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DETERMINATION OF VALANCE QUARK DISTRIBUTIONS IN HIGHER ORDER OF PERTURBATIVE QCD*

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We perform N^{3}LO QCD analysis of the world data on the unpolarized DIS structure functions to determine α_s and valance quark distributions with fully correlated errors. Due to increasing the accuracy of DIS experiments and reduction the experimental errors on determining the strong coupling constant, the higher order analysis is required. The Padé approximations are used to estimate the 4-loop anomalous dimension. We consider the non-singlet sector of parton distributions in our calculations. A comparison of the N^{3}LO/N^{2}LO with N^{3}LO/NLO ratios for up and down valence quark distributions indicate outstanding influence of the higher order analysis.

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1. Introduction

The parton distribution functions (PDFs) are quite reliable tools to calculate the nucleon structure function. DIS processes have played and still play a very important role for our understanding of QCD and of nucleon structure. In fact, DIS structure functions have been the subject of detailed theoretical and experimental investigations. Today, with high-precision data from the electron–proton collider HERA and in view of the outstanding importance of hard scattering processes at proton–(anti)proton colliders like the TEVATRON and the forthcoming Large Hadron Collider (LHC) at

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CERN, a quantitative understanding of deep-inelastic processes is indispensable. To predict the rates of the various processes, a set of universal PDFs is required. On the other hand, all calculations of high energy processes with initial hadrons, whether within the standard model or exploring new physics, require PDFs as an essential input. These distribution functions can determine by QCD global fits to all the available DIS and related hard-scattering data. In this paper we use Jacobi polynomials expansion method which gives us a reliable and highly predictive approach to determine the non-singlet PDFs.

2. Higher order QCD corrections

The global parton analysis of deep inelastic scattering (DIS) and the related hard scattering data are generally performed at next-to-leading order (NLO) and next-to-next-to-leading order ($\mathrm{N^2LO}$) [1–3]. Generally speaking, leading order (LO) description is not satisfactory enough and because of that the NLO approximation is used which works quite better. The (massless) evolution of the PDFs and some important processes can now be calculated up to NNLO and some (few) calculations have been carried out even to higher orders. Recently, much effort has been invested in computing $\mathrm{N^3LO}$ QCD corrections to a wide variety of partonic processes and, therefore, we need to generate parton distributions also at $\mathrm{N^3LO}$ such that the theory can be applied in a consistent manner [4, 5]. Analyzing DIS at $\mathrm{N^3LO}$ is important as we may be able to investigate the hierarchy LO $\rightarrow$ NLO $\rightarrow$ $\mathrm{N^2LO}$ $\rightarrow$ $\mathrm{N^3LO}$ in the process in which there are perhaps the most precise data. Recently, there has been a significant attempt to enhance the precision of the structure function data from the different experiments which are used to generate an updated version of the PDFs.

To carry out the analogous analysis at $\mathrm{N^3LO}$ we need both the relevant anomalous dimension as well as the coefficient functions. However, the 3-loop deep inelastic coefficient functions are known [6], the anomalous dimensions are not yet available at 4-loop order. In spite of the unknown 4-loop anomalous dimension, one can obtain the non-singlet parton distributions and Λ_{QCD} by estimating the uncalculated fourth-order corrections to the non-singlet anomalous dimension. On the other hand, the 3-loop coefficient functions are known and it is now possible to distinguish the effect of the 4-loop anomalous dimension in our calculations if it is compared to the effect which is arising from the higher order of coefficient functions. In this case the 4-loop anomalous dimension may be obtained from the Padé approximations as mathematical tools which can be used to estimate perturbative coefficients in quantum field theory [7–10]. In this way and in Mellin-N space it is easy to obtain the following results for the 4-loop anomalous dimension [4]

$$
\begin{aligned}
\widetilde{\gamma}^{\,[1/1]}(N) &\equiv [1/1](N) = \frac{\gamma^{(2)^2}(N)}{\gamma^{(1)}(N)}\,, \\
\widetilde{\gamma}^{\,[0/2]}(N) &\equiv [0/2](N) = \frac{2\gamma^{(1)}(N)\gamma^{(2)}(N)}{\gamma^{(0)}(N)} - \frac{\gamma^{(1)^3}(N)}{\gamma^{(0)^2}(N)}\,. \qquad (1)
\end{aligned}
$$

Here, we use these anomalous dimensions to perform a $\mathrm{N^3LO}$ global parton analysis for more precise data sets for structure function measurements. As for the results reported in $\mathrm{N^2LO}$ [1], we use similar parameterization and data sets [1, 4, 5]. To extend our analysis to the $\mathrm{N^3LO}$ approximation, we use the 4-loop expression for α_{s} in the $\overline{\mathrm{MS}}$ scheme and employ the matching conditions when evolving through the heavy quark thresholds.

The energy dependence of the strong coupling constant is given by the renormalization group equation

$$
Q^2 \frac{\partial \alpha_{\mathrm{s}}\left(Q^2\right)}{\partial Q^2} = \beta\left(\alpha_{\mathrm{s}}\left(Q^2\right)\right)\,. \qquad (2)
$$

The perturbative expansion of the β function has been fully calculated to 4-loop approximation:

$$
\beta\left(\alpha_{\mathrm{s}}\left(Q^2\right)\right) = -\beta_0\alpha_{\mathrm{s}}^2\left(Q^2\right) - \beta_1\alpha_{\mathrm{s}}^3\left(Q^2\right) - \beta_2\alpha_{\mathrm{s}}^4\left(Q^2\right) - \beta_3\alpha_{\mathrm{s}}^5\left(Q^2\right) + \mathcal{O}\left(\alpha_{\mathrm{s}}^6\right)\,. \qquad (3)
$$

The full expression for the β_i coefficients can be found in [11–13]. In the 4-loop approximation the running coupling constant is given by

$$
\begin{aligned}
\alpha_{\mathrm{s}}\left(Q^2\right) = {} & \frac{1}{\beta_0 L} - \frac{1}{\beta_0^3 L^2}\beta_1 \ln L + \frac{1}{\beta_0^3 L^3}\left(\frac{\beta_1^2}{\beta_0^2}\left(\ln^2 L - \ln L - 1\right) + \frac{\beta_2}{\beta_0}\right) \\
& + \frac{1}{\beta_0^4 L^4}\left(\frac{\beta_1^3}{\beta_0^3}\left(-\ln^3 L + \frac{5}{2}\ln^2 L + 2\ln L - \frac{1}{2}\right)\right) - \frac{1}{\beta_0^4 L^4}\left(3\frac{\beta_1\beta_2}{\beta_0^2}\ln L + \frac{\beta_3}{2\beta_0}\right), \qquad (4)
\end{aligned}
$$

where $L = \ln\left(Q^2/\Lambda^2_{\overline{\mathrm{MS}}}\right)$.

3. Conclusion and results

We have carried out the $\mathrm{N^3LO}$ QCD analysis to obtain the PDFs and $\alpha_{\mathrm{s}}(M_Z^2)$ by including the available experimental data on DIS. In our higher order QCD analysis, the Padé approximations have been used to estimate the 4-loop anomalous dimension. By parameterizing the non-singlet PDFs at $Q_0^2 = 4\ \mathrm{GeV}^2$ and using the Padé [1/1] approximation, the following optimum global $\mathrm{N^3LO}$ fit has been obtained [4]

$$
\begin{aligned}
xu_v &= 5.095x^{0.791}(1-x)^{4.026}\left(1 + 0.094x^{0.5} + 1.110x\right)\,, \qquad (5) \\
xd_v &= 3.413x^{0.809}(1-x)^{3.768}\left(1 + 0.139x^{0.5} - 1.120x\right)\,. \qquad (6)
\end{aligned}
$$

We should notice here that there is a very good compatibility between Padé [1/1] and [0/2] approximations in 4-loop order. In Fig. 1 we plot the N^{3}LO/N^{2}LO and N^{3}LO/NLO ratios for up and down valence quark distributions, at $Q^2 = 10$ and 10^4 GeV^2. It is clear that the N^{3}LO/N^{2}LO ratios for the up and down valence quark distributions are closer to 1 in comparison with N^{3}LO/NLO ratio. Moreover, these ratios are closer to 1 at $Q^2 = 10^4$ GeV^2 than at $Q^2 = 10$ GeV^2. The `MATHEMATICA` code which contains our model (KKT09) to extract $xu_v(x, Q^2)$ and $xd_v(x, Q^2)$ in wide range of x and Q^2 up to 4-loop can be found in [14].

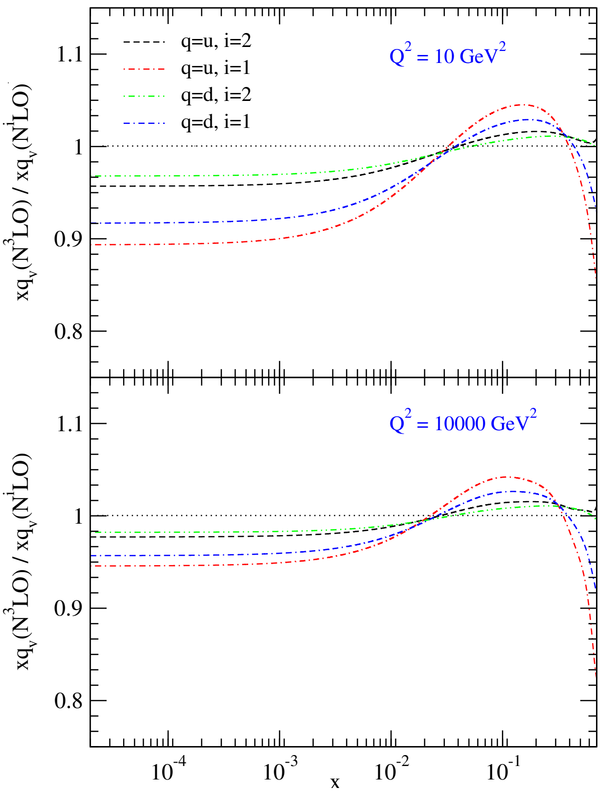

Fig. 1. A comparison of the up and down valence quark distributions obtained in the N^{3}LO with those obtained in the N^{2}LO and NLO QCD analysis. The comparison is shown for two values of Q^2.

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