NAÏVE SOLUTION OF THE LITTLE HIERARCHY PROBLEM AND ITS PHYSICAL CONSEQUENCES*

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We argue that adding gauge-singlet real scalars to the Standard Model can both ameliorate the little hierarchy problem and provide a realistic source of Dark Matter. Masses of the scalars should be in the 1–3 TeV range, while the lowest cutoff of the (unspecified) UV completion of the model must be $\gtrsim 5$ TeV, depending on the Higgs boson mass and the number of singlets present. The scalars couple to the Majorana mass term for right-handed neutrinos implying one massless neutrino. The resulting mixing angles are consistent with the tri-bimaximal mixing scenario.

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1. Introduction

Our intention is to construct economic extension of the Standard Model (SM) for which the little hierarchy problem is ameliorated while preserving all the successes of the SM. We will consider only those extensions that interact with the SM through renormalizable interactions. Since quadratic divergences of the Higgs boson mass are dominated by top-quark contributions, it is natural to consider extensions of the scalar sector, so that they can reduce the top contribution (as they enter with an opposite sign). The extensions we consider, although renormalizable, shall be treated as effective low-energy theories valid below a cutoff energy ~ 5–10 TeV; we will not discuss the UV completion of this model.

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2. The little hierarchy problem

The quadratically divergent 1-loop correction to the Higgs boson (h) mass was first calculated by Veltman [1]

$$\delta^{(\mathrm{SM})} m_h^2 = \left[3m_t^2/2 - \left(6m_W^2 + 3m_Z^2\right)/8 - 3m_h^2/8\right] \Lambda^2/\left(\pi^2 v^2\right) \,, \qquad (1)$$

where Λ is a UV cutoff, that we adopt as a regulator, and $v \simeq 246$ GeV denotes the vacuum expectation value of the scalar doublet (SM logarithmic corrections are small since we assume $v \ll \Lambda \lesssim 10$ TeV). The SM is considered here as an effective theory valid up to the physical cutoff Λ, the scale at which new physics enters.

Precision tests of the SM (mainly from the oblique T_{obl} parameter [2]) require a light Higgs boson, $m_h \sim 120$–170 GeV. The correction (1) can then exceed the mass itself even for small values of Λ, *e.g.* $\delta^{(\mathrm{SM})} m_h^2 \simeq m_h^2$ for $m_h = 130$ GeV already for $\Lambda \simeq 600$ GeV. That suggests extensions of the SM with a typical scale at 1 TeV, however no indication of such low energy new physics have been observed. This difficulty is known as the little hierarchy problem.

Here our modest goal is to construct a simple modification of the SM within which δm_h^2 (the total correction to the SM Higgs boson mass squared) is suppressed up to only $\Lambda \lesssim 3$–10 TeV. Since (1) is dominated by the fermionic (top quark) terms, the most economic way of achieving this is by introducing new scalars φ_i whose 1-loop contributions reduce the ones derived from the SM. In order to retain SM predictions we assume that φ_i are singlets under the SM gauge group. Then it is easy to observe that the theoretical expectations for all existing experimental tests remain unchanged if $\langle\varphi_i\rangle = 0$ (which we assume hereafter), in particular the SM expectation of a light Higgs is preserved.

The most general scalar potential implied by $Z_2^{(i)}$ independent symmetries $\varphi_i \to -\varphi_i$ (imposed in order to prevent $\varphi_i \to hh$ decays) reads:

$$\begin{aligned} V(H,\varphi_i) \;=\; & -\mu_H^2|H|^2 + \lambda_H|H|^4 + \sum_{i=1}^{N_\varphi} \left(\mu_\varphi^{(i)}\right)^2 \varphi_i^2 \\ & + \frac{1}{24} \sum_{i,j=1}^{N_\varphi} \lambda_\varphi^{(ij)} \varphi_i^2 \varphi_j^2 + |H|^2 \sum_{i=1}^{N_\varphi} \lambda_x^{(i)} \varphi_i^2 \,. \qquad (2) \end{aligned}$$

In the following numerical computations we assume for simplicity that $\mu_\varphi^{(i)} = \mu_\varphi$, $\lambda_\varphi^{(ij)} = \lambda_\varphi$ and $\lambda_x^{(i)} = \lambda_x$, in which case (2) has an $\mathcal{O}(\mathcal{N}_\varphi)$ symmetry (small deviations from this assumption do not change our results qualitatively). The minimum of V is at $\langle H\rangle = v/\sqrt{2}$ and $\langle\varphi_i\rangle = 0$ when $\mu_\varphi^2 > 0$ and

$\lambda_x, \lambda_H > 0$ which we now assume. The masses for the SM Higgs boson and the new scalar singlets are $m_h^2 = 2\mu_H^2$ and $m^2 = 2\mu_\varphi^2 + \lambda_x v^2$ ($\lambda_H v^2 = \mu_H^2$), respectively.

Positivity of the potential at large field strengths requires $\lambda_H \lambda_\varphi > 6\lambda_x^2$ at the tree level. The high energy unitarity (known [3] for $N_\varphi = 1$) implies $\lambda_H \leq 4\pi/3$ (the SM requirement) and $\lambda_\varphi \leq 8\pi$, $\lambda_x < 4\pi$. These conditions, however, are derived from the behavior of the theory at energies $E \gg m$, where we do not pretend our model to be valid, so that neither the stability limit nor the unitarity constraints are applicable within our pragmatic strategy, which aims at a modest increase of Λ to the 3–10 TeV range.

The existence of φ_i generates additional radiative corrections[1] to m_h^2. Then the extra contribution to m_h^2 reads

$$\delta^{(\varphi)} m_h^2 = -\left[N_\varphi \lambda_x / \left(8\pi^2\right)\right]\left[\Lambda^2 - m^2 \ln\left(1 + \Lambda^2/m^2\right)\right] . \tag{3}$$

Adopting the parameterization $|\delta m_h^2| = |\delta^{(\mathrm{SM})} m_h^2 + \delta^{(\varphi)} m_h^2| = D_t m_h^2$, we can determine the value of λ_x needed to suppress δm_h^2 to a desired level (D_t) as a function of m, for any choice of m_h and Λ; examples are plotted in Fig. 1 for $N_\varphi = 6$. It should be noted that (in contrast to SUSY) the logarithmic terms in (3) can be relevant in canceling large contributions to δm_h^2. It is important to note that the required value of λ_x decreases as the number of singlets N_φ grows. When $m \ll \Lambda$, the λ_x needed for the amelioration of the hierarchy problem is insensitive to m, D_t or Λ; as illustrated in Fig. 1; analytically we find up to terms $\mathcal{O}\left(m^4/\Lambda^4\right)$

$$\begin{aligned}\lambda_x \;\simeq\; & N_\varphi^{-1}\left\{4.8 - 3(m_h/v)^2 + 2D_t[2\pi/(\Lambda/\ \mathrm{TeV})]^2\right\} \\ & \times\left[1 - m^2/\Lambda^2 \ln\left(m^2/\Lambda^2\right)\right] .\end{aligned} \tag{4}$$

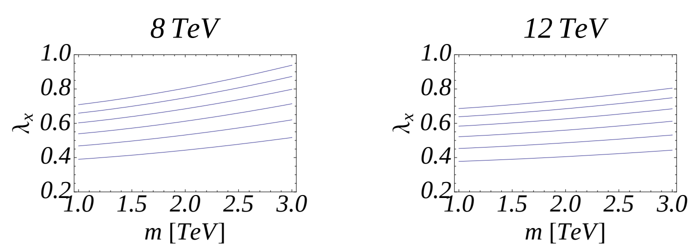

Fig. 1. Plot of λ_x corresponding to $D_t = 0$ and $N_\varphi = 6$ as a function of m for $\Lambda = 8$ TeV and 12 TeV (as indicated above each panel). The various curves correspond to $m_h = 130$, 150, 170, 190, 210, 230 GeV (starting with the uppermost curve).

[1] The Λ^2 corrections to m^2 can also be tamed within the full model with additional fine tuning, but we will not consider them here, see [4].

Since we consider $\lambda_x \sim \mathcal{O}(1)$ effects of higher order corrections [5] to (1) should be considered as well (see also [6]). In general, the fine tunning condition reads (m_h was chosen as a renormalization scale):

$$\left|\delta^{(\mathrm{SM})} m_h^2 + \delta^{(\varphi)} m_h^2 + \Lambda^2 \sum_{n=1} f_n(\lambda_x, \ldots) \left[\ln(\Lambda/m_h)\right]^n\right| = D_t m_h^2 \,, \tag{5}$$

where the coefficients $f_n(\lambda_x, \ldots)$ can be determined recursively [5], with the leading contributions being generated by loops containing powers of λ_x: $f_n(\lambda_x, \ldots) \sim [\lambda_x/(16\pi^2)]^{n+1}$. To estimate these effects we can consider the case where $\delta^{(\mathrm{SM})} m_h^2 + \delta^{(\varphi)} m_h^2 = 0$ at one loop then, keeping only terms $\propto \lambda_x^2$, we find (using [5]), at 2 loops, $D_t \simeq (\Lambda/(4\pi^2 m_h))^2 \; \ln(\Lambda/m_h)$ (note that $N_\varphi \lambda_x \simeq 4$). Requiring $D_t \lesssim 1$ implies $\Lambda \lesssim 3$–5 TeV for $m_h = 130$–230 GeV.

It must be emphasized that in the model proposed here the hierarchy problem is softened (by lifting the cutoff) only if λ_x, Λ and m are appropriately fine-tuned; this fine tuning, however, is significantly less dramatic than in the SM. In order to illustrate the necessary amount of tunning, it is useful to calculate the Barbieri–Giudice [7] parameter

$$\Delta_{\lambda_x} \equiv \left(\lambda_x/m_h^2\right)\left(\partial m_h^2/\partial \lambda_x\right) = \left|\delta^{(\varphi)} m_h^2\right| / m_h^2 \,. \tag{6}$$

It turns out that the minimal value of Δ_{λ_x} obtained while scanning over λ_x, Λ and m ($0.2 \leq \lambda_x \leq 6$, 1 TeV $\leq m \leq$ 10 TeV and 10 TeV $\leq \Lambda \leq$ 20 TeV) is substantial: $\Delta_{\lambda_x} \gtrsim 200$.

3. Dark Matter

The singlets φ_i also offer a natural source for Dark Matter (DM) (for $N_\varphi = 1$, see [8]). Using standard techniques for cold DM [9] we estimate its present abundance Ω_{DM}, assuming for simplicity that all the φ_i are equally abundant (*e.g.* as in the $O(N_\varphi)$ limit). Ω_{DM} is determined by the thermally averaged cross-section for φ_i annihilation into SM final states $\varphi_i\varphi_i \to$ SM SM, which in the non-relativistic approximation, and for $m \gg m_h$, reads

$$\langle \sigma_i v \rangle \simeq \lambda_x^2/(8\pi m^2) + \lambda_x^2 v^2 \Gamma_h(2m)/\left(8m^5\right) \simeq \left[1.73/\left(8\pi\right)\right] \lambda_x^2/m^2 \,. \tag{7}$$

The first contribution in (7) originates from the hh final state (keeping only the s-channel Higgs exchange; the t and u channels can be neglected since $m \gg m_h$) while the second one comes from all other final states; $\Gamma_h(2m) \simeq 0.48$ TeV $(2m/1 \text{ TeV})^3$ is the Higgs boson width calculated for its mass equal $2m$.

From this the freeze-out temperature $x_{\mathrm{f}} = m/T_{\mathrm{f}}$ is given by

$$x_{\mathrm{f}} = \ln\left[0.038\, m_{\mathrm{Pl}}\, m \,\langle \sigma_i v \rangle / (g_\star x_{\mathrm{f}})^{1/2}\right] \,, \tag{8}$$

where $g_\star$ is the number of relativistic degrees of freedom at annihilation and $m_{\rm Pl}$ denotes the Planck mass. In the range of parameters relevant for our purposes, $x_{\rm f} \sim 12$–50 and $m \sim 1$–2 TeV, so that this is indeed a case of cold DM. Then the present density of φ_i is given by

$$\Omega_\varphi^{(i)} h^2 = 1.06 \times 10^9 x_{\rm f} / \left(g_\star^{1/2} m_{\rm Pl} \langle \sigma_i v \rangle \ {\rm GeV} \right) . \tag{9}$$

The condition that the φ_i's account for the observed DM abundance, $\Omega_{\rm DM} h^2 = \sum_{i=1}^{N_\varphi} \Omega_\varphi^{(i)} h^2 = 0.106 \pm 0.008$ [2], can be used to fix $\langle \sigma_i v \rangle$, which implies a relation $\lambda_x = \lambda_x(m)$ through (7). Using this in the condition $|\delta m_h^2| = D_t m_h^2$, we find a relation between m and Λ (for a given D_t), which is plotted in Fig. 2 for $N_\varphi = 6$. It should be emphasized that it turns out to be possible to find Λ, λ_x and m such that *both* the hierarchy is ameliorated to the desired level *and* such that $\Omega_\varphi h^2$ agrees with the DM requirement (we use a 3σ interval). It is also instructive to mention that the singlet mass (as required by the DM) scales with their multiplicity as $N_\varphi^{-3/2}$, therefore growing N_φ implies smaller scalar mass, *e.g.* changing N_φ from 1 to 6 leads to the reduction of mass by a factor ~ 15.

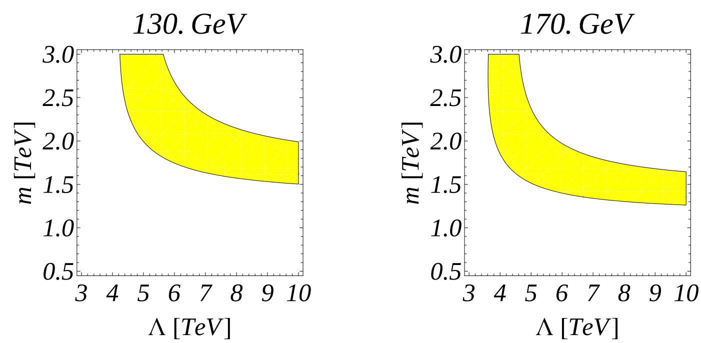

Fig. 2. The allowed region in the (m, Λ) plane for $D_t = 0$, $N_\varphi = 6$ and $\sum_{i=1}^{N_\varphi} \Omega_\varphi^{(i)} h^2 = 0.106 \pm 0.008$ at the 3σ level for $m_h = 130$ GeV and 170 GeV (as indicated above each panel).

4. Neutrino masses and mixing angles

We now consider implications of the existence of φ_i for the leptonic sector, which we assume consists of the SM fields and three right-handed neutrino fields ν_{iR} $(i = 1, 2, 3)$ that are also gauge singlets. For simplicity here we limit ourself to the case of only one singlet. The relevant Lagrangian is then

$$\mathcal{L}_Y = -\bar{L} Y_l H l_R - \bar{L} Y_\nu \tilde{H} \nu_R - \tfrac{1}{2} \overline{(\nu_R)^c} M \nu_R - \varphi \overline{(\nu_R)^c} Y_\varphi \nu_R + {\rm h.c.} , \tag{10}$$

where $L=(\nu_L, l_L)^T$ is a SM lepton SU(2) doublet and l_R a charged lepton singlet (we omit family indices); we will assume that the see-saw mechanism is responsible for smallness of three light neutrino masses, and therefore we require $M \gg M_D \equiv Y_\nu v/\sqrt{2}$. Since the symmetry of the potential under $\varphi \to -\varphi$ should be extended to (10) we require

$$L \to S_L L, \qquad l_R \to S_{l_R} l_R, \qquad \nu_R \to S_{\nu_R}\nu_R\,, \tag{11}$$

where the unitary matrices S_{L,l_R,ν_R} obey

$$S_L^\dagger Y_l S_{l_R} = Y_l\,, \quad S_L^\dagger Y_\nu S_{\nu_R} = Y_\nu\,, \quad S_{\nu_R}^T M S_{\nu_R} = +M\,, \quad S_{\nu_R}^T Y_\varphi S_{\nu_R} = -Y_\varphi\,. \tag{12}$$

In the following we will adopt a basis in which M and Y_l are real and diagonal; for simplicity we will also assume that M has no degenerate eigenvalues. Then the last two conditions in (12) imply that S_{ν_R} is real and diagonal (so ± 1). It is easy to see that for 3 neutrino species there are two possibilities (up to permutations of the basis vectors): we either have $S_{\nu_R}=\pm\mathbb{1},\ Y_\varphi=0$, or, more interestingly,

$$S_{\nu_R} = \epsilon\,\mathrm{diag}(1,1,-1)\,; \qquad Y_\varphi = \begin{pmatrix} 0 & 0 & b_1 \\ 0 & 0 & b_2 \\ b_1 & b_2 & 0 \end{pmatrix}\,, \qquad \epsilon=\pm 1\,, \tag{13}$$

where $b_{1,2}$ are, in general, complex. To satisfy the first conditions in (12) one needs $S_{l_R} = S_L$ with

$$S_L = \mathrm{diag}(s_1, s_2, s_3)\,, \qquad |s_i| = 1\,. \tag{14}$$

Diagonalizing (to leading order in M^{-1}) the neutrino mass matrix in terms of the light (n) and heavy (N) eigenstates leads to,

$$\begin{aligned} \mathcal{L}_m &= -(\bar n M_n n + \bar N M N/2) \quad \text{with} \\ M_n &= \mu^* P_R + \mu P_L,\ \mu = -4 M_D M^{-1} M_D^T\,, \end{aligned} \tag{15}$$

where n and N are related to ν_R and ν_L through $\nu_L = n_L + (M_D M^{-1}) N_L$ and $\nu_R = N_R - (M^{-1} M_D^T) n_R$.

The remaining condition in (12) leads to ten inequivalent solutions for Y_ν [2]. Of those, assuming no more than one massless neutrino and the absence of $\varphi \to n_i n_j$ decays, only one is interesting; it corresponds to $s_{1,2,3} = \epsilon$ (see (13)). Since $\det Y_\nu = 0$, the symmetry implies one massless neutrino.

To compare our results with experimental constraints on the leptonic mixing angles, we use the so-called tri-bimaximal [11] lepton mixing matrix where $\theta_{13}=0$, $\theta_{23}=\pi/4$ and $\theta_{12}=\arcsin(1/\sqrt{3})$. We find that the form of Y_ν consistent with this (up to axes permutations) is

[2] The conditions (12) where also discussed in [10].

$$Y_\nu = \begin{pmatrix} a & b & 0 \\ -a/2 & b & 0 \\ -a/2 & b & 0 \end{pmatrix}, \qquad \begin{array}{l} m_1 = -3v^2a^2/M_1 \\ m_2 = -6v^2b^2/M_2 \\ m_3 = 0 \end{array}, \tag{16}$$

where a and b are real (for simplicity) parameters. The resulting mass spectrum agrees with the observed pattern of neutrino mass differences, see *e.g.* [12]. If φ is a candidate for DM we should guarantee its stability. For the solution (13) only N_3 and φ are odd under the Z_2 symmetry hence the φ will be absolutely stable if $m < M_3$.

It is worth noticing that in the presence of Y_φ there exist three sources of 1-loop contributions to the quadratic divergence in corrections to the φ mass: (1) those generated by $|H|^2 \sum_{i=1}^{N_\varphi} \lambda_x^{(i)} \varphi_i^2$, (2) those from the quartic φ coupling and (3) the additional one from the Yukawa coupling $\varphi\overline{(\nu_R)^c} Y_\varphi \nu_R$. The presence of ν_R can be used [4] to ameliorate the little hierarchy problem associated with m thereby "closing" the solution to the little hierarchy problem in a spirit analogous to supersymmetry.

5. Conclusions

We have shown that the addition of real scalar singlets φ_i to the SM may soften the little hierarchy problem (by lifting the cutoff Λ to multi TeV range). This scenario also offers realistic candidates for DM. In the presence of right-handed neutrinos this model allows for a light neutrino mass matrix texture that is consistent with experimental data, in which case there should be one massless neutrino.

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