2- AND 3-LOOP HEAVY FLAVOR CORRECTIONS TO TRANSVERSITY*

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We calculate the two- and three-loop massive operator matrix element (OME) contributing to the heavy flavor Wilson coefficients of transversity. We obtain the complete result for the two-loop OME and compute the first thirteen Mellin moments at three-loop order. As a by-product of the calculation, the moments $N = 1$ to 13 of the complete two-loop and the T_F-part of the three-loop transversity anomalous dimension are obtained.

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1. Framework

The transversity distribution belongs to the three twist-2 parton distribution functions (PDFs), together with those for unpolarized and polarized deep-inelastic scattering. It is a flavor non-singlet, chiral-odd distribution and can be measured in semi-inclusive deep-inelastic scattering (SIDIS) and via the polarized Drell–Yan process (for a review see Ref. [1]). Different experiments perform transversity measurements at the moment, *cf.* Refs [2]. Recently, a first phenomenological parameterization has been given for the transversity up- and down-quark distributions in Ref. [3], the moments of which are in qualitative agreement with first lattice calculations [4].

The scattering cross-section for semi-inclusive deeply inelastic charged lepton–nucleon scattering $lN \to l'h + X$ is given by

$$\frac{d^3\sigma^{\mathrm{SIDIS}}}{dxdydz} = \frac{4\pi\alpha_{\mathrm{em}}^2 s}{Q^4} \sum_{a=q,\overline{q}} e_a^2 x \Biggl\{ \frac{1}{2}\left[1+(1-y)^2\right] F_a\left(x,Q^2\right) D_a\left(z,Q^2\right) - (1-y)\,|\boldsymbol{S}_\perp||\boldsymbol{S}_{h\perp}| \cos\left(\phi_S+\phi_{S_h}\right) \Delta_{\mathrm{T}} F_a\left(x,Q^2\right) \Delta_{\mathrm{T}} D_a\left(z,Q^2\right) \Biggr\}, \quad (1)$$

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after the $\boldsymbol{P}_{h\perp}$-integration has been performed, [1]. We consider, for definiteness, only scattering cross-sections free of $\boldsymbol{k}_\perp$-effects to refer to twist-2 quantities. x and y denote the Bjorken variables, z the fragmentation variable, $q^2 = -Q^2$ the space-like 4-momentum transfer, $\alpha_{\rm em}$ the fine structure constant, e_a the quark charge, and s the c.m.s. energy squared. $\boldsymbol{S}_\perp$ and $\boldsymbol{S}_{h\perp}$ are the transverse spin vectors of the incoming nucleon N and the measured hadron h. $F_a\left(z,Q^2\right), \Delta_{\rm T} F_a\left(z,Q^2\right)$ and $D_a\left(z,Q^2\right), \Delta_{\rm T} D_a\left(z,Q^2\right)$ denote the unpolarized and transversity structure- and fragmentation functions, respectively. The angles ϕ_{S,S_h} are measured in the plane transverse to the $\gamma^* N$ axis between the x-axis and the respective vector. In process (1) the spin of the *transversely* polarized hadron h has to be measured.

The transversity distribution may also be measured in the transversely polarized Drell–Yan processes. In Mellin space the scattering cross-section is given by [5]

$$\frac{d\Delta_{\rm T}\sigma^{\rm DY}}{d\phi} = \frac{\alpha_{\rm em}^2}{9s}\cos\left(2\phi\right)\Delta_{\rm T} H\left(N,M^2\right)\Delta_{\rm T} C_q^{\rm DY}\left(N,M^2\right)\,, \qquad (2)$$

where N denotes the Mellin variable and ϕ is the azimuthal angle of one of the final state leptons $l^\pm$ relative to the axis defined by the transverse polarizations.

$$\Delta_{\rm T} H\left(N,Q^2\right) = \sum_q e_q^2\left[\Delta_{\rm T} q_1\left(N,Q^2\right)\Delta_{\rm T}\overline{q}_2\left(N,Q^2\right) + \Delta_{\rm T}\overline{q}_1\left(N,Q^2\right)\Delta_{\rm T} q_2\left(N,Q^2\right)\right]$$

is a combination of transversity parton distributions for the incoming light (anti-)quarks, and $\Delta_{\rm T} C_q^{\rm DY}\left(N,M^2\right)$ denotes the Wilson coefficient of the Drell–Yan process, with M^2 the invariant mass of the produced lepton pair.

Like in the case of unpolarized and polarized deep-inelastic processes transversity receives heavy flavor corrections in higher orders in QCD. These are given by the corresponding heavy flavor Wilson coefficients. As for other non-singlet quantities [6, 7], these corrections start at $O\left(a_{\rm s}^2\right)$, with $a_{\rm s} = \alpha_{\rm s}/\left(4\pi\right)$. In SIDIS one can tag $Q\bar{Q}$-production in the same way as in the deep-inelastic process, [8]. A measurement is possible in high luminosity experiments. In the Drell–Yan process, on the other hand, heavy flavor contributions emerge inclusively since there the final-state l^+l^--pairs are measured in the first place. The calculation of the heavy quark Wilson coefficients for $Q^2 \gg m^2$ proceeds in the same way as in unpolarized and polarized deep-inelastic scattering [6, 7, 9, 10].

The complete Wilson coefficients for transversity can be decomposed into a light- and a heavy quark contribution

$$C_q^{\rm TR}\left(x,\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2}\right) = C_q^{\rm TR,light}\left(x,\frac{Q^2}{\mu^2}\right) + H_q^{\rm TR}\left(x,\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2}\right)\,. \qquad (3)$$

As shown in [6], the heavy quark Wilson coefficient for hard processes factorizes into the light quark Wilson coefficients and the massive operator matrix element $A_{qq,Q}^{\rm TR}$ at large enough scales $Q^2 \gg m^2$. We apply this to the heavy flavor Wilson coefficient for transversity[1]

$$\begin{aligned} H_q^{\rm TR}\left(x,\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2}\right) &= C_q^{\rm TR,light}\left(x,\frac{Q^2}{\mu^2}\right)\otimes A_{qq,Q}^{\rm TR}\left(x,\frac{m^2}{\mu^2}\right) \\ &= a_{\rm s}^2\left[\Delta_{\rm T}A_{qq,Q}^{(2),{\rm NS,TR}}+\Delta_{\rm T}\hat{C}_q^{(2)}\left(N_f\right)\right] \\ &\quad +a_{\rm s}^3\left[\Delta_{\rm T}A_{qq,Q}^{(3),{\rm NS,TR}}\left(N_f+1\right)+\hat{C}_q^{(3)}\left(N_f\right)\right. \\ &\quad \left.+\,\Delta_{\rm T}A_{qq,Q}^{(2),{\rm NS,TR}}\otimes\Delta_{\rm T}C_q^{(1)}\right]. \end{aligned} \tag{4}$$

The aim of this article is to present a computation of the renormalized two- and three-loop heavy-flavor operator matrix elements contributing to transversity. Details of the calculation are given in Ref. [11]. The operator matrix element $\langle q|O^{\rm NS,TR}|q\rangle$ is given by a two-point Green's function containing a closed loop of a heavy quark Q and external massless quarks q. The local operator is given by [12]

$$O_{F,a;\mu\mu_1\ldots\mu_n}^{\rm NS,TR} = i^n \boldsymbol{S}\left[\overline{\psi}\gamma_5\sigma_{\mu\mu_1}D_{\mu_2}\ldots D_{\mu_n}\frac{\lambda_a}{2}\psi\right] - \text{trace terms}\,. \tag{5}$$

Here, $\sigma_{\mu\nu}=(i/2)\left[\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu\right]$, $\boldsymbol{S}$ denotes symmetrization of the Lorentz indices and D_μ is the covariant derivative. The Green's function in $D=4+\varepsilon$ dimensions obeys the following vector decomposition

$$\begin{aligned} \hat{G}_{\mu,q,Q}^{ij,{\rm TR,NS}} &= J_N\left\langle q|O_{F,a;\mu\mu_1\ldots\mu_n}^{\rm NS,TR}|q\right\rangle = \delta_{ij}\left\{\Delta_\rho\sigma^{\mu\rho}\hat{\hat{A}}_{qq,Q}^{\rm TR,NS}\left(\frac{\hat{m}^2}{\mu^2},\varepsilon,N\right)\right. \\ &\quad \left.+c_1\Delta^\mu+c_2p^\mu+c_3\gamma^\mu\not{p}+c_4\not{\Delta}\not{p}\Delta^\mu+c_5\not{\Delta}\not{p}p^\mu\right\}(\Delta\cdot p)^{N-1} \end{aligned} \tag{6}$$

contracting the OME with a source term $J_N=\Delta^{\mu_1}\ldots\Delta^{\mu_N}$, with $\Delta^2=0$ and p the parton momentum. It determines the un-renormalized massive OME

$$\begin{aligned} \hat{\hat{A}}_{qq,Q}^{\rm TR,NS}\left(\frac{\hat{m}^2}{\mu^2},\varepsilon,N\right) &= \frac{-i\,\delta^{ij}}{4N_c\,(\Delta\cdot p)^{N+1}\,(D-2)}\left\{{\rm Tr}\left[\not{\Delta}\not{p}p^\mu\hat{G}_{\mu,q,Q}^{ij,{\rm TR,NS}}\right]\right. \\ &\quad \left.-\Delta\cdot p\,{\rm Tr}\left[p^\mu\hat{G}_{\mu,q,Q}^{ij,{\rm TR,NS}}\right]+i\Delta\cdot p\,{\rm Tr}\left[p^\mu\hat{G}_{\mu,q,Q}^{ij,{\rm TR,NS}}\right]\right\}. \end{aligned} \tag{7}$$

[1] We use the notation $\hat{f}(N_f)=f(N_f+1)-f(N_f)$.

A total of 129 diagrams contribute, which were generated using `QGRAF` [13]. These were projected onto $\hat{\hat{A}}_{qq,Q}^{\rm TR,NS}$, *cf.* [9], using codes written in `FORM` [14]. After tensor reduction, the loop integrals are of the tadpole-type, since the single external quark is on-shell and massless. The integrals were then evaluated using `MATAD` [15]. The renormalization of the OMEs is described in Ref. [9].

After mass- and charge renormalization one obtains the massive OMEs in the on-mass-shell scheme, *cf.* [9],

$$\Delta_{\rm T} A_{qq,Q}^{(2),{\rm NS},\overline{\rm MS}} = \frac{\beta_{0,Q}\gamma_{qq}^{(0),{\rm TR}}}{4}\ln^2\Big(\frac{m^2}{\mu^2}\Big) + \frac{\hat{\gamma}_{qq}^{(1),{\rm TR}}}{2}\ln\Big(\frac{m^2}{\mu^2}\Big) + a_{qq,Q}^{(2),{\rm TR}} - \frac{\beta_{0,Q}\gamma_{qq}^{(0),{\rm TR}}}{4}\zeta_2\,, \tag{8}$$

$$\begin{aligned}\Delta_{\rm T} A_{qq,Q}^{(3),{\rm NS},\overline{\rm MS}} = & -\frac{\gamma_{qq}^{(0),{\rm TR}}\beta_{0,Q}}{6}\Big(\beta_0+2\beta_{0,Q}\Big)\ln^3\Big(\frac{m^2}{\mu^2}\Big) + \frac{1}{4}\Big\{2\gamma_{qq}^{(1),{\rm TR}}\beta_{0,Q} \\ & -2\hat{\gamma}_{qq}^{(1),{\rm TR}}\Big(\beta_0+\beta_{0,Q}\Big) + \beta_{1,Q}\gamma_{qq}^{(0),{\rm TR}}\Big\}\ln^2\Big(\frac{m^2}{\mu^2}\Big) + \frac{1}{2}\Big\{\hat{\gamma}_{qq}^{(2),{\rm TR}} \\ & -\Big(4a_{qq,Q}^{(2),{\rm TR}} - \zeta_2\beta_{0,Q}\gamma_{qq}^{(0),{\rm TR}}\Big)(\beta_0+\beta_{0,Q}) + \gamma_{qq}^{(0),{\rm TR}}\beta_{1,Q}^{(1)}\Big\}\ln\Big(\frac{m^2}{\mu^2}\Big) \\ & +4\overline{a}_{qq,Q}^{(2),{\rm TR}}(\beta_0+\beta_{0,Q}) - \gamma_{qq}^{(0)}\beta_{1,Q}^{(2)} - \frac{\gamma_{qq}^{(0),{\rm TR}}\beta_0\beta_{0,Q}\zeta_3}{6} - \frac{\gamma_{qq}^{(1),{\rm TR}}\beta_{0,Q}\zeta_2}{4} \\ & +2\delta m_1^{(1)}\beta_{0,Q}\gamma_{qq}^{(0),{\rm TR}} + \delta m_1^{(0)}\hat{\gamma}_{qq}^{(1),{\rm TR}} + 2\delta m_1^{(-1)}a_{qq,Q}^{(2),{\rm TR}} + a_{qq,Q}^{(3),{\rm TR}}\end{aligned} \tag{9}$$

at 2- and 3-loops. Here, ζ_k denotes the Riemann ζ-function and $\gamma_{qq}^{(l),{\rm TR}}$ are the transversity anomalous dimensions for $l = 0,1,2$ in LO [16], NLO [5,17], and NNLO [18]. For the other quantities we refer to [9]. The new terms being calculated are $a_{qq,Q}^{(2),{\rm TR}}(N)$, $\overline{a}_{qq,Q}^{(2),{\rm TR}}(N)$ and $a_{qq,Q}^{(3),{\rm TR}}(N)$, and for the higher values of N, $\hat{\gamma}_{qq}^{(2),{\rm TR}}(N)$.

2. Results

2.1. Massive operator matrix elements

At $O(a_s^2)$ the massive operator matrix elements for transversity $\Delta_{\rm T} A_{qq,Q}^{(2),{\rm NS},\overline{\rm MS}}$ are obtained for general values of N, *cf.* Eq. (8). The un-renormalized OME is computed to $O(\varepsilon)$ to also extract the function $\overline{a}_{qq,Q}^{(2),{\rm TR}}(N)$. The new terms at 2-loops are $a_{qq,Q}^{(2),{\rm TR}}$ and $\bar{a}_{qq,Q}^{(2),{\rm TR}}$, *cf.* Eqs (8), (9):

$$a_{qq,Q}^{{\rm TR},(2)}(N) = C_F T_F\Big\{-\frac{8}{3}S_3 + \frac{40}{9}S_2 - \Big[\frac{224}{27}+\frac{8}{3}\zeta_2\Big]S_1 + 2\zeta_2 + \frac{(24+73\,N+73\,N^2)}{18N\,(1+N)}\Big\}, \tag{10}$$

$$\overline{a}_{qq,Q}^{\mathrm{TR},(2)}(N) = C_F T_F \Biggl\{ -\left[\frac{656}{81} + \frac{20}{9}\zeta_2 + \frac{8}{9}\zeta_3\right] S_1 + \left[\frac{112}{27} + \frac{4}{3}\zeta_2\right] S_2 - \frac{20}{9} S_3 + \frac{4}{3} S_4 + \frac{1}{6}\zeta_2 + \frac{2}{3}\zeta_3 + \frac{\left(-144 - 48\,N + 757\,N^2 + 1034\,N^3 + 517\,N^4\right)}{216 N^2 (1+N)^2} \Biggr\}, \tag{11}$$

with $S_k \equiv S_k(N)$ denoting the single harmonic sums.

At $O(a_s^3)$ the moments $N = 1$ to 13 were computed for the massive OME, as *e.g.*

$$\begin{aligned}
\Delta_{\mathrm{T}} A_{qq,Q}^{(3),\mathrm{NS},\overline{\mathrm{MS}}}(13) = C_F T_F \Biggl\{ &\left[\frac{1751446}{110565} C_A - \frac{7005784}{1216215} T_F (N_f + 2)\right] \ln^3\left(\frac{m^2}{\mu^2}\right) \\
&- \left[\frac{20032048197492631}{2193567563187000} C_F + \frac{137401473299}{8027019000} C_A + \frac{93611152819}{3652293645} T_F\right] \ln^2\left(\frac{m^2}{\mu^2}\right) \\
&+ \Biggl[\left(\frac{170583232732904244998 3}{263491335690022440000} + \frac{7005784}{45045}\zeta_3\right) C_F + \left(\frac{338545448824819123 7}{65807026895610000} \right. \\
&\left. - \frac{7005784}{45045}\zeta_3\right) C_A - \frac{458114791076413771}{6580702689561000} N_f T_F - \frac{217179304}{3648645} T_F\Biggr] \ln\left(\frac{m^2}{\mu^2}\right) \\
&- \left(\frac{7005784}{135135} B_4 - \frac{3502892}{15015}\zeta_4 + \frac{81735983092}{243486243}\zeta_3 - \frac{5537627829952273383742505249 3}{1220808056519011969008 00000}\right) C_F \\
&+ \left(\frac{3502892}{135135} B_4 - \frac{3502892}{15015}\zeta_4 + \frac{4061479439}{12162150}\zeta_3 - \frac{34868969747438825567756 47}{129350292066011016 00000}\right) C_A \\
&- \left(\frac{279922752632160355860697}{35571330318153029400 00} - \frac{56046272}{1216215}\zeta_3\right) T_F N_f \\
&+ \left(\frac{291526550302760070155303}{71142660636306058800 00} - \frac{14011568}{173745}\zeta_3\right) T_F \Biggr\},
\end{aligned} \tag{12}$$

where

$$B_4 = -4\zeta_2 \ln^2(2) + \tfrac{2}{3}\ln^4(2) - \tfrac{13}{2}\zeta_4 + 16\mathrm{Li}_4\left(\tfrac{1}{2}\right).$$

Like for the massive OMEs in case of unpolarized deep-inelastic scattering the structure of $\Delta_{\mathrm{T}} A_{qq,Q}^{(3),\mathrm{NS},\overline{\mathrm{MS}}}(N)$ is widely known for general values of N, except for the finite part $a_{qq,Q}^{(3),\mathrm{NS,TR}}$ and the 3-loop anomalous dimension $\hat{\gamma}_{qq}^{(2),\mathrm{TR}}(N)$. One notices the cancellation of all ζ_2 terms in $\Delta_{\mathrm{T}} A_{qq,Q}^{(3),\mathrm{NS},\overline{\mathrm{MS}}}(N)$ after renormalization.

2.2. Anomalous dimensions

The transversity anomalous dimension is given by

$$\gamma_{qq}^{\mathrm{TR}}(N, a_s) = \sum_{i=1}^{\infty} a_s^i \gamma_{qq}^{(i),\mathrm{TR}}(N). \tag{13}$$

From Eq. (9) one may determine the complete 2-loop anomalous dimension [5, 17] and the T_F-part of the 3-loop anomalous dimension [18]. We agree with the results given in [5, 17] and confirm the T_F-contributions for the moments $N = 1$ to 8 given in Ref. [18]. Furthermore, we newly obtain $\hat{\gamma}_{qq}^{(2),\mathrm{TR}} = \gamma_{qq}^{(2),\mathrm{TR}}(N_f+1) - \gamma_{qq}^{(2),\mathrm{TR}}(N_f)$ for $N = 9$ to 13, as *e.g.*

$$\hat{\gamma}_{qq}^{(2),\mathrm{TR}}(N=13) = -C_F T_F\left[\frac{3671331901540714157001 7}{13174566784501 1220000}C_F - \frac{14011568}{45045}\right.$$
$$\left.\times (C_F - C_A)\zeta_3 + \frac{66409807459266571}{3290351344780500}T_F(1+2N_f) + \frac{6571493644375020121}{65807026895610000}C_A\right]. \quad (14)$$

2.3. A remark on the Soffer bound

If the Soffer inequality [19]

$$|\Delta_{\mathrm{T}} f\left(x, Q^2\right)| \leq \tfrac{1}{2}\left[f\left(x, Q^2\right) + \Delta f\left(x, Q^2\right)\right] \quad (15)$$

holds for the non-perturbative PDF in Eq. (15) one may check its generalization from $f_i \to F_i$ for the corresponding structure functions. This includes the non-singlet evolution operator (Eq. (6), Ref. [20]) and the heavy flavor Wilson coefficient. At perturbative scales, it holds for the evolution operator [11], generalizing a NLO result from [5] to the moments $N = 1$ to 13 at NNLO. For the heavy quark Wilson coefficient in SIDIS we only know the massive OME so far. As shown in Ref. [11], a final conclusion can only be drawn knowing the yet undetermined massless Wilson coefficients. The difference $\left[A_{qq,Q}^{\mathrm{V}} - A_{qq,Q}^{\mathrm{TR}}\right](x)$ of the massive OMEs, shows a sign change to negative values for Q^2/m^2 in the physical range. For large scales $Q^2/m^2 \gg 1$ positive values are obtained.

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