

## CAN WE DISTINGUISH DIRAC AND MAJORANA NEUTRINOS PRODUCED IN MUON DECAY?\*

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Neutrinos produced in the muon decay scatter on electrons in the near (without oscillation) and in the far detector (after oscillation) and the number of produced muons is observed. In the frame of the Standard Model the cross-section for muon production does not depend on the neutrino nature. The situation is different, if beyond the SM neutrino interactions are present. We use the Fermi contact model, where we allow only one additional coupling, the charged scalar right-handed coupling which appear in variety of models. No bounds on the new scalar coupling for Majorana neutrino are found. The cross-sections for muon production is different for Dirac and Majorana neutrinos giving in principle possibility to distinguish their nature. The differences between the Dirac and Majorana cross-sections appear in the near and in the far detector. The cross-section for both types of neutrinos is different even if neutrinos do not oscillate, but the difference is larger after oscillation, even in the vacuum.

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### 1. Introduction

Neutrino oscillations showed that neutrinos have non zero masses — then the next important question is their nature — are they Dirac or Majorana particles? [1]. In practice, only relativistic neutrinos are observed and, in the Standard Model (SM), it has been proved [2] that such neutrino are practically indistinguishable. The aim of this work is to give some hints for experimental determination of neutrinos nature if physics beyond the SM describes their interaction. We assume that the New Physics (NP) is given by the scalar right-handed chiral interaction. Such NP interaction exists in a number of models and has the greatest influence on different

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behaviour of Dirac and Majorana neutrinos. At first, we consider inverse muon decay process with neutrinos coming from a pion decay. From existing experimental data, bounds on parameters describing New Physics (NP) can be found. In the next section, beam of neutrinos coming from muon decays is used to produce muons after scattering with the electrons ( $\nu + e^- \rightarrow \nu + \mu^-$ ) in a near detector (without neutrino oscillation) and in a far detector (after neutrino oscillation). The number of produced muons is different for Dirac and Majorana neutrinos, giving, in principle, possibility to distinguish their nature. Finally, we make some conclusions.

## 2. Muon decay

We assume that the muon decay and inverse muon decay processes are described by contact interaction:

$$\mathcal{L} = \frac{4G_F}{\sqrt{2}} \sum_{\gamma=S,V,T} \sum_{\epsilon,\delta=R,L} g_{\epsilon\delta}^{\gamma} \langle \bar{e}_{\epsilon} | \Gamma^{\gamma} | \nu_e \rangle \langle \bar{\nu}_{\mu} | \Gamma_{\gamma} | \mu_{\delta} \rangle + \text{h.c.}, \quad (1)$$

which is most general form of Lorentz invariant four-fermion Lagrangian [3]. To describe production of the neutrino mass eigenstates we parameterize coupling constants in Eq. (1) as follows:

$$\mathcal{L} = \frac{4G_F}{\sqrt{2}} \sum_{\gamma=S,V,T} \sum_{\epsilon,\delta=R,L} \sum_{j,k} g_{\epsilon\delta}^{\gamma} U_{ej}^{\gamma\epsilon} \left( U_{\mu k}^{\gamma\delta} \right)^* \langle \bar{e}_{\epsilon} | \Gamma^{\gamma} | \nu_j \rangle \langle \bar{\nu}_k | \Gamma_{\gamma} | \mu_{\delta} \rangle + \text{h.c.}, \quad (2)$$

where  $i, j = 1, 2, 3$ . The matrices  $U^{\gamma\delta}$  may not be unitary. This type of parametrization cover many models, which are considered in literature as an extension of the SM *e.g.* Left-Right symmetric model [4], models with additional Higgs doublets and triplets or models with exotic fermions [6]. In further analysis, we assume that only the parameters  $g_{LL}^V$  and  $g_{LL}^S$  are non zero. There are several reasons why we want to keep just one NP parameter  $g_{LL}^S$  as non zero. At first there are plenty of such models, the experimental bound on the  $g_{LL}^S$  is still weak giving enough large space for the NP, the difference between Dirac and Majorana behaviour is visible and finally with only two non-vanishing parameters the whole analysis is simpler [5]. We denote  $U^{VL} = U$  and  $U^{SL} = V$ , and for simplicity we assume that these matrices are unitary, which is exactly the case if we compare three Dirac with three Majorana neutrinos. For some models, *e.g.* see-saw ones, where heavy Majorana neutrinos appear, both matrices are non-unitary.

### 2.1. Bounds on the parameters $g_{LL}^V$ and $g_{LL}^S$

To find the bounds on the parameters  $g_{LL}^V$  and  $g_{LL}^S$  we consider a beam of neutrinos produced in pion decay [7] which scatter on electrons and produce

muons. As the  $\pi \rightarrow e\nu$  decay channel is very small, we can assume that the beam of neutrinos is described by the following density matrix:

$$\varrho_\nu(\lambda, i; \xi, k) = \begin{pmatrix} p_{++} & 0 \\ 0 & 1 - p_{++} \end{pmatrix} U_{\mu i}^* U_{\mu k}, \quad (3)$$

where [8]

$$p_{++} < 0.002. \quad (4)$$

Then, for Dirac neutrinos, the cross-section for inverse muon decay process  $\nu_\mu + e^- \rightarrow \nu_e + \mu^-$  is the following:

$$\sigma = \frac{G_F^2}{\pi s} (s - m_\mu^2)^2 \left( |g_{LL}^V|^2 (1 - p_{++}) + \frac{2s + m_\mu^2}{24s} |g_{LL}^S|^2 \left| (VU^\dagger)_{\mu, \mu} \right|^2 p_{++} \right). \quad (5)$$

and in the case of Majorana neutrinos it reads:

$$\begin{aligned} \sigma = & \frac{G_F^2}{\pi s} (s - m_\mu^2)^2 \left( \left\{ |g_{LL}^V|^2 + g_{LL}^V g_{LL}^S \operatorname{Re} \left[ (UV^T)_{\mu, e} (UV^T)_{e, \mu}^* \right] \right. \right. \\ & + \frac{1}{4} |g_{LL}^S|^2 \left| (VU^T)_{e, \mu} \right|^2 \left. \right\} (1 - p_{++}) + \frac{2s + m_\mu^2}{6s} \left\{ |g_{LL}^V|^2 \left| (UU^T)_{e, \mu} \right|^2 \right. \\ & + g_{LL}^V g_{LL}^S \operatorname{Re} \left[ (UU^T)_{e, \mu}^* (UV^\dagger)_{\mu, \mu} (UV^T)_{\mu, e} \right] \\ & \left. \left. + \frac{1}{4} |g_{LL}^S|^2 \left| (VU^\dagger)_{\mu, \mu} \right|^2 \right\} p_{++} \right). \end{aligned} \quad (6)$$

The experimental value of the ratio  $S$  was measured [8]:

$$S = \frac{\sigma}{\left( \frac{G_F^2 (s - m_\mu^2)^2}{\pi s} \right)} = 0.958 \pm 0.054. \quad (7)$$

On the other hand, we have the constraint on the parameters which follows from normalization of the total muon decay width and definition of the Fermi constant. For Dirac neutrinos, it reads:

$$|g_{LL}^V|^2 + \frac{1}{4} |g_{LL}^S|^2 = 1. \quad (8)$$

The same condition for Majorana neutrinos is given by:

$$|g_{LL}^V|^2 + \operatorname{Re} \left[ g_{LL}^V g_{LL}^{S*} (UV^T)_{e\mu} (UV^T)_{\mu e}^* \right] + \frac{1}{4} |g_{LL}^S|^2 = 1. \quad (9)$$

In the case of Dirac neutrino, the bound does not depend strongly on matrix  $V$ , and we obtain  $g_{LL}^S < 0.62$ . The Majorana case is more complicated. Fig. 1 shows the allowed region for all possible values of matrix  $V$ .

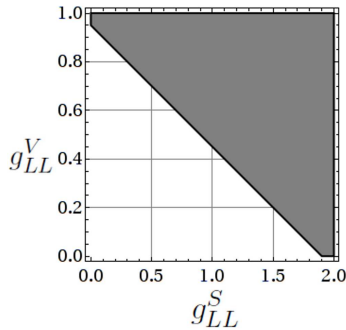


Fig. 1. Allowed region for parameters  $g_{LL}^S$  and  $g_{LL}^V$  for all possible values of matrix  $V$ .

### 3. Neutrinos from muon decay

To describe in a proper way the beam of neutrinos produced in the muon decay we need to construct a density matrix [9]. We do it in the following way:

$$\varrho^{\nu 1}(i, \lambda; i', \lambda') = \frac{1}{N} \int_{\Omega} d\Omega \sum_{k, \lambda_e, \lambda_{\nu_2}} \varrho_{i', k, \lambda', \lambda_e, \lambda_{\nu_2}}^{i, k, \lambda, \lambda_e, \lambda_{\nu_2}}, \quad (10)$$

where  $\Omega$  is the part of the phase space of all particles in the muon decay except for the considered neutrino and

$$\varrho_{i', k', \lambda', \lambda'_e, \lambda'_{\nu_2}}^{i, k, \lambda, \lambda_e, \lambda_{\nu_2}} = \frac{1}{N} \sum_{\lambda_{\mu} \lambda'_{\mu}} A_{i, k}(\lambda_{\mu}; \lambda_e \lambda, \lambda_{\nu_2}) A_{i', k'}^*(\lambda'_{\mu}; \lambda'_e, \lambda', \lambda'_{\nu_2}) \rho_{\lambda'_{\mu}, \lambda_{\mu}}. \quad (11)$$

Here  $A_{i, k}(\lambda_{\mu}; \lambda_e \lambda, \lambda_{\nu_2})$  is the amplitude for muon decay assuming that interactions are determined by the Lagrangian given by Eq. (2). The  $i, k$  ( $i', k'$ ) label the produced neutrinos mass states,  $\rho_{\lambda'_{\mu}, \lambda_{\mu}}$  is the density matrix for initial muon and  $\lambda$ 's denote helicities of particles. To describe the oscillation phenomenon in the vacuum we calculate the density matrix at the distance  $L = T$  from the production site, adapting the standard plain wave approximation [10]:

$$\varrho(L) = \exp(-iHL) \varrho \exp(iHL). \quad (12)$$

Finally, we calculate the detection cross-section using the same Lagrangian (2) as for the production process. We assume that the neutrinos are detected by the inverse muon decay. In the initial beam coming from  $\mu^-$  decay, there are two kinds of neutrinos: muon neutrinos and electron antineutrinos

(using the SM language) or neutrinos with two opposite helicities (using the Majorana language), so muons can be produced in two processes:

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e, \quad (13)$$

$$\bar{\nu}_e + e^- \rightarrow \mu^- + \bar{\nu}_\mu. \quad (14)$$

In each case the cross-section is given by:

$$\sigma_n = \int \sum_{\substack{\lambda_e, \lambda_l, \lambda_{\nu_l}, \\ \lambda_\nu, \lambda'_{\nu'}, i, i'}} \frac{1}{128\pi^2 s} \frac{p_f}{p_i} B_i^*(\lambda_\nu, \lambda_e; \lambda_l, \lambda_{\nu_l}) \varrho(L)_{i\lambda_\nu}^{i'\lambda'_{\nu'}} B_{i'}(\lambda'_{\nu'}, \lambda_e; \lambda_l, \lambda_{\nu_l}) d\Omega, \quad (15)$$

where  $B_{i'}(\lambda'_{\nu'}, \lambda_e; \lambda_l, \lambda_{\nu_l})$  is an amplitude for the suitable detection process. Calculating the total cross-section for the Majorana neutrinos, both reactions (13) and (14) are automatically included, but in the Dirac case we have to add cross-sections for these two reactions and normalize them correctly in order to compare the result with the Majorana case:

$$\sigma = \alpha\sigma_1 + (1 - \alpha)\sigma_2, \quad (16)$$

where  $\alpha$  is the percentage of suitable neutrinos in the beam,  $\sigma_1$  and  $\sigma_2$  are the cross-sections of (13) and (14), respectively. In Fig. 2, we have plotted the allowed regions for the Dirac and Majorana cross-sections as a functions of a parameter  $g_{LL}^S$ . We see that knowing the value of the cross-section of Eq. (16), we can distinguish Dirac and Majorana neutrinos only if we have independent information about the value of the scalar coupling constant from some other source. However, such situation is difficult to imagine, so

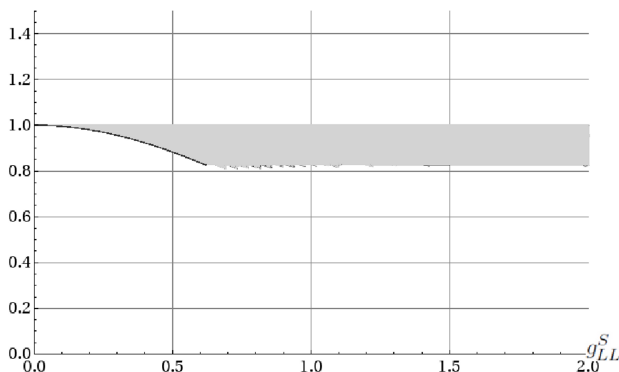


Fig. 2. Allowed values for the detection cross-section at  $L = 0$  and neutrino energy  $E = 12$  GeV. The dark grey line represents the Dirac cross-section, the light grey region represents the Majorana case.

in practice, distinguishing both neutrino nature in the near detector will be very difficult. Fig. 3 shows the cross-section at distance  $L = 10000$  km. We see now that Majorana neutrinos oscillate in a different way due to the interference terms in Majorana amplitudes that are not present in the Dirac case. Allowed region for Majorana neutrinos cross-section is bigger than that for the Dirac neutrinos, so it may be possible to distinguish the neutrino nature even if we do not know the exact value of the  $g_{LL}^S$ , we only need to know that it is non zero.

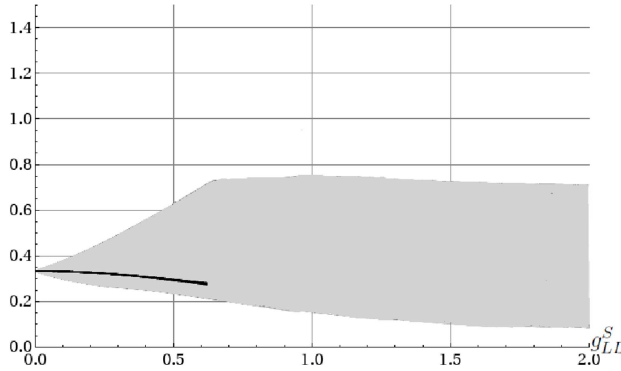


Fig. 3. Allowed values for the detection cross-section at  $L = 10000$  km and neutrino energy  $E = 12$  GeV. The dark grey line represents the Dirac cross-section, the light grey region represents possible value of the cross-section for the Majorana neutrinos.

#### 4. Conclusions

We have calculated the bounds on parameters  $g_{LL}^V$  and  $g_{LL}^S$  in the case of Dirac and Majorana neutrinos. We constructed a density matrix which contains all information about the neutrinos produced in muon decay, this matrix was used to calculate the detection cross-sections. We have shown that in the case of NP it may be possible to distinguish Dirac and Majorana neutrinos. To that end, if there is no neutrino oscillation in the near detector we need to know the parameter  $g_{LL}^S$  from independent source. In the far detector, after neutrino oscillation, there is a chance to find the neutrinos nature even if we do not know exact value of the NP scalar coupling.

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