bhabha scattering. The calculation has been carried out by employing the OPP method [11,12]. Partial results were already available for small-angle Bhabha scattering [24] and s-channel annihilation processes at large angles [25]. The impact of the full calculation for this class of corrections could be rather small in high-energy regimes [26], while it is probably more important at small energies. Further phenomenological studies are in progress and they will be presented in future publications.

As a by-product, we have also computed the one-loop QED corrections to $e^-e^+ \to \mu^-\mu^+\gamma$[1]. This process is an important background for the determination of the pion form factor and provides an independent calibration for a measurement of the hadronic production cross-section. In addition, it represents an interesting application of the reduction method in the presence of two mass scales inside loop diagrams.

2.1. Virtual corrections

The one-loop QED corrections to the process $e^-e^+ \to e^-e^+\gamma$ involve only 76 Feynman diagrams. Representative graphs are shown in Fig. 1.

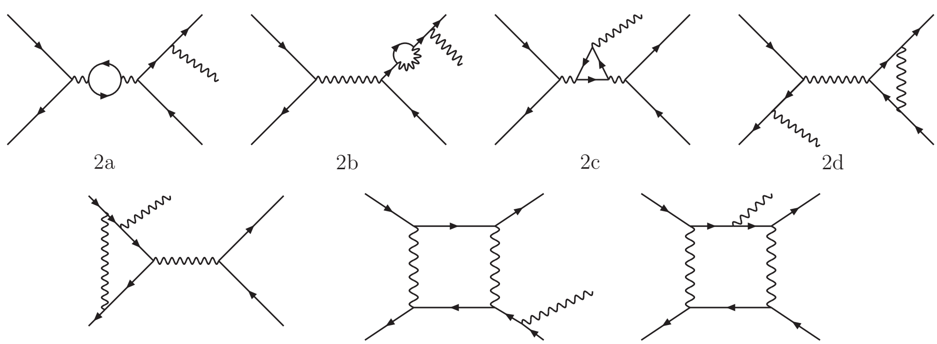

Fig. 1. Representative one-loop diagrams for $e^-e^+ \to e^-e^+\gamma$.

The calculation of the complete one-loop corrections requires the evaluation of pentagon diagrams of class 2g, the most challenging and time consuming part of this calculation. Note also that, due to Furry's theorem, diagrams of class 2c cancel in the sum. For further details about the calculation, we refer the reader to the original paper [23].

[1] This corrections have been also calculated independently by a different group [27].

The one-loop amplitudes, generated through QGRAF [28], have been processed with FORM [29] routines in order to produce a FORTRAN 95 output. The latter has then been used as an input for two computer implementations of the OPP technique which numerically evaluate the scattering amplitudes at any given phase-space point.

The first implementation employs the routines of the publicly available package CutTools [30] for the numerical determination of the coefficients which multiply the basis integrals. These routines have been combined with QCDLoop [31]. The second version of the calculation is used for cross-checking our results and it makes use of an independent code [32] for the reduction of the tensor integrals, which includes an optional optimization of the OPP technique based on the Discrete Fourier Transform [33]. The basis integrals are evaluated using the code OneLOop [34], written by A. van Hameren.

The interference of the one-loop amplitude $\mathcal{M}_{1-\mathrm{loop}}$ with the complex-conjugate tree-level amplitude $\mathcal{M}^{\star}_{\mathrm{tree}}$ is evaluated in dimensional regularization and subdivided into four contributions:

$$\mathcal{I}^{\mathrm{V}}_{\mathrm{NLO}} = \frac{1}{4}\sum_{\mathrm{spins}} 2\,\mathrm{Re}\,\left(\mathcal{M}_{1-\mathrm{loop}}\mathcal{M}^{\star}_{\mathrm{tree}}\right) = \mathcal{CC}_4 + \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{UV}_{\mathrm{ct}}\ . \quad (1)$$

We call $\mathcal{CC}_4$ the four-dimensional cut-constructible term, namely the contribution proportional to scalar boxes, triangles, bubbles and tadpoles [35,36]. The notation $\mathcal{R}_1 + \mathcal{R}_2$ defines the so-called rational part [37], while $\mathcal{UV}_{\mathrm{ct}}$ is the contribution induced by the ultraviolet counterterms.

We carried out the ultraviolet renormalization in the on-mass-shell scheme. Throughout our computation we have retained the full dependence on the fermion masses, introducing in particular the appropriate ultraviolet and $\mathcal{R}_2$-type mass counterterm diagrams depicted in Fig. 2.

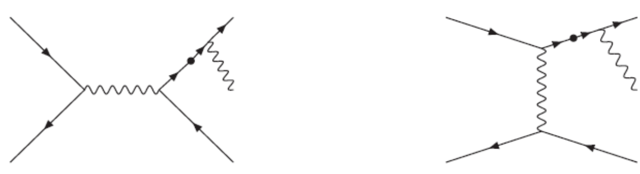

Fig. 2. Representative mass-counterterm diagrams for $e^-e^+ \to e^-e^+\gamma$. Black dots stand for mass-counterterm insertions, necessary for performing renormalization and computing the rational term $\mathcal{R}_2$ in the massive case.

The rational part $\mathcal{R}_1$ is computed by using different strategies in the two versions of the calculation: CutTools employs the mass-shift procedure introduced in Ref. [11], while the second approach uses the counterterm-based method described in Ref. [12]. The results obtained with the two independent implementations for the cut-constructible term and the component $\mathcal{R}_1$ are in very good agreement. The rational part $\mathcal{R}_2$ is calculated employing *ad hoc* Feynman rules for QED described in Ref. [37], that have been recently extended to QCD in Ref. [38].

2.2. Stability tests and results

In order to check the stability of our results, we have performed the so-called $N = N$ test, monitoring the agreement between the numerical values of the numerator function $\bar{N}(\bar{q})$ before and after the decomposition in terms of inverse propagators [30]. When the numerical agreement in the comparison does not reach a given limit set by the user, the code automatically triggers the use of the more time-consuming multi-precision routines [39]. We observe more than 9 digits of agreement between the results obtained in double precision, requiring a 10^{-5} relative precision for the $N = N$ test, and those we got after forcing multi-precision in the reduction program for reaching a 10^{-15} relative precision.

As a second test on our calculation, we check the cancellation of the poles. In fact, after including the ultraviolet counterterm $\mathcal{UV}_{\mathrm{ct}}$ and the infrared pole coming from real emission, all poles cancel as they should.

Finally, as described above, we have performed the calculation of all contributions by means of two independent codes.

As a further test of the stability of our results, we calculated the amplitudes for phase-space configurations in which the electron, or the positron, is (almost) parallel to the emitted photon [23]: while we observe the appearence of collinear divergencies, the virtual corrections still follow a smooth curve, thus suggesting a good stability of our results even in this particular kinematic configuration.

3. Conclusions

We recently evaluated the complete NLO virtual QED corrections to the hard bremsstrahlung processes $e^-e^+ \to e^-e^+\gamma$ and $e^-e^+ \to \mu^-\mu^+\gamma$, that are relevant for the determination of the luminosity at low-energy electron–positron colliders.

The result of our calculation, performed with the OPP method of reduction, is the implementation of hard-bremsstrahlung emission corrections, both for Bhabha scattering and muon-pair production, into a FORTRAN 95 code which employs the publicly available packages CutTools for the extraction of the coefficients of the scalar integrals and QCDLoop for evaluating the needed scalar integrals.

The numerical stability of our results has been tested in several ways: we compared our results in double and multiple precision; we checked the cancellation of infrared divergencies arising from virtual and real corrections; finally we compared the results obtained by using two independent codes. For LO calculations, we find an agreement of at least 12 digits; for NLO results, we estimate 9 digits of accuracy.

The typical order of magnitude of the CPU time of the FORTRAN 95 code we have developed is $\mathcal{O}(10^{-1})$ seconds for each phase-space point.

The numerical code that we developed, supplemented with the finite part for real-emission diagrams, will allow for a phenomenological study of the two hard-bremsstrahlung processes $e^-e^+ \to e^-e^+\gamma$ and $e^-e^+ \to \mu^-\mu^+\gamma$, including the calculation of the cross-sections and the relevant distributions. This work in currently in progress. In addition, our results can be compared with the corrections already implemented in MC generators in order to assess the associated theoretical uncertainty.

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