THE QCD NLO PARTON SHOWER FOR THE INITIAL-STATE[*] [**]

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We present a new scheme of a fully exclusive QCD NLO parton shower for the initial state. The scheme is based on the collinear factorization, but at the same time it provides fully exclusive events with four-momenta of all emitted partons. We show a first working prototype of such a MC code for the subset of graphs for the Non Singlet evolution and we show that on the inclusive level it reproduces the standard $\overline{\mathrm{MS}}$ DGLAP results.

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The LHC experiments will start collecting data in November this year (2009). This moment will open a new era in the high precision QCD measurements. In order to fully exploit these results, comparable in precision QCD Monte Carlo (MC) event generator programs are necessary. However, the current QCD MC programs are based on the improved Leading Order (LO) evolution equations as the building blocks (as far as the differential distributions are concerned; the overall, inclusive, normalization can easily be corrected to higher order precision), *cf.* for example [1–3][1]. The LO-level precision may be not enough for the LHC, where, as a rule of thumb, one needs at least NLO theoretical precision.

It is therefore justified to ask whether it is possible to develop a new, genuine NLO, scheme for the QCD MC event generator? Such a scheme would: provide more precise predictions of the perturbative QCD; allow for

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[1] This should be contrasted with the inclusive, analytical, calculations which are routinely done at the NLO level, and in some cases even in the NNLO approximation [4].

better treatment of the heavy quark mass thresholds; offer a new method of transferring parton distributions from HERA to LHC energies; offer a better control of parton luminosities and much more.

Such a new NLO MC event generator should be based on complete NLO calculations both in resumed, evolution, part as well as in fixed-order, hard matrix element, part. However, as we already mentioned, the MC parton shower (PS) featuring complete NLO evolution does not exist yet!

In this presentation we will demonstrate that it is feasible to construct such a NLO parton shower. We will describe prototype of such a MC for the case of initial state radiation which we have developed recently within the `KRKmc` project [5–8][2].

In a nutshell the `KRKmc` parton shower can be described as follows. It is based on Feynman Diagrams and Lorentz-invariant phase space. It uses the scheme of collinear factorization [12,13] and, therefore, it implements *exactly* the NLO $\overline{\mathrm{MS}}$ DGLAP evolution [14]. The key novelty of the `KRKmc` is the fact that it provides fully unintegrated parton density functions (with complete kinematical information about all the generated partons) with NLO evolution done by MC itself, using new, exclusive NLO kernels.

In this presentation we will show first working prototype of such a NLO parton shower. It is done for the case of Non Singlet evolution and limited only to the C_{F}^2 part of the kernel.

These new exclusive kernels have been calculated using the same methodology as in the original calculation by Curci, Furmanski and Petronzio [14]. The main difference is that we had to perform the calculations for various choices of the evolution time: in addition to the original choice of the virtuality (q^2) we used also transverse momentum ($\boldsymbol{k}_\perp$) and rapidity. The exclusive (unintegrated) kernels have different form depending on the choice of the evolution time (even though after integration and summation over all Feynman graphs, the inclusive kernels are identical).

In Fig. 1 we show all the contributions to the Non Singlet LO and NLO evolution kernels. As we mentioned earlier, in this work we restrict ourselves to the C_{F}^2 part (two "bremsstrahlung-like" graphs) labeled with the dotted blue line marked box. It is in a sense the most complicated part because it involves the subtraction of the soft counter-term (labeled as $1-\mathbf{P}$ in the plot). The projection operator $\mathbf{P}$ plays the crucial role in the factorization scheme. It extracts the collinear singularity by means of three consecutive operations: spin projection, kinematical on-shell projection and pole-part projection. The $1-\mathbf{P}$ part of the double bremsstrahlung graph (shown in Fig. 1) is therefore a pure NLO correction.

[2] Let us mention that there are some other projects aiming at constructing parton showers beyond the LO as well, *cf. eg.* [9–11].

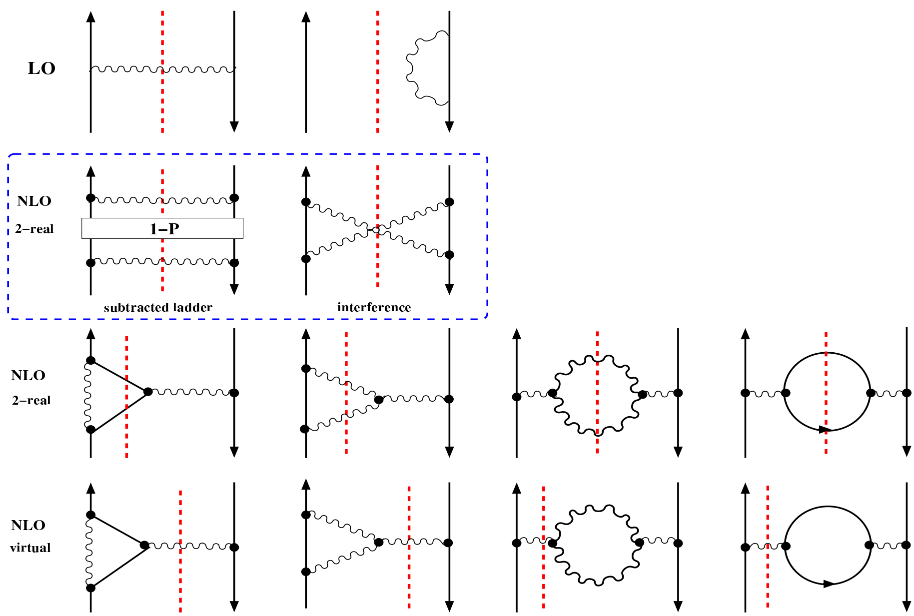

Fig. 1. Contributions to the NLO Non Singlet evolution kernel. The C_F^2 part labeled with the dotted blue line marked box.

Having reorganized the kernels, next we had to change the regularization scheme. The dimensional regularization, used for the collinear singularities, is not good for the MC as it would lead to very high parton multiplicities. Instead, we introduced a geometrical cut-off Δ:

$$\frac{1}{\epsilon} \Rightarrow \int\limits_0^{Q^2} \frac{dq^2}{q^2} \left(\frac{q^2}{Q^2}\right)^{\epsilon} \Rightarrow \int\limits_{\Delta^2}^{Q^2} \frac{dq^2}{q^2} .$$

Note, that the above replacement is so simple because the collinear singularity has the form dq^2/q^2 which can be factorized out of the rest of the formula. Note also, that this change of regularization procedure has to be done in such a way that it does not violate the $\overline{\mathrm{MS}}$ scheme.

Next thing to be reorganized is the factorization procedure itself. The reason for it is that the DGLAP equation mixes orders of perturbative expansion. The NLO DGLAP kernel, denoted as P, is in fact a sum of LO and NLO pieces.

$$P = \alpha P^{\mathrm{LO}} + \alpha^2 P^{\mathrm{NLO}} .$$

Therefore, P^k terms in the expansion of $\sum P^k$ are a mixture of various orders in α. In the actual MC we have to use the true expansion in α^k instead.

This is shown graphically in Fig. 2 for the case of $k=4$ (expansion in α is truncated at α^4). The contributions which contain only the LO-labeled pink boxes constitute the LO crude MC. The other pieces, in each of the curly brackets separately, are introduced as the MC weights, to be applied on the top of the LO term (from the same curly bracket!).

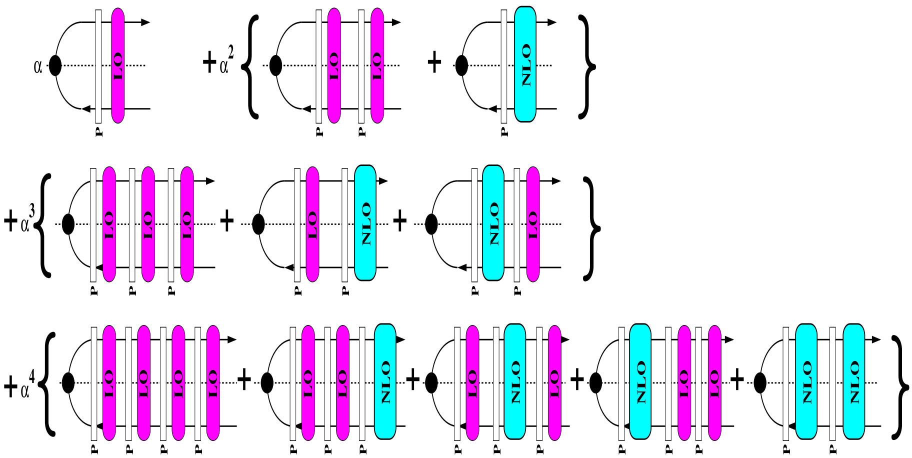

Fig. 2. The expansion of $P+P^2+P^3+P^4$ in powers of coupling constants (truncated at α^4).

Having reorganized the calculation of the Feynman diagrams, the regularization scheme and the factorization procedure, the last thing to be reorganized is the phase-space. Namely, we found it necessary to resign from the ordering in the evolution time in the underlying LO crude MC. Instead we used the Bose-symmetric form. The ordering is an approximate feature of the LO matrix element and, therefore, it is not strict at the NLO level and we have to explicitly sum over entire phase space. Let us illustrate it on the case of triple emission. The fully differential formula corresponding to the second line of Fig. 2 has the form

$$\begin{aligned} D_3^{L+N}(t,x) \;&\sim\; \frac{1}{3!}\int\limits_{k_{\min}}^{k_{\max}} \left(\prod_{i=1}^{3}\frac{d^3k_i}{2k_i^0}\right)\delta_{x_0-x=\alpha_1+\alpha_2+\alpha_3}\rho_3^{L+N}(k_3,k_2,k_1)\,, \\ \rho_3^{L+N}(k_3,k_2,k_1) \;&=\; \sum_{\pi}\Big(\rho_3^{L}(k_{\pi_3},k_{\pi_2},k_{\pi_1})+\rho_{3a}^{N}(k_{\pi_3},k_{\pi_2},k_{\pi_1}) \\ &\qquad +\rho_{3b}^{N}(k_{\pi_3},k_{\pi_2},k_{\pi_1})\Big)\,, \end{aligned} \qquad (1)$$

$$\begin{aligned}
\rho_3^L(k_3,k_2,k_1) &= \rho^L(k_3|x_2)\,\rho^L(k_2|x_1)\,\rho^L(k_1|x_0)\,\theta_{|\boldsymbol{k}_3|>|\boldsymbol{k}_2|>|\boldsymbol{k}_1|}\,,\\
\rho_{3a}^N(k_3,k_2,k_1) &= \rho^L(k_3|x_2)\,b_2^{\theta N}(k_2,k_1|x_0)\,\theta_{|\boldsymbol{k}_3|>|\boldsymbol{k}_2|}\,,\\
\rho_{3b}^N(k_3,k_2,k_1) &= b_2^{\theta N}(k_3,k_2|x_1)\,\rho^L(k_1|x_0)\,\theta_{|\boldsymbol{k}_3|>|\boldsymbol{k}_1|}\,.
\end{aligned} \tag{2}$$

The ρ^L ($b_2^{\theta N}$) corresponds to the LO (NLO) contribution, grey/pink (light grey/blue) box, in Fig. 2. The overall distribution for triple emission, denoted as ρ_3^{L+N} is a sum of three contributions: LO*LO*LO (ρ_3^L), LO*NLO (ρ_{3a}^{LN}) and NLO*LO (ρ_{3b}^N). It is the ρ_{3b}^N one where the violation of strict ordering is seen in a most transparent way: there is no reason why $|\boldsymbol{k}_2|$ should be bigger than $|\boldsymbol{k}_1|$.

At this moment we are ready to construct the MC algorithm. As the crude MC we use the LO distributions, like the ρ_3^L in Eq. (1). The LO MC can be of the standard Markovian type or of the non-Markovian type [8, 15], if the constraint on final x and parton type is needed. The NLO corrections are then introduced in the form of weights. The three-emission weight corresponding to the equation (1) looks as follows

$$\begin{aligned}
w &= 1 + w_{3a}^N + w_{3b}^N\,,\\
w_{3a}^N &= \frac{b_2^{\theta N}(\tilde{k}_2,\tilde{k}_1|x_0)}{\rho^L(\tilde{k}_2|x_1)\,\rho^L(\tilde{k}_1|x_0)}\theta_{\tilde{t}_2>t_{\mathrm{M}}}\,,\\
w_{3b}^N &= \frac{b_2^{\theta N}(\tilde{k}_3,\tilde{k}_2|x_1)}{\rho^L(\tilde{k}_3|x_2)\,\rho^L(\tilde{k}_2|x_1)}\theta_{\tilde{t}_3>t_{\mathrm{M}}}\\
&\quad + \frac{b_2^{\theta N}(\tilde{k}_3,\tilde{k}_1|x_1^{\pi_b^\bullet})}{\rho^L(\tilde{k}_3|x_2)\,\rho^L(\tilde{k}_1|x_0)}\;\frac{\rho^L(\tilde{k}_2|x_0)}{\rho^L(\tilde{k}_2|x_1)}\theta_{\tilde{t}_3>t_{\mathrm{M}}}\,.
\end{aligned} \tag{3}$$

We see explicitly the two terms due to the NLO*LO contribution ρ_{3b}^N corresponding to the two kinematical regions: $|\boldsymbol{k}_2| > |\boldsymbol{k}_1|$ and $|\boldsymbol{k}_1| > |\boldsymbol{k}_2|$. For explanation of the rest of the notation in Eqs (1)–(3) we refer reader to Refs [5, 6].

Let us close presentation of our new exclusive MC PS scheme with a more technical remark: there is a fundamental difference between the standard inclusive calculations and the MC PS calculation in the treatment of the lower limit of the phase space integrals. In the analytical calculation of the $\overline{\mathrm{MS}}$ kernel the lower limit of the internal degree of freedom, integrated out during the procedure of constructing the NLO kernel, is set to 0. On the contrary, in the MC it is limited by some t_0 for all partons. This mismatch must lead to a discrepancies in the numerical results. In this work we decided to resolve this conflict by "artificial" lowering of the t_0 limit, below the actual start of the evolution, labeled as $t_{\rm M}$ in Eq. (3), and by performing a LO pre-evolution in this extended region. This way we maintain exact agreement with the $\overline{\rm MS}$ result. This set-up is illustrated in Fig. 3.

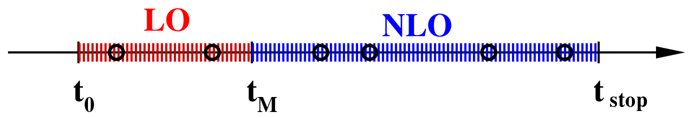

Fig. 3. The pre-evolution and "true" evolution ranges.

We have implemented the above scheme of the exclusive NLO evolution in the MC program KRKmc. In Fig. 4 we compare its results with a standard NLO DGLAP MC evolution. Both evolutions are implemented as weights on the top of the same LO Markovian MC algorithm. Evolution ranges from 10 GeV to 1 TeV. LO pre-evolution starts at 1 GeV from $\delta(1-x)$ distribution. The entry $r = 1$ ($r = 2$) corresponds to contribution from terms with one (two) NLO "insertions". Graphically, the $r = 2$ contribution comes from the last term in the bottom line of Fig. 2 and the $r = 1$ one from six other terms with one light grey/blue "NLO"-box.

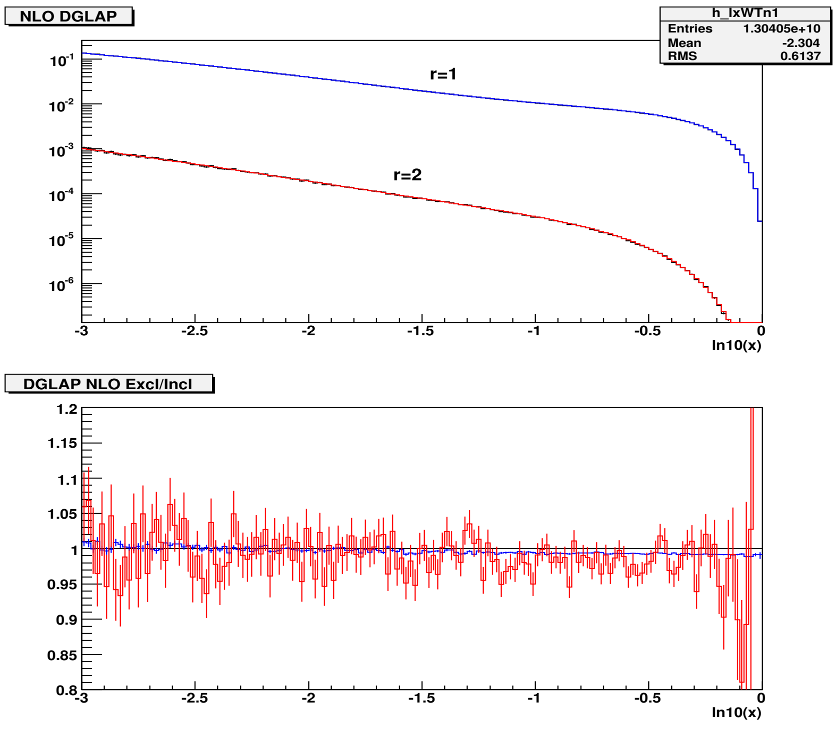

Fig. 4. Comparison of exclusive and standard DGLAP NLO evolutions.

As one can see the agreement is very good, within the statistical errors. This way we demonstrated for the first time ever that the QCD NLO parton shower can be constructed (although for a limited set of Feynman diagrams)!

We are currently in the process of adding the rest of graphs from Fig. 1, omitted in this work. This way the Non Singlet evolution will be completed. Once also the singlet evolution is added, we will be ready to construct the complete event generator for the Drell–Yan-type processes at LHC and/or DIS at HERA.

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