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PROGRESS IN THE PREDICTION OF $g-2$ OF THE MUON*

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I review recent progress in the prediction of the muon $g-2$. The main issue are those contributions which cannot be calculated by perturbative means: the hadronic higher order effects which come into play at $O(\alpha^2)$ and the hadronic light–by–light scattering contribution at the order of $O(\alpha^3)$.

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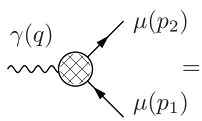
1. The anomalous magnetic moment of the muon: Status

Here I give a very short sketch of the status of the muon anomalous magnetic moment. For a much more detailed recent review and original references I refer to [1] (also see [2]). In theory, the anomalous magnetic moment a_μ is defined by the matrix element



$$= (-\mathrm{i}e)\, \bar{u}(p_2) \left[\gamma^\mu F_{\mathrm{E}}(q^2) + \mathrm{i} \frac{\sigma^{\mu\nu} q_\nu}{2m_\mu} F_{\mathrm{M}}(q^2) \right] u(p_1)\;,$$

$$F_{\mathrm{E}}(0) = 1\,; \qquad F_{\mathrm{M}}(0) = a_\mu\,.$$

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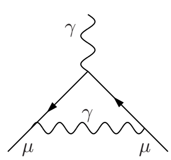
The non-zero a_μ is responsible for the Larmor precession, which, at the magic energy ~ 3.1 GeV, is directly proportional to the applied magnetic field $\vec{B}$:

$$\vec{\omega}_a = \frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\vec{\beta}\times\vec{E}\right]_{\text{at ``magic } \gamma\text{''}}^{E\sim 3.1 \text{ GeV}} \simeq \frac{e}{m}\left[a_\mu \vec{B}\right] .$$

A first muon $g-2$ experiment based on this principle was put into practice with a muon storage ring at CERN. More recently, a substantially improved experiment E821 was implemented at Brookhaven National Laboratory (BNL) which reached a 14-fold improvement with a precision of 0.54 ppm [3].

1.1. Standard Model prediction for a_μ

The dominating QED contribution to a_μ has been computed (or estimated) up to 5 loops. Growing coefficients in the α/π expansion reflect the presence of large $\ln \frac{m_\mu}{m_e} \simeq 5.3$ terms coming from electron loops. The QED contributions are of dramatically increasing complexity, starting with one leading order 1-loop diagram:



$$a_\mu^{(2)} = \frac{\alpha}{2\pi}$$

calculated first by Schwinger in 1948 and which yields the first 3 significant digits of the full result. Higher loop QED contributions are obtained by adding internal photon lines and lepton loops in internal photon lines in all possible ways. The 2-loop contribution is given by 7 diagrams (Peterman 1957, Sommerfield 1957). At 3 loops we have 72 universal (one species of leptons only) diagrams (Lautrup, Peterman, de Rafael 1974, Laporta, Remiddi 1996)[1]. At 4 loops the complexity is already overwhelming with about 1000 diagrams (Kinoshita 1999, Kinoshita, Nio 2004)[2]. For 5 loops only leading

[1] The complete 3-loop problem has been solved analytically(Remiddi *et al.*, Remiddi, Laporta 1996 (after a 27 years effort).

[2] The 4-loop contribution (Kinoshita *et al.*, Aoyama, Hayakawa, Kinoshita and Nio 2007) was evaluated mostly numerically in a 30 years heroic effort. The result for the $(\alpha/\pi)^4$ coefficient is given by $A_1^{(8)} = -1.9144(35)$ (error from Monte Carlo integration). A recent reevaluation led to a shift by 0.19 (10%), a 7σ shift in the prediction of a_e!. Note that the universal $O(\alpha^4)$ contribution to a_μ is sizable, a 6 standard deviation effect, given the current experimental accuracy. Therefore, the precise knowledge of this term is crucial for the comparison between theory and experiment.

terms have been estimated (Karshenboim 1993, Czarnecki, Marciano 2000, Kinoshita, Nio 2005). Results are given in the following table

TABLE I

Number of loops	C_i $[(\alpha/\pi)^n]$	$a_\mu^{\rm QED} \times 10^{11}$
1	+0 5	116140973.289 (43)
2	+0 765 857 410(26)	413217.620 (14)
3	+24 050 512 28(46)	30141.905 (1)
4	+130 8105(85)	380.807 (25)
5	+663 0(20.0)	4.483 (137)
Total		116584718.104 (0.147)

We thus obtain[3]

$$a_\mu^{\rm QED} = 116\,584\,718.104\ (0.043)\ (0.014)\ (0.025)\ (0.137) \times 10^{-11}\,,$$

with errors from $\alpha_{\rm inp}$, m_e/m_μ, α^4 numerics and missing α^5 terms. The current uncertainty is well below the 60×10^{-11} experimental error from E821 [3].

The weak contributions come from diagrams which exhibit internal W, Z and Higgs lines. The Higgs contribution is tiny! At 1-loop we have $a_\mu^{\rm weak(1)} = (194.82 \pm 0.02) \times 10^{-11}$ (Brodsky, Sullivan 1967, ..., Bardeen, Gastmans, Lautrup 1972). Kukhto *et al.* 1992 pointed out potentially large 2-loop terms $\propto G_{\rm F} m_\mu^2 \frac{\alpha}{\pi} \ln \frac{M_Z}{m_\mu}$, which however cancel by quark–lepton duality and the related absence of the triangle anomaly (Peris, Perrottet, de Rafael 1995, Czarnecki, Krause, Marciano 1996, Knecht, Peris, Perrottet, de Rafael 2002, Czarnecki, Marciano, Vainshtein 2002). The full 2-loop result is given by $a_\mu^{\rm weak(2)} = (-42.08 \pm 1.5\ [m_H, m_t] \pm 1.0\ [\rm had]) \times 10^{-11}$ (Heinemeyer, Stöckinger, Weiglein 2004, Gribouk, Czarnecki 2005). The most recent evaluation with improved hadronic part, beyond the quark parton model (QPM), reads [4]

$$a_\mu^{\rm weak} = (153.2 \pm 1.0\ [\rm had] \pm 1.5\ [m_H, m_t, 3\text{–loop}]) \times 10^{-11}\,.$$

1.2. Theory versus experiment: Do we see New Physics?

Table II summarizes the muon anomaly results from theory and experiment. Given are also the Hadronic Vacuum Polarization (HVP) and the hadronic light-by-light (LbL) contributions, which we discuss below. There

[3] We use $\alpha^{-1}(a_e) = 137.035999084(51)[0.37$ ppb], derived via the recent high precision measurement of the electron anomaly $a_e^{\rm exp} = 0.001\,159\,652\,180\,73(28)$ (Gabrielse *et al.* 2008) and work of Kinoshita *et al.* 2004/2007.

exists a 3.2σ deviation between theory and experiment. This discrepancy is not so easy to explain, because a_μ is most sensitive to nearby new physics, *i.e.*, relatively light new states which collider experiments have largely excluded already. The only exceptions are states predicted in SUSY models with R-parity conservation and/or in type II two Higgs doublet models. In these models also heavier states may produce substantial effects because they allow for a strongly enhanced muon Yukawa coupling ($\tan\beta$-enhancement), without spoiling the applicability of perturbation theory (see [1]).

TABLE II

Standard Model theory and experiment comparison [in units 10^{-10}].

Contribution		Value	Error	Reference
QED incl. 4-loops + LO 5-loops		11 658 471.81	0.02	[1]
Leading HVP		690.3	5.3	[5]
Subleading HVP		−10.0	0.2	[6]
Hadronic LbL		11.6	3.9	[1]
Weak incl. 2-loops		15.32	0.22	[4]
Theory		11 659 179.0	6.5	—
Experiment		11 659 208.0	6.4	[3]
Theor.–Exp.	$3.2\,\sigma$ deviation	−29.0	9.0	—

2. Improvements due to the new $\pi^+\pi^-$ data from BaBar

The biggest uncertainty in the prediction of a_μ comes from the hadronic contribution to the photon Vacuum Polarization (VP). The latter non-perturbative contribution is determined via a dispersion integral from the total cross-section of e^+e^--annihilation into hadrons and especially from pion pair production, which is dominating this contribution. Here, one of the very recent highlights is the determination of the cross-section of the reaction $e^+e^- \to \pi^+\pi^-$ from the ratio of the Initial State Radiation (ISR) cross-sections of $e^+e^- \to \pi^+\pi^-\gamma$ and $e^+e^- \to \mu^+\mu^-\gamma$ with the BaBar detector at the SLAC B factory [7,8]. The low energy $\pi^+\pi^-$ cross-section measurement via radiative return has been pioneered previously with the KLOE detector at the Φ factory DAΦNE [9]. Precision measurements based on ISR require a precise understanding of radiative corrections effects [10]. BaBar allows one to measure the spectrum in one experiment over a large energy range, from threshold up to about 2 GeV. Common ISR effects drop out from the ratio and assuming that other radiation effects, like Final State Radiation (FSR) and higher order effects, are sufficiently well understood, the resulting ratio $R(s)$ is obtained with unprecedented precision.

The integrated result from threshold to 1.8 GeV is $a_\mu^{\pi\pi(\gamma),\mathrm{LO}} \times 10^{10} = (514.1 \pm 2.2 \pm 3.1)$ [8], where the errors are statistical and systematic ones. This value is larger than that from a combination of previous e^+e^- data (503.5 ± 3.5), but it is in good agreement with the updated value from τ-decay (515.2 ± 3.4) [11, 12]. The τ-decay $\pi^0\pi^-$ isovector spectral-function, related to the $I = 1$ component of the e^+e^--annihilation $\pi^+\pi^-$ form-factor, has been measured with high statistics recently by the Belle Collaboration [13]. This new measurement after applying the appropriate isospin corrections, is in much better agreement with the e^+e^- data than previous ALEPH/OPAL data, but in fair agreement with CLEO. Also, additional isospin breaking corrections have been included in the new analysis by Davier *et al.* [11]. One new ingredient is a calculation of the photonic corrections in combination with an effective chiral resonance Lagrangian model [14], the other are mass and width corrections between the charged (τ) and the neutral (e^+e^-) ρ, which have been advocated some time ago in [5, 15]. Remarkably, the discrepancy in a_μ between τ-based and e^+e^--based evaluations decrease from 2.4σ previously to 1.5σ only. The now much better agreement is mainly due to the shift $\delta a_\mu = 10.6 \times 10^{-10}$ to higher values by the new BaBar $\pi\pi$ cross-section. But substantial differences in the distributions persist. Other isospin breaking aspects have been discussed also in [16, 17].

What also remains unclear is whether normalization uncertainties, in particular those of the τ spectral-functions, have been estimated properly in all cases. Possible problems with the normalization of the cross-sections have been suggested in a recent reanalysis within the effective resonance Lagrangian approach [18]. In $e^+e^- \to$ hadrons, a new much more precise measurement of $R(s)$ is also available for the three energy points 2.6, 3.07 and 3.65 GeV from BES II at 3.5% precision [19]. My estimate is

$$a_\mu^{\mathrm{had,LO}} = (690.3 \pm 5.3) \times 10^{-10}\,.$$

Davier *et al.* including the new BaBar data and using a more progressive error analysis obtain $a_\mu^{\mathrm{had,LO}} = (695.5 \pm 4.1) \times 10^{-10}$. For other results see Fig. 3 below. It should be noted that the consistency of different experiments is far from being satisfactory. The reduction of errors is usually not the result of additional data, but results from the art of handling errors and/or from replacing data by pQCD results extensively (so called "theory driven" analyses). Higher order effects have not changed and account for

$$a_\mu^{\mathrm{had,HO}} = (-10.0 \pm 0.2) \times 10^{-10}\,.$$

3. The hadronic light-by-light scattering contribution

Available hadronic LbL scattering results have been reviewed recently in [1, 20]. For a reevaluation based on earlier work see [21]. A new evaluation of the pseudoscalar contribution, the first relaxing from the pole approximation, is given in [22], within the large-N_c QCD approach. A first calculation of this contribution in AdS/QCD (in the pole approximation) may be found in [23].

The hadronic LbL scattering contribution is the biggest challenge for theory. What we know for sure is that the hadronic part of $\gamma\gamma \to \gamma\gamma$ is of fully non-perturbative nature: what is a quark loop in perturbative QCD, in reality is dominated by single pseudoscalar exchanges as shown in Fig. 1.

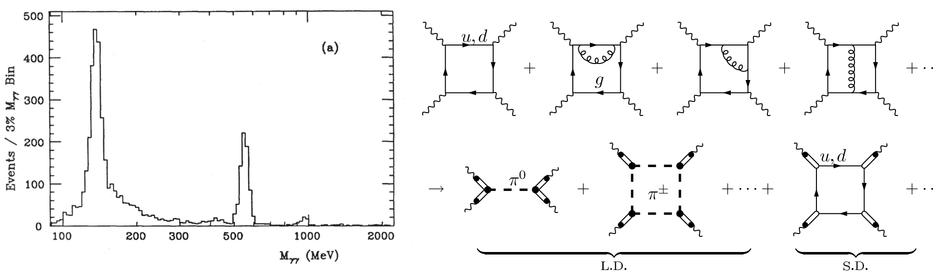


Fig. 1. The spectrum of invariant $\gamma\gamma$ masses obtained with the Crystal Ball detector. The three rather pronounced spikes seen are the $\gamma\gamma \to$ pseudoscalar (PS) $\to \gamma\gamma$ excitations: PS $= \pi^0, \eta, \eta'$. pQCD fails and must be replaced by the appropriate low energy effective expansion (right).

In fact, the leading contribution we are looking for, in low energy effective QCD is represented by diagrams which exhibit vertices with the full off-shell $\pi^0\gamma\gamma$ form-factor. What do we know about the latter?

- The general form-factor $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(s, s_1, s_2)$ is largely unknown. As a multi-scale quantity it is not of the simple type "low energy effective theory plus perturbative tail" like in the single scale HVP problem. Besides "all scales low" and "all scales high" we have mixed region where only asymptotic operator product expansion (OPE) arguments may help. In any case, the effective separation suggested in Fig. 2 oversimplifies the situation.

- The on-shell value $e^2\, \mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, 0, 0) = \frac{e^2 N_c}{12\pi^2 f_\pi} = \frac{\alpha}{\pi f_\pi} \approx 0.025~\mathrm{GeV}^{-1}$, a constant, is well determined by the $\pi^0 \to \gamma\gamma$ decay rate, which may be derived from the Wess–Zumino–Witten (WZW) effective Lagrangian.

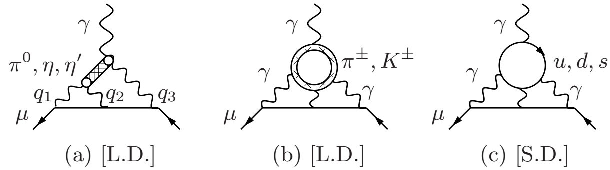


Fig. 2. Hadronic light-by-light scattering diagrams in a low energy effective model description. Diagrams (a) and (b) represent the long distance [L.D.] contributions at momenta $p \leq \Lambda$, diagram (c) involving a quark loop yield the leading short distance [S.D.] part at momenta $p \geq \Lambda$ with $\Lambda \sim 1$ to 2 GeV a ultra-violet cut-off.

- Direct measurements of $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, -Q^2, 0)$ are available from experiments with CELLO, CLEO and BaBar, which investigated the process $e^+e^- \to e^+e^-\pi^0$.

While the CELLO and CLEO measurements of the $\gamma^*\gamma\pi^0$ form factor $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, -Q^2, 0)$ at high space-like Q^2 are fairly well described by the Brodsky–Lepage interpolating formula

$$\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, -Q^2, 0) \simeq \frac{1}{4\pi^2 f_\pi} \frac{1}{1+(Q^2/8\pi^2 f_\pi^2)} \sim \frac{2f_\pi}{Q^2}\,,$$

or improvements of it (see [1]), the recent BaBar measurement [24] seems to exhibit a different high energy behavior, which may be fitted by a massive quark loop with effective mass about 130 MeV. The quark loop exhibits a square log enhancement $\propto \frac{m_q^2}{Q^2} \ln^2\left(Q^2/m_q^2\right)$ (see also [25]). Experimental clarification is urgently needed here.

Note that with the unambiguous effective WZW coupling taken constant at both vertices, we get an infinite result $a_\mu^{(6)}(\mathrm{LbL}, \pi^0) = \left[\frac{N_c^2}{48\pi^2}\frac{m_\mu^2}{F_\pi^2}\ln^2\frac{\Lambda}{m_\mu} + \ldots\right]$ $\times\ (\alpha/\pi)^3$ and thus we need a model for the off-shell form-factor. Of course, a Vector Meson Dominance (VMD) type damping renders the contribution finite. However, what is the precise model to be used is still controversial.

Elaborate evaluations have been performed by a number of groups: Hayakawa, Kinoshita, Sanda (HKS) 1995, Bijnens, Pallante, Prades (BPP) 1995, Hayakawa, Kinoshita (HK) 1998. All are based on the effective Lagrangian approach which implements VMD ideas in a way consistent with the low energy symmetries of QCQ, extending chiral perturbation theory to include the vector and axialvector resonances. Later Knecht, Nyffeler (KN) 2002 [26], and Melnikov, Vainshtein (MV) 2004 [27] adopted the large N_c QCD approach, which exploits quark–hadron duality in a way , which avoids the notorious problem of mismatch between low energy effective theory and the pQCD regime. All these analyses were based on the pole approximation, which later was criticized because of the kinematic inconsistency [2] (see also MV). In $g-2$ the external photon is at zero momentum. This implies that only $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2,-Q^2,0)$ and not $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0)$ is consistent with the $g-2$ kinematics. Unfortunately, the relevant off-shell form factor is not known and, in fact, not measurable and the CELLO/CLEO constraint does not apply! Melnikov, Vainshtein[4] stress that, in the chiral limit, the vertex with external photon must be non-dressed, *i.e.*, use $\mathcal{F}_{\pi^0\gamma^*\gamma}(0,0,0)$ to avoid a kinematic inconsistency. Because of the missing VDM damping the result increases by about 30%. As argued in [2], I think the chiral limit in this case is not a reasonable approximation. Any kind of VMD implementation in any case predicts a damping. However, one now is confronted with the problem that the unknown form-factor $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2,-Q^2,0)$ comes into play, with a far off-shell pion in the deep Euclidean region, *per se* not observable.

Finally, let us consider the evaluation of $a_\mu^{\rm LbL}$ in the large-N_c framework as pioneered by Knecht and Nyffeler [26] and later adopted by Melnikov and Vainshtein [27]. Both were using a large-N_c inspired $\pi^0\gamma^*\gamma^*$ form-factor in the pion–pole approximation. Relaxing from the latter, we are using an off-shell generalization of the KN LDM+V form-factor

$$\begin{aligned}
\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(p_\pi^2,q_1^2,q_2^2) &= \frac{F_\pi}{3}\,\frac{\mathcal{P}(q_1^2,q_2^2,p_\pi^2)}{\mathcal{Q}(q_1^2,q_2^2)}\,,\\
\mathcal{P}(q_1^2,q_2^2,p_\pi^2) &= h_7+h_6\,p_\pi^2+h_5\,(q_2^2+q_1^2)+h_4\,p_\pi^4+h_3\,(q_2^2+q_1^2)\,p_\pi^2\,,\\
&\quad +h_2\,q_1^2\,q_2^2+h_1\,(q_2^2+q_1^2)^2+q_1^2\,q_2^2\,(p_\pi^2+q_2^2+q_1^2))\\
\mathcal{Q}(q_1^2,q_2^2) &= (q_1^2-M_1^2)\,(q_1^2-M_2^2)\,(q_2^2-M_1^2)\,(q_2^2-M_2^2)\,,
\end{aligned}$$

where all constants are constrained by short distance expansions (OPE), except for $h_3+h_4=2\,c_{\rm VT}$ and h_6, which are new constants coming into play. The first $c_{\rm VT}=M_{V_1}^2\,M_{V_2}^2\,\chi/2$ is related to the magnetic susceptibility χ via the vector–tensor correlator $\Pi_{\rm VT}(0)=-(\langle\bar\psi\psi\rangle_0)/2\,\chi$. Evaluations of χ range from $\chi[\mathrm{GeV}^{-2}]\ =-2.7$ (Ball *et al.* 2003), -3.3 (LMD), -8.2 (Ioffe, Smilga 1984) to -8.9 (Vainshtein 2003). First off-shell calculations used

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[4] MV use the KN model (LMD+V form factor):

$$F_{\pi\gamma^*\gamma^*}(q_1^2,q_2^2)=\frac{4\pi^2F_\pi^2}{N_c}\;\frac{q_1^2q_2^2(q_1^2+q_2^2)-h_2q_1^2q_2^2+h_5(q_1^2+q_2^2)+(N_cM_1^4M_2^4/4\pi^2F_\pi^2)}{(q_1^2+M_1^2)(q_1^2+M_2^2)(q_2^2+M_1^2)(q_2^2+M_2^2)}\,,$$

where M_1 and M_2 are identified with the ρ and ρ' mass, respectively, and $h_5=6.93$ GeV^4, with two modifications: (1) $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(q_2^2,q_2^2,0)=1$, *i.e.*, a undressed soft photon (non-renormalization of ABJ anomaly) in the chiral limit, (2) $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(q_2^2,q_1^2,q_3^2)\simeq\mathcal{F}_{\pi^0\gamma^*\gamma^*}(m_\pi^2,q_1^2,q_3^2)=$ KN with $h_2=0\pm20$ GeV^2 (KN) *versus.* $h_2=-10$ GeV^2 (MV) fixed by twist 4 in OPE ($1/q^4$), and (3) $a_1[f_1,f_1^*]$ including f_1,f_1^* explicitly. My criticism concerns the fact that the KN Ansatz only covers the $(0,q_1^2,q_2^2)$-plane, while consistent kinematics depends on 3 variables.

$h_6 = (-5 \pm 5)$ GeV^4 (positivity required, WZW bounded, QPM compatible), which yields rather stable results [5], while in [22] $h_6 = (+5 \pm 5)$ GeV^4 was advocated, arguing with LDM *versus* LDM+V smoothness. Results are rather sensitive to the choice of h_6 and better estimates are required.

My own calculation[5] using $h_3 \in [-10, 10]$ GeV^{-2} yields $a_\mu(\mathrm{LbL}; X) \times 10^{11} = 93.91 \pm 12.40$ for $X = \pi^0, \eta, \eta'$, 28.13 ± 5.63 for $X = a_1, f_1', f_1$ and -5.98 ± 1.20 for $X = a_0, f_0', f_0$. In [1], based on Nyffeler 2009, we adopted

$$a_\mu^{\mathrm{LbL;had}} = (116 \pm 39) \times 10^{-11} \,,$$

which we use as our estimate. Knecht, Nyffeler 2002 found 83 ± 12 for π^0, η, η' in the pole approximation, and corrected the previously wrong sign of this contribution, Bijnens, Prades 2007 now advocate 110 ± 40 [28] as their value. Some compromise values were proposed in [21] (PdRV 0209 Table III).

TABLE III

Summary of results (see also [1, 20, 21]).

Contribution	BPP	HKS	MV	PdRV	N/JN
π^0, η, η'	85 ± 13	82.7 ± 6.4	114 ± 10	114 ± 13	99 ± 16
π, K loops	-19 ± 13	-4.5 ± 8.1	0 ± 10	-19 ± 19	-19 ± 13
Axial vectors	2.5 ± 1.0	1.7 ± 1.7	22 ± 5	15 ± 10	22 ± 5
Scalars	-6.8 ± 2.0	—	—	-7 ± 7	-7 ± 2
Quark loops	21 ± 3	9.7 ± 11.1	—	2.3	21 ± 3
Total	83 ± 32	89.6 ± 15.4	136 ± 25	105 ± 26	116 ± 39

A summary of the status of the hadronic LbL contributions is given in Table III. Is this the final answer? How to improve these results? If no convincing progress should be possible the hadronic LbL uncertainty soon

[5] I am using the new representation presented in [1]: for any one particle exchange and any hadronic form-factor one can write a 3-dimensional integral over the moduli $Q_1 = |Q_1|$, $Q_2 = |Q_2|$ and $t = \cos\theta$ ($t = (Q_1 \cdot Q_2)/(Q_1 Q_2)$, $Q_3 = -(Q_1 + Q_2)$)

$$\begin{aligned} a_\mu(\mathrm{LbL}; \pi^0) \;\; = \;\; & -\frac{2\alpha^3}{3\pi^2} \int\limits_0^\infty \mathrm{d}Q_1 \mathrm{d}Q_2 \int\limits_{-1}^{+1} \mathrm{d}t \, \sqrt{1-t^2} \, Q_1^3 \, Q_2^3 \\ & \times \left(F_1 \, P_6 \, I_1(Q_1, Q_2, t) + F_2 \, P_7 \, I_2(Q_1, Q_2, t) \right) \,, \end{aligned}$$

where, *e.g.* for one pion exchange, $P_6 = 1/(Q_2^2 + m_\pi^2)$, and $P_7 = 1/(Q_3^2 + m_\pi^2)$ denote the Euclidean single particle exchange propagators. I_1 and I_2 are known integration kernels. The non-perturbative factors in case of π^0 exchange are $F_1 = \mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(q_2^2, q_1^2, q_3^2)\, \mathcal{F}_{\pi^{0*}\gamma^*\gamma}(q_2^2, q_2^2, 0)$, $F_2 = \mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)\, \mathcal{F}_{\pi^{0*}\gamma^*\gamma}(q_3^2, q_3^2, 0)$.

could turn into the limitation for more precise $g-2$ tests. We are looking for new ideas to get rid of existing model dependencies. In the long term, only lattice QCD is expected to provide an unambiguous answer here.

4. Summary and outlook

The present status and how we got there is shown in Fig. 3. The muon $g-2$ is a beautiful example which illustrates the principle "the closer you look the more there is to see". Note that the anomalous magnetic moment of the muon, by itself a tiny 0.116 % effect, now is measured with a precision 5×10^{-7}. Note that a 2σ to 3σ "discrepancy" persisted after the first of the three independent high precision measurement from the E821 experiment was released, in spite of many changes an improvements on the theory side as well as in the experimental determination of $R(s)$. Very likely upcoming experiments at the large hadron collider LHC at CERN will clarify what kind of effect we "see" in the deviation between theory and experiment, provided it is a real deviation and not just a statistical fluctuation or due to underestimating the uncertainties.

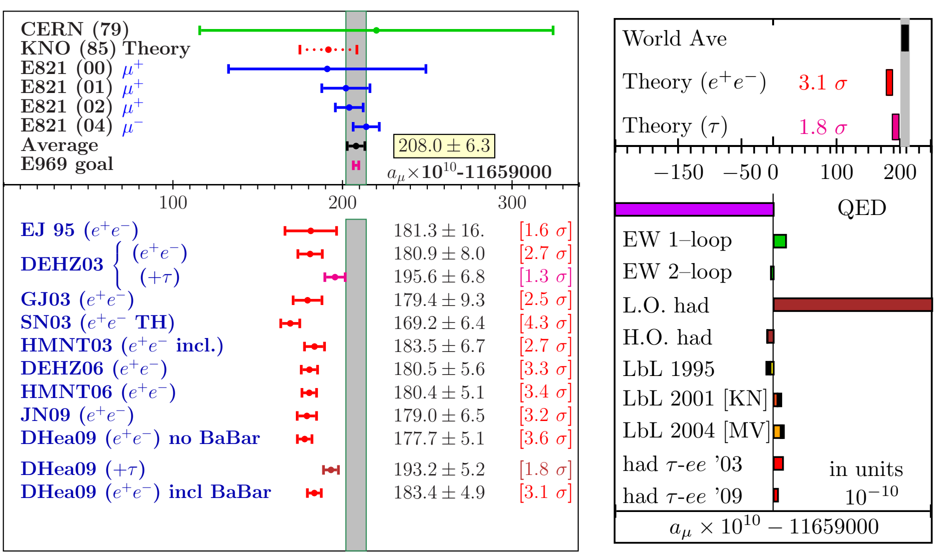

Fig. 3. Present status of experiments and theoretical predictions. Given theory results (left) essentially only differ by $a_\mu^{\mathrm{had}(1)}$. The increase in precision is due to better $R(s)$ data and by more progressive error treatments and more extensive use of pQCD. The importance of various small effects is also illustrated (right). For references to results not discussed in the text see [1].

What do we expect for the future?

(1) We are hoping for a follow up experiment at Fermilab or JPAC/Japan.

(2) Improved hadronic VP is needed [29] since it is still dominating the present theory error:

— Continuation of the experimental program of $R(s)$ measurements is indispensable: VEPP-2000, DAFNE-2, BES III, CLEOc, Belle.

— Lattice QCD will provide results within a few years to cross check and hopefully improve hadronic VP calculations up to 2 GeV.

(3) We urgently need better understanding and control of hadronic FSR effects [30, 31]. Dedicated experimental studies are indispensable.

(4) Hadronic LbL is the touchstone of theory:

— Progress is possible: sort out "the" realistic resonance Lagrangian, as the true low energy effective version of QCD, by global fit strategies. For first such attempts see [18].

— Lattice QCD calculations can provide in steps important results to cross check model calculations. This is a very long term project [32–34].

— More experiments on hadronic $\gamma^*\gamma^* \to \gamma^*\gamma^*$ are mandatory.

(5) An independent check of the 4-loop QED contribution is highly desirable, also within the context of the new a_e extreme precision measurements and the determination of $\alpha(a_e)$.

(6) Progress in calculating 5-loop QED is important for future progress.

The muon $g-2$ has provided deep inside into the world of quantum fluctuations as predicted by the electroweak Standard Model (SM). It remains one of the big challenges for the frontier of precision tests of the SM and beyond. It also remains one of the outstanding monitors for the discovery of new physics. Indeed we are waiting for news from the LHC to learn more about the present 3σ deviation between theory and experiment (see *e.g.* [35]).

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