TARGET MASS CORRECTION FOR THE POLARIZED NUCLEON STRUCTURE FUNCTION*

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We use the phenomenological valon model to extract polarized parton densities and polarized proton structure function. Since the analytical result at large values of x and low values of Q^2 is not in good agreement with available experimental data, we employ target mass correction to increase the reliability of the calculations. New comparison confirms a better agreement with the experimental data.

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1. Introduction

Lepton scattering off nucleon targets plays an essential role in the investigation of the proton as a composite particle which consists of quarks and gluons. Experimental data which are obtained from deep inelastic scattering (DIS) experiments are used to extract parton distributions. By increasing the variety of data and also their precision, it is necessary to improve the theoretical analysis. This can be done by adding some corrections to the available analytical expressions. One of the most important effects is the target mass correction (TMC). This correction is written in terms of powers of M^2/Q^2. Here M is the mass of the target and Q is the energy which is allocated to an intermediate particle probing the nucleon. The TMC has an expansion in terms of the different order of twist operators. They arise in the calculation when partonic correlations and gluon contributions are important. The power corrections in M^2/Q^2 can be theoretically justified using the operator product expansion (OPE) theorem [1].

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2. Theoretical framework

The TMC shows up in the hadronic matrix element of a product $j_\mu(x)j_\nu(y)$ of two currents. Specifically we consider the amplitude of Compton scattering:

$$T_{\mu\nu} = i\int d^4x e^{iqx}\langle PS|T(j_\mu(x)j_\nu(0))|PS\rangle\,. \tag{1}$$

Calculating the $j_\mu(x)j_\nu(y)$ product at very short distances will lead to divergences. In order to avoid them we need to employ the OPE. The TMC is largest at low Q^2 and large x values, where there is no good agreement between the analytical result and the experimental data for the polarized proton structure function. In DIS, the Bjorken variable, x, is defined as:

$$x = \frac{Q^2}{2P\cdot q} = \left(\frac{Q^2}{2M\nu}\right)_{\rm rest}\,. \tag{2}$$

Here P and q are the four momenta of the target particle and the photon propagator and ν is the difference between the energy of incoming and outgoing particle. So far we have assumed the states in (1) to be massless quarks. In order to take into account the mass of the target particle we will use the Natchmann variable defined as:

$$\xi = \frac{2x}{1+\sqrt{1+\frac{4x^2M^2}{Q^2}}}\,, \tag{3}$$

instead of the Bjorken variable x. It can be easily seen that the Natchmann variable at large values of Q^2 is approaching x. The mass correction to the polarized structure function $g_1(x,Q^2)$ [2], represented in terms of moments of matrix elements, can be written as the second and third twist operators in the following way [3]:

$$\begin{aligned} g_1^{\rm TMC}\left(n,Q^2\right) &= a_n + \frac{M^2}{Q^2}\frac{n(n+1)}{(n+2)^2}(na_{n+2}+4d_{n+2}) \\ &+ \left(\frac{M^2}{Q^2}\right)^2\frac{n(n+1)(n+2)}{2(n+4)^2}(na_{n+4}+8d_{n+4}) \\ &+ \left(\frac{M^2}{Q^2}\right)^3\frac{n(n+1)(n+2)(n+3)}{6(n+6)^2}(na_{n+6}+12d_{n+6})\,, \end{aligned} \tag{4}$$

where we have kept terms up to the third order in the mass expansion. The a_n and d_n twist operators can been written as [4]:

$$a_n = \int_0^1 dx\frac{\xi^{n+1}}{x^2}\left\{\left[\frac{x}{\xi} - \frac{n^2}{(n+2)^2}\frac{M^2}{Q^2}x\xi\right]g_1\left(x,Q^2\right)\right.$$

$$-\frac{M^2}{Q^2}x^2\frac{4n}{n+2}g_2\left(x,Q^2\right)\Bigg\}, \qquad (n=3,5,\ldots), \tag{5}$$

$$\begin{aligned} d_n &= \int\limits_0^1 dx\frac{\xi^{n+1}}{x^2}\Bigg\{\frac{x}{\xi}g_1\left(x,Q^2\right) \\ &\quad + \left[\frac{n}{n-1}\frac{x^2}{\xi^2}-\frac{n}{n+1}\frac{M^2}{Q^2}x^2\right]g_2\left(x,Q^2\right)\Bigg\}, \qquad (n=3,5,\ldots). \end{aligned} \tag{6}$$

The structure function $g_2(x,Q^2)$ in the equations above can be calculated in terms of $g_1(x,Q^2)$ as follows:

$$g_2\left(x,Q^2\right) = -g_1\left(x,Q^2\right)+\int\limits_x^1 \frac{g_1\left(y,Q^2\right)}{y}dy\,. \tag{7}$$

3. Polarized structure function and TMC

To evaluate the TMC effect, we need first to calculate the polarized structure function $xg_1(x,Q^2)$. Within the $\overline{\mathrm{MS}}$ factorization scheme, the NLO contributions to $g_1(x,Q^2)$ are given by [5]

$$\begin{aligned} g_1^p\left(x,Q^2\right) &= \frac{1}{2}\sum_q e_q^2\Big\{\delta q\left(x,Q^2\right)+\delta\bar{q}\left(x,Q^2\right) \\ &\quad +\frac{\alpha_s\left(Q^2\right)}{2\pi}\left[\delta C_q\otimes(\delta q+\delta\bar{q})+2\delta C_g\otimes\delta g\right]\Big\}. \end{aligned} \tag{8}$$

The convolution symbol $\otimes$ is defined as

$$\left(C\otimes q\right)\left(x,Q^2\right) = \int\limits_x^1 \frac{dy}{y}C\left(\frac{x}{y}\right)q\left(y,Q^2\right)\,. \tag{9}$$

The polarized structure function can be related to the Mellin moments using the Jaccobi polynomials expansion [6]. In this case, the unknown parameters of the phenomenological valon model [7] can be extracted and the polarized proton structure function xg_1 is calculable. The plot of xg_1 in the NLO approximation at $Q^2=2$ GeV^2 has been depicted in Fig. 1. A comparison with experimental data [8] has also been done. Eq. (7) will give us the structure function $g_2(x,Q^2)$. Then the coefficients a_n and d_n can be calculated using Eqs (5)–(6). In our constituent quark model, $xg_1(x,Q^2)$ does not have a simple structure and we have not been able to calculate directly the integral $\int_0^1 \frac{g_1(y,Q^2)}{y}dy$. To calculate this integral, we assign a polynomial function

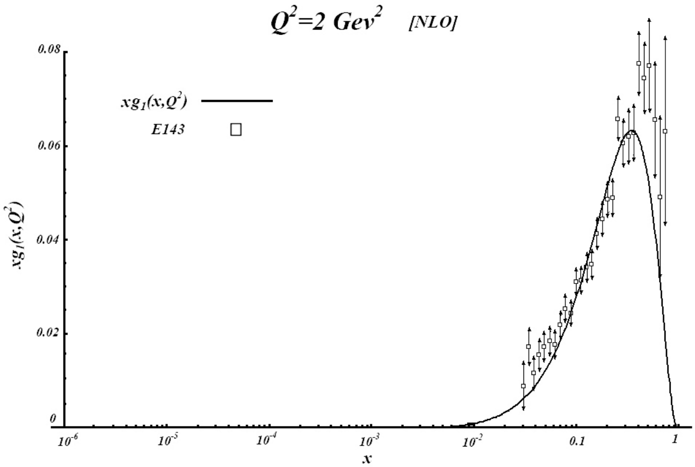

Fig. 1. The plot of xg_1^p at $Q^2 = 2$ GeV^2 in the NLO approximation. Comparison with the experimental data has also been done.

of y to it which contains some unknown parameters. The parameters will be obtained by fitting to the experimental data. To calculate the integral in Eq. (5) and Eq. (6) we use the numerical solution of Newton–Cotes [9]. In Fig. 2, including the TMC effects, we have plotted the structure function $xg_1(x, Q^2)$ at $Q^2 = 2$ GeV^2 in the NLO approximation. As it can be seen, employing TMC shifts the $xg_1(x, Q^2)$ plot upwards and we get better agreement with the experimental data.

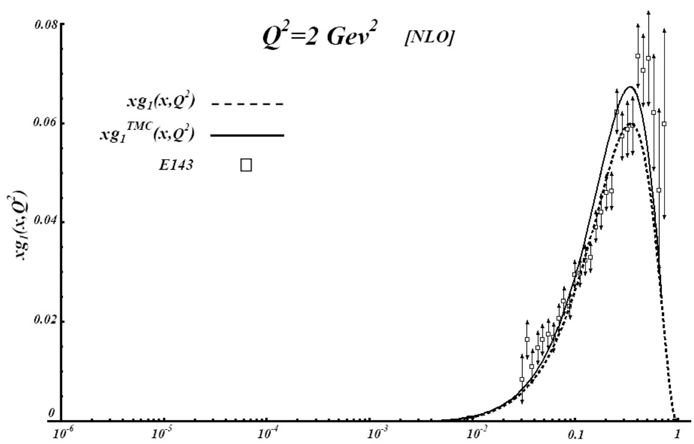

Fig. 2. Comparison between xg_1 and $xg_1^{\rm TMC}(x, Q^2)$ at $Q^2 = 2$ GeV^2 in the NLO approximation. Experimental data have been adopted from [8].

4. Conclusions

We have shown that the TMC effect is important at large values of x and low values of Q^2. But as it can be seen in Fig. 2, the area under the curve of the $xg_1^{\rm TMC}(x,Q^2)$ function (first moment of $g_1^{\rm TMC}(x,Q^2)$) is larger than the area which is related to $xg_1(x,Q^2)$. This will not satisfy the required constraints. To obey the constraints, we need to extract the unknown parameters from a structure function which involves the TMC effect. This means that we should fit $xg_1^{\rm TMC}(x,Q^2)$ to the experimental data rather than $xg_1(x,Q^2)$. Due to multiple integrals, the calculations will not be easy and we should find a numerical solution. In this regard some guide lines can be found in [10, 11]. A full calculation will be done as a new research job in future.

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