# QED PENTAGON CONTRIBUTIONS TO $e^+e^- \to \mu^+\mu^-\gamma^*$

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We report on a numerical implementation of the QED one-loop 5-point functions. These functions contribute to the NLO corrections to the hard-bremsstrahlung process $e^+e^- \to \mu^+\mu^-\gamma$, which is an important background process for accurate predictions in experiments at high luminosity meson factories such as DAFNE and PEP-II. The calculation is implemented using publicly available tools and incorporates several numerical and analytical cross-checks. Numerical precision and stability is demonstrated by preliminary test runs with KLOE and BarBar kinematical cuts.

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## 1. Introduction

The evaluation of 5-point function contributions is essential for accurate predictions for some important QED processes. These functions contribute to NLO corrections to $\mu^+\mu^-\gamma$ production and NNLO corrections to Bhabha scattering.

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Bhabha scattering is used as a reference process for luminosity measurements at all electron–positron colliders. The full NNLO calculation would be incomplete without contributions from radiative loop corrections. So far only bremsstrahlung from external legs (lowest order) has been taken into account [1] and implemented in [2–4].

The muon pair production with real photon emission ($e^+e^- \to \mu^+\mu^-\gamma$) is an important background and normalization reaction in the measurement of the pion form-factor:

$$R_{\rm exp} = \frac{\sigma(e^+e^- \to \pi\pi\gamma)}{\sigma(e^+e^- \to \mu^+\mu^-\gamma)}\,. \tag{1}$$

It is necessary for precise determination of the anomalous magnetic moment of the muon, $(g-2)_\mu$, at high luminosity meson factories such as DAFNE and PEP-II.

In this respect contributions from previously unconsidered QED 5-point functions become important for accurate predictions in experiments like BaBar and KLOE.

We have developed and tested a toolchain for the evaluation of QED 5-point functions contributions[1]. The methods of computation and the numerical stability are discussed in the following sections.

## 2. Computation method

We have used the DIANA package [5, 6] for the Feynman diagrams generation, which relies on QGRAF [7]. There are 50 one-loop diagrams contributing to the process $e^+e^- \to \mu^+\mu^-\gamma$. Four diagrams with 5-point loop integrals are most difficult and time-consuming in numerical evaluation.

The DIANA output for loop and tree diagrams was further processed analytically in FORM [8] to generate FORTRAN 77 output for the cross-section. For a numerical evaluation of loop integrals we use LoopTools [9, 10], which is based on the FF [11] package. LoopTools implements two reduction schemes for pentagons: the Passarino–Veltman scheme [12] and Denner/Dittmaier scheme [13, 14]. We decided to use the second scheme, because it gives better precision for 5-point integrals.

We performed several analytical and numerical tests. The IR parts of pentagons and boxes were calculated analytically with the Mellin–Barnes method using AMBRE [15] and MB method [16]. These IR parts were used to verify the correctness of the IR part of LoopTools results. It was done by subtracting the analytical expression for the IR part from LoopTools answer and checking that the resulting expression does not depend on the regularization parameter.

[1] See also a paper on the same subject by G. Ossola.

We have also checked numerical values of 5-point integrals at selected sets of phase-space points using analytical expansions provided by `hexagon` [17]. The numerical implementation of five and six point functions based on the same ideas [18, 19] as in `hexagon` is under development in our group and will be used in the future for completely independent numerical cross-checks.

Additionally, for a crosscheck we compared our expressions with outputs of FeynArts [20] and FormCalc [10, 21].

For the Monte Carlo simulation and the phase space integration the generated FORTRAN 77 subroutines were linked with the PHOKHARA [22, 23] event generator.

## 3. Numerical tests

The most challenging and time consuming part of the computation is the evaluation of the 5-point integrals. Contributions from boxes, triangles and self-energies do not present any difficulties.

So we consider here only a subset of the full number of diagrams contributing to the $e^+e^- \to \mu^+\mu^-\gamma$ process, which consists of gauge invariant combinations of one pentagon and two corresponding boxes. There are four such combinations, one of them is shown in Fig. 1.

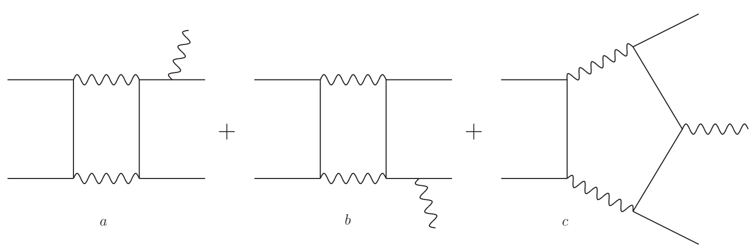


Fig. 1. Gauge invariant combination of a pentagon and two boxes.

The PHOKHARA kinematical settings for test runs are shown in Table I.

TABLE I

Kinematical cuts of KLOE and Babar experiments.

| | KLOE | BaBar |
|---|---|---|
| $E_{\text{CMS}}$ | 1.02 GeV | 10.56 GeV |
| $Q^2$ | 0.0447–1.06 GeV$^2$ | 0.0447–50 GeV$^2$ |
| $E_{\min,\gamma}$ | 0.02 GeV | 3 GeV |
| $\theta_\mu$ | 50°–130° | 40°–140° |
| $\theta_\gamma$ | 0°–15° and 165°–180° | 20°–138° |

They correspond to configurations of KLOE and Babar experiments. $E_{\mathrm{CMS}}$ is the collision energy and $E_{\min,\gamma}$ is the minimal energy of the tagged photon. The angles are measured between directions of the corresponding particle and the $e^-$-beam axis.

The gauge invariance test checks that invariant combinations of diagrams cancel when amplitudes are contracted with external photon momenta instead of polarization vectors.

In our case, such invariance test is presented among four and five point loop integrals (*e.g.* Fig. 1). The test was carried out with two different working precisions: double and quadruple. The relative accuracy is defined as:

$$A = \max \frac{\sum_i \Re\left(M^i_{\mathrm{loop}} M^\dagger_{\mathrm{tree}}\right)}{\min \Re\left(M^i_{\mathrm{loop}} M^\dagger_{\mathrm{tree}}\right)} \,. \tag{2}$$

Table II shows the accuracy as defined in 2 for $2.5\times10^6$ generated events. It is clearly seen that using quadruple precision gives much better accuracy.

TABLE II

Gauge invariance test $A$ as defined in 2 with KLOE and BaBar settings for different working precisions.

| $A$ | KLOE | BaBar |
|---|---|---|
| Double precision | $10^{-2}$ | $10^{-5}$ |
| Quadruple precision | $10^{-12}$ | $10^{-10}$ |

As it was pointed out in the previous section, the interference amplitudes were prepared using different software sets. Thereafter both results were cross-checked, this consisted as an additional check for gauge invariance and normal numerical calculations.

We performed several Monte Carlo simulations to check the accuracy and stability of the evaluation of 5-point functions. All plotted points were histogramed using one of the subroutines which is encoded in PHOKHARA and provides the calculation of Monte Carlo errors. All the values *i.e.* $\theta_\mu$, $\theta_\gamma$ and $Q^2$ were produced in 200 bins.

The contribution of the discussed gauge invariant combinations to the $\theta_\mu$ dependence with BaBar settings is shown in Fig. 2. The correction amounts to about 0.5% of the tree-level result shown in Fig. 3. The antisymmetry of the muon angular dependence is due to the asymmetric kinematical cut on $\theta_\gamma$ (see Table I).

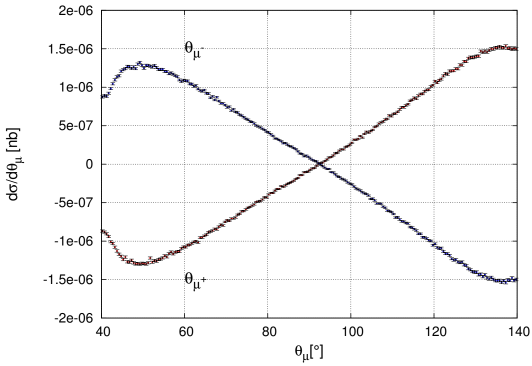


Fig. 2. 5-point gauge invariant contribution to $e^+e^- \to \mu^+\mu^-\gamma$ at BaBar. The $d\sigma/d\theta_{\mu^-}$ and $d\sigma/d\theta_{\mu^+}$ are shown.

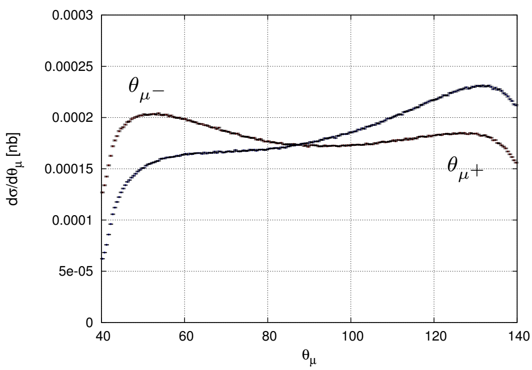


Fig. 3. Tree-level contribution to $e^+e^- \to \mu^+\mu^-\gamma$ at BaBar. The $d\sigma/d\theta_{\mu^-}$ and $d\sigma/d\theta_{\mu^+}$ are shown.

The similar plot for KLOE in Fig. 4 is symmetric in accordance with the cuts. The contribution is also about 0.5% of the LO result shown in Fig. 5.

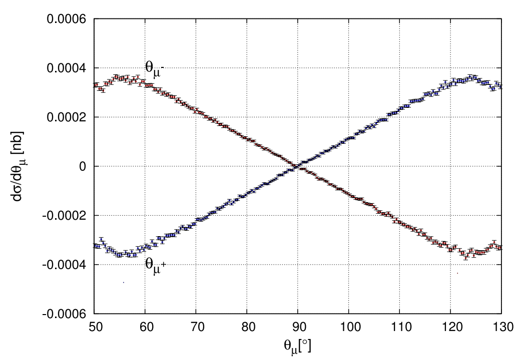


Fig. 4. 5-point gauge invariant contribution to $e^+e^- \to \mu^+\mu^-\gamma$ at KLOE. The $d\sigma/d\theta_{\mu^-}$ and $d\sigma/d\theta_{\mu^+}$ are shown.

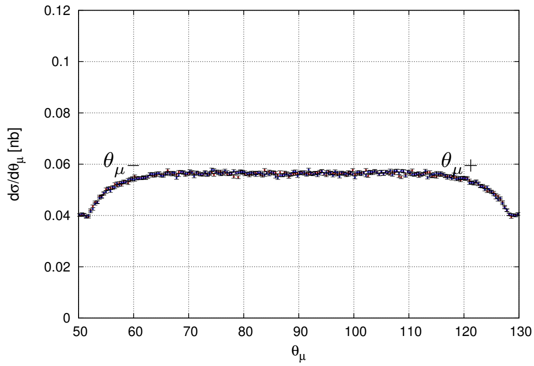


Fig. 5. Tree-level contribution to $e^+e^- \rightarrow \mu^+\mu^-\gamma$ at KLOE. The $d\sigma/d\theta_{\mu^-}$ and $d\sigma/d\theta_{\mu^+}$ are shown.

## 4. Conclusions

We have developed a toolchain for 1-loop QED 5-point functions evaluation. The correctness of the answer is assured by a number of various analytical and numerical tests. In the next step we plan to merge the toolkit with the hard part of the considered processes.

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