

MÖSSBAUER ANTINEUTRINOS: SOME BASIC CONSIDERATIONS*

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Basic aspects of phononless resonant capture of monoenergetic electron antineutrinos (Mössbauer antineutrinos) emitted in bound-state β -decay in the ${}^3\text{H}$ – ${}^3\text{He}$ system are considered. It is shown that stochastic magnetic relaxation phenomena as well as the direct influence of solid-state effects on the energy of the $\bar{\nu}_e$ will cause line broadening by a factor of more than 10^{13} . Lattice expansion and contraction after the transformation of the nucleus will drastically reduce the probability for phononless transitions. Thus, the observation of Mössbauer $\bar{\nu}_e$ of the ${}^3\text{H}$ – ${}^3\text{He}$ system will most probably be unsuccessful. As a possible alternative, the rare-earth system ${}^{163}\text{Ho}$ – ${}^{163}\text{Dy}$ is briefly discussed.

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1. Introduction

After the discovery of the Mössbauer effect with γ -transitions in nuclei [1], the conditions for observing Mössbauer antineutrinos, *i.e.*, the recoilfree resonant emission and absorption of electron antineutrinos ($\bar{\nu}_e$), have been discussed in various publications [2–6]. The basic concept is to use 18.6 keV $\bar{\nu}_e$ that are emitted in the bound-state β -decay of ${}^3\text{H}$ to ${}^3\text{He}$ [7, 8] and are resonantly captured in the reverse bound-state process in which ${}^3\text{He}$ is transformed to ${}^3\text{H}$ [3–5, 9]. Since bound-state β -decay is a two-body process, the emitted $\bar{\nu}_e$ has a fixed energy $E_{\bar{\nu}_e} = Q + B_z - E_R$. The $\bar{\nu}_e$ -energy is determined by the Q value, the binding energy B_z of the atomic orbit the electron is emitted into, and by the recoil energy E_R of the atom formed after the decay. Also the reverse reaction is a two-body process. The required $\bar{\nu}_e$ -energy is given by $E'_{\bar{\nu}_e} = Q + B_z + E'_R$, where E'_R is the recoil energy of the atom after the transformation of the nucleus. To prevent the recoil, ${}^3\text{H}$ as well as ${}^3\text{He}$ are considered to be imbedded in Nb metal lattices.

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The absorption cross-section σ_R at resonance for Mössbauer antineutrinos can be written as [1,3]

$$\sigma_R = 2\pi \left(\frac{\lambda}{2\pi} \right)^2 s^2 (\alpha^2 f^2 \delta), \quad (1)$$

where λ is the wavelength of the antineutrinos, s takes into account statistical factors (nuclear spins, isotopic abundance, *etc.*) and is considered to be of the order of unity, α is the ratio of bound-state to continuum-state β decays ($\alpha \approx 0.005$ for the ${}^3\text{H}$ - ${}^3\text{He}$ system), f is the probability that no phonons are excited in the Nb lattice when the $\bar{\nu}_e$ is emitted or captured, and $\delta = \Gamma/\Gamma_{\text{exp}}$, with Γ the natural linewidth of the weak decay and Γ_{exp} the experimental width due to line broadening.

In the present paper, we will demonstrate that — in contradiction to a recent publication [10] — it will not be possible to observe $\Gamma = h/2\pi\tau = 1.17 \times 10^{-24}$ eV, $\tau = 17.81$ y being the lifetime of ${}^3\text{H}$. Magnetic relaxation phenomena in Nb metal as well as the *direct* influence of the variation of the binding energies (due to the random distributions of ${}^3\text{H}$ and ${}^3\text{He}$ in the Nb lattices) on the energy of the $\bar{\nu}_e$ will cause $\Gamma_{\text{exp}} \gg \Gamma$ by many orders of magnitude. In addition, the factor f^2 in Eq. (1) may be as small as $\sim 10^{-7}$ due to lattice expansion and contraction processes which — being not present in conventional Mössbauer spectroscopy with photons — have been overlooked until recently [9, 11]. With the ideas and techniques available at present, the observation of $\bar{\nu}_e$ will most probably be unsuccessful. We briefly consider the rare-earth system ${}^{163}\text{Ho}$ - ${}^{163}\text{Dy}$ as an alternative.

2. Phononless transitions

Concerning $\bar{\nu}_e$, phononless transitions are jeopardized by two kinds of lattice-excitation processes:

1. momentum transfer due to emission/capture of a $\bar{\nu}_e$,
2. lattice expansion and contraction when the nuclear transformation occurs during which the $\bar{\nu}_e$ is emitted or absorbed. This possibility is not present in usual Mössbauer spectroscopy with photons [9, 11].

Considering first the momentum transfer, in the Debye-model and in the limit of very low temperatures T , the recoilfree fraction f_r is given by

$$f_r(T \rightarrow 0) = \exp \left\{ -\frac{E^2}{2Mc^2} \frac{3}{2k_B\theta} \right\}, \quad (2)$$

where θ is the effective Debye temperature, k_B is the Boltzmann constant, and $\frac{E^2}{2Mc^2}$ is the recoil energy which would be transmitted to a free atom of mass M . With $\theta \approx 800$ K [9], $f_r^2 \approx 0.07$ for $T \rightarrow 0$.

Considering process 2, the $\bar{\nu}_e$ itself takes part in the nuclear processes transforming one chemical element into a different one. ^3H and ^3He are differently bound in the Nb lattice and use different amounts of lattice space. Thus, when the nuclear processes occur, the lattice will expand or contract. The lattice deformation energies for ^3H and ^3He in the Nb lattice are $E_{\text{L}}^{^3\text{H}} = 0.099$ eV and $E_{\text{L}}^{^3\text{He}} = 0.551$ eV, respectively [12]. Assuming again an effective Debye temperature of $\theta \approx 800$ K one can estimate — in analogy to the situation with momentum transfer — that the probability f_{L} that this lattice deformation will *not* cause lattice excitations is smaller than $f_{\text{L}} \approx \exp \left\{ -\frac{E_{\text{L}}^{^3\text{He}} - E_{\text{L}}^{^3\text{H}}}{k_{\text{B}}\theta} \right\} \approx 1 \times 10^{-3}$. Thus, the total probability for phononless emission and consecutive phononless capture of $\bar{\nu}_e$ is $f^2 = f_{\text{r}}^2 \cdot f_{\text{L}}^2 \approx 7 \times 10^{-8}$, a very tiny probability indeed.

3. Linewidth

Basically, there are three important types of line-broadening effects [13–15], which prevent the observation of the natural width of $\bar{\nu}_e$ in the ^3H – ^3He system.

3.1. Homogeneous broadening

Homogeneous broadening is caused by electromagnetic relaxation, *e.g.*, by spin–spin interactions between the nuclear spins of ^3H and ^3He and with the spins of the Nb nuclei. Contrary to the claim in Ref. [10], such magnetic relaxations are stochastic processes and *cannot* be described by a periodic energy modulation of an excited hyperfine state [9,11]. Stochastic processes lead to sudden, irregular transitions between hyperfine-split states originating from many magnetic spins. Thus the wave function of the $\bar{\nu}_e$ is determined by random phase changes which lead to line broadening characterized by the time–energy uncertainty relation [9]. As a consequence, the broadened lines *cannot* be decomposed into multiple sharp lines. For the system ^3H – ^3He in Nb metal, $\Gamma_{\text{exp}} \approx 4 \times 10^{13} \Gamma$, thus $\delta_{\text{H}} \approx 2.5 \times 10^{-14}$ [9,11].

3.2. Inhomogeneous broadening

This type of broadening is caused by stationary effects, *e.g.*, by lattice defects, impurities and, in particular, by the random distributions of ^3H and ^3He on the tetrahedral interstitial sites in Nb metal. Thus the periodicity of the lattice is destroyed and the binding energies E_{B} of ^3H and ^3He will vary within the Nb lattice. Typical values for E_{B} are in the eV range [12]. In conventional Mössbauer spectroscopy with photons, using the best single crystals, such variations in E_{B} cause energy shifts of $\approx 10^{-12}$ eV. Since, in the nuclear transformations, the $\bar{\nu}_e$ energy is *directly* affected by E_{B} , one has

to expect that the variations of the $\bar{\nu}_e$ energy are much larger than 10^{-12} eV, probably in the 10^{-6} eV regime. Thus, inhomogeneous line broadening is estimated to give $\delta_I \ll 10^{-12}$ [9, 11], probably $\delta_I \approx 10^{-18}$.

3.3. Relativistic effects

Another contribution to line broadening in an imperfect lattice is due to relativistic effects. An atom vibrating around its equilibrium position in a lattice exhibits a mean-square velocity $\langle v^2 \rangle$. According to Special Relativity Theory this causes a time-dilatation, *i.e.*, a reduction in frequency (energy E) of the $\bar{\nu}_e$: $\Delta E = -v^2 E / (2c^2)$. Being proportional to $(v/c)^2$, this reduction is often called Second-Order Doppler shift (SOD).

Within the Debye model, the SOD between source (s) at temperature T_s and target (t) at temperature T_t is given by

$$(\Delta E/E) = \frac{9k_B}{16Mc^2}(\theta_s - \theta_t) + \frac{3k_B}{2Mc^2} [T_s f(T_s/\theta_s) - T_t f(T_t/\theta_t)] , \quad (3)$$

with

$$f(T/\theta) = 3 \left(\frac{T}{\theta} \right)^3 \int_0^{\theta/T} \frac{x^3}{\exp(x) - 1} dx . \quad (4)$$

At low temperatures (*e.g.*, in a liquid-He bath at 4.2 K), the temperature-dependent term in Eq. (3) can usually be neglected if source and target are at about the same temperature. However, even in the low-temperature limit, the first term in Eq. (3) which is caused by the zero-point motion (zero-point energy) cannot be neglected. It is *not* required that the chemical bonds (*i.e.* the Debye temperatures) of ^3H and ^3He have to be the same in the metal matrix. However, the chemical bond for ^3H (and also for ^3He) has to be the same in source and target [5]. The critical point is, that a variation of the binding energies E_B of ^3H and ^3He in an imperfect lattice will result in a variation of the effective Debye temperatures and thus also in a variation of the zero-point energies. If the effective Debye temperature varies by only 1 K, $(\Delta E/E) \approx 2 \times 10^{-14}$, corresponding to a lineshift of 3×10^{14} times the natural width Γ . Thus, this broadening effect is expected to give $\delta_{\text{SOD}} \approx 3 \times 10^{-15}$ comparable to or even smaller than δ_H [9].

With $\lambda \approx 0.67 \times 10^{-8}$ cm for the wavelength of the $\bar{\nu}_e$, $\alpha^2 \approx 2.5 \times 10^{-5}$, $f^2 \approx 7 \times 10^{-8}$ and $\delta_H \leq 2.5 \times 10^{-14}$, we get from Eq. (1) an optimistic upper limit of

$$\sigma_R \leq 3 \times 10^{-43} \text{ cm}^2 . \quad (5)$$

Thus, in particular the small values for δ (various types of broadening) make it impossible to reach the natural width and together with the small value for f (probability for phononless transitions) it appears that

Mössbauer $\bar{\nu}_e$ cannot be observed with the ^3H – ^3He system. We have reached this conclusion by only taking into account some basic principles. Technological difficulties, which are expected to be enormous too [16], have not been considered in the present paper.

4. Rare-earth systems — an alternative?

In Ref. [3], the system ^{163}Ho – ^{163}Dy was estimated to be the next-best case. Compared to ^3H – ^3He , there are three major advantages:

1. The Q value of 2.6 keV (thus the $\bar{\nu}_e$ energy) is very low, the mass of the nuclei is large. As a consequence, the recoilfree fraction is expected to be ≈ 1 .
2. Due to the similar chemical behaviour of the rare earths, the lattice deformation energies of ^{163}Ho and ^{163}Dy can be expected to be similar and could lead to a probability of phononless transitions which is larger by up to seven orders of magnitude.
3. Due to the large mass, the relativistic effects mentioned in Section 3.3, will be smaller.

The main disadvantage will be the large magnetic moments due to the $4f$ electrons of the rare-earth atoms. Thus, line broadening effects due to magnetic relaxation phenomena will again be decisive for a successful observation of Mössbauer $\bar{\nu}_e$. Profound technological knowledge concerning the fabrication of high-purity materials and single crystals in large amounts will be required [3]. Further investigations are necessary.

5. Interesting experiments

Several basic questions and interesting experiments could be addressed if Mössbauer $\bar{\nu}_e$ could be observed [9].

1. Mössbauer $\bar{\nu}_e$ could lead to a better understanding of the true nature of neutrino oscillations [17–23]. Considering the evolution of the neutrino state in time only, neutrino oscillations are a non-stationary phenomenon and Mössbauer $\bar{\nu}_e$ would not oscillate because of their extremely narrow energy distribution. An evolution of the neutrino wave function in space and time, however, would make oscillations possible in both the non-stationary and also in the stationary case (Mössbauer $\bar{\nu}_e$).
2. If Mössbauer $\bar{\nu}_e$ oscillate, the low energy (18.6 keV) would allow us to use ultra-short base lines (≈ 10 m instead of 1500 m) to determine accurate oscillation parameters, *e.g.*, Θ_{13} , Δm_{12}^2 and Δm_{31}^2 [24].

The question of mass hierarchy could be settled: for normal (inverted) hierarchy, the phase of atmospheric-neutrino oscillations advances (is retarded) by $2\pi \sin^2 \Theta_{12}$ for every solar oscillation [25, 26].

3. Oscillating Mössbauer $\bar{\nu}_e$ could be used to search for the conversion $\bar{\nu}_e \rightarrow \nu_{\text{sterile}}$ [27]. For $\Delta m^2 \approx 1$ eV, the oscillation length would only be ~ 5 cm!
4. Gravitational redshift $\bar{\nu}_e$ measurements could only be performed if an experimental linewidth of $\Gamma_{\text{exp}} \approx 10^{-10}$ eV (or smaller) could be reached [9]. Unfortunately, our estimates in this paper show that with the ${}^3\text{H}$ – ${}^3\text{He}$ system, this is unrealistic.

6. Conclusions

Several basic aspects of the ${}^3\text{H}$ – ${}^3\text{He}$ system have been considered for a possible observation of Mössbauer $\bar{\nu}_e$. In contradiction to the claim of Ref. [10] it will not be possible to reach the natural linewidth because of homogeneous broadening — due to stochastic relaxation processes — and inhomogeneous broadening mainly due to the variations of binding energies and zero-point energies and their *direct* influence on the energy of the $\bar{\nu}_e$. In addition, the probability for phononless emission and detection will drastically be reduced due to lattice expansion and contraction after the transformation of the nucleus. The observation of Mössbauer $\bar{\nu}_e$ of the ${}^3\text{H}$ – ${}^3\text{He}$ system will most probably *not* be possible. The rare-earth system ${}^{163}\text{Ho}$ – ${}^{163}\text{Dy}$ offers several advantages, in particular a large probability of phononless emission and detection. However, magnetic relaxation processes and technological requirements still have to be investigated in detail to decide if the ${}^{163}\text{Ho}$ – ${}^{163}\text{Dy}$ system is a successful alternative. Interesting experiments could be performed if Mössbauer $\bar{\nu}_e$ were observed.

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