UNCOSMOLOGY*

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We discuss some cosmological features of a hypothetical type of new physics characterized by begin asymptotically free in the UV regime and conformally invariant in the IR. We show that nucleosynthesis data generates non-trivial constrains this type of models.

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1. The basic idea

The "unparticle" proposal [1] is based on the assumption that there is a type of New Physics (NP) with the peculiar properties of being asymptotically free in the Ultraviolet (UV) and conformally invariant in the Infrared (IR). It is assumed that the NP interacts weakly with the Standard Model (SM) through the exchange of a set of heavy mediators of mass $M_{\mathcal{U}}$. Though not explicitly stated it is also tacitly assumed that the theory will have an ultraviolet completion, whose details (aside form the existence of the above-mentioned mediators) are left unspecified. The NP sector has two relevant high-energy scales: $M_{\mathcal{U}}$ and $\Lambda_{\mathcal{U}}$ the scale at which conformal invariance sets in. One must have $M_{\mathcal{U}} > \Lambda_{\mathcal{U}}$ and we will also assume $\Lambda_{\mathcal{U}} > v = 246$ GeV, the electroweak scale. The basic example for this type of NP is provided by a model proposed by Banks and Zaks [2]. In the following we will denote the UV phase of the NP as the Banks–Zaks ($\mathcal{BZ}$) phase, and the IR phase as the unparticle ($\mathcal{U}$) phase.

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At energies below $M_{\mathcal{U}}$ all mediator effects are virtual and generate effective interactions between the new physics and the Standard Model (SM) sectors. The dominant interactions are assumed to be of the form

$$\mathcal{L}(\mathrm{UV}) = c_{\mathcal{BZ}} M_{\mathcal{U}}^{-k} \mathcal{O}_{\mathrm{SM}} \mathcal{O}_{\mathcal{BZ}} \,, \tag{1}$$

where $\mathcal{O}_{\mathrm{SM}}$ is a local, gauge-invariant operator constructed out of the SM fields and their derivatives and, similarly, $\mathcal{O}_{\mathcal{BZ}}$ denotes a local operator constructed of the NP fields in the $\mathcal{BZ}$ phase. In the infrared these type of effective interactions suffer from strong renormalization effects leading to the replacement $\mathcal{O}_{\mathcal{BZ}} \to \Lambda_{\mathcal{U}}^{d_{\mathcal{BZ}}-d_{\mathcal{U}}} \mathcal{O}_{\mathcal{U}}$, where $d_{\mathcal{BZ}}$ and $d_{\mathcal{U}}$ denote the scaling dimensions of the operators $\mathcal{O}_{\mathcal{BZ}}$ and $\mathcal{O}_{\mathcal{U}}$, respectively. We then have

$$\mathcal{L}(\mathrm{IR}) = c_{\mathcal{U}} M_{\mathcal{U}}^{-k} \mathcal{O}_{\mathrm{SM}} \left(\Lambda_{\mathcal{U}}^{d_{\mathcal{BZ}}-d_{\mathcal{U}}} \mathcal{O}_{\mathcal{U}} \right) . \tag{2}$$

The construction of the operator $\mathcal{O}_{\mathcal{U}}$ is in general a difficult task, and requires detailed knowledge of the NP theory. However, for most calculations this is not needed since one is interested in operator correlators and these are very strongly constrained by conformal invariance. If one considers the effects of a single operator $\mathcal{O}_{\mathcal{U}}$ then for the purposes of calculating cross-sections and for the effects on standard cosmology one only needs the following density of states,

$$\int d^4x e^{iqx} \left\langle [\mathcal{O}_{\mathcal{U}}(x), \mathcal{O}_{\mathcal{U}}(0)] \right\rangle = A_{d_{\mathcal{U}}} \theta(q^2) (q^2)^{d_{\mathcal{U}}-2} \left[\theta(q^0) + \frac{1}{e^{\beta|q_0|}-1} \right] , \tag{3}$$

where $A_n = (4\pi)^{3-2n}/[2\Gamma(n)\Gamma(n-1)]$ and $\langle \cdots \rangle$ denotes the thermal average at temperature $1/\beta$, and where we assumed $\mathcal{O}_{\mathcal{U}}$ has bosonic character. For collider applications one simply lets $\beta \to \infty$. Using this expression one can determine the effects of this type of NP on various experimentally interesting processes in terms of a few parameters (k, $d_{\mathcal{U}}$, $M_{\mathcal{U}}$ and $\Lambda_{\mathcal{U}}$). In the following we will study some of the effects of this type of NP on standard cosmology

2. Thermodynamics

The conformal invariance requirement amounts to the assumption that the NP beta function vanishes at $g = g_* \neq 0$. In this case the β function and g will behave qualitatively as in Fig. 1.

For such theories the trace of the energy-momentum tensor [3] obeys

$$\left\langle \theta^\mu_\mu \right\rangle = \rho_{\mathcal{U}} - 3P_{\mathcal{U}} = (\beta/2g) \left\langle N \left[F_a^{\mu\nu} F_{a\,\mu\nu} \right] \right\rangle = (\beta/2g) b T^{4+\delta} \tag{4}$$

(N denotes normal ordering). It follows that $\langle \theta^\mu_\mu \rangle$ will vanish in the IR (since it is $\propto \beta$) and we expect $\rho_{\mathcal{U}} \simeq 3P_{\mathcal{U}}$ at low temperatures. The leading corrections are produced by $\langle N [F_a^{\mu\nu} F_{a\,\mu\nu}] \rangle$, as indicated above, with δ the anomalous dimension of this operator.

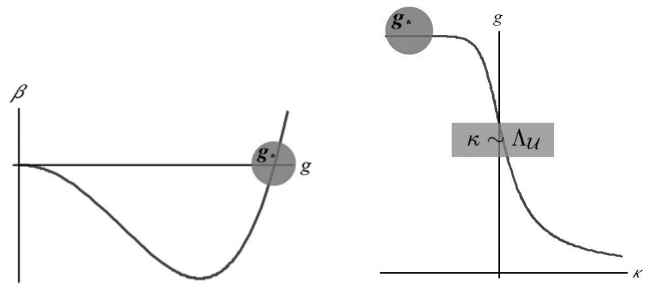

Fig. 1. Renormalization group behavior of unparticle theories.

Using then standard thermodynamic relations we find

$$\rho_{\mathcal{U}} = \sigma T^4 + A(1+3/\delta)T^{4+\delta}\,; \qquad P_{\mathcal{U}} = \sigma T^4/3 + (A/\delta)T^{4+\delta} \tag{5}$$

valid in the IR regime. In the UV limit, we will also have $\rho_{\rm NP} \propto T^4$ (up to logarithmic corrections) since the theory is asymptotically free. Then

$$\rho = \frac{3}{\pi^2} g_{\rm NP} T^4\,; \qquad g_{\rm NP} = \begin{cases} g_{\mathcal{BZ}} & \mathcal{BZ}\text{ phase} \\ g_{\mathcal{U}} & \mathcal{U}\text{ phase} \end{cases}\,, \tag{6}$$

where $g_{\rm NP}$ will be referred to as the number of Relativistic Degrees of Freedom (RDF). For $\mathcal{BZ}$ models we have $g_{\mathcal{BZ}} \sim 100$, both for the original example as well as others that have been studied using lattice Monte Carlo methods [4]. In the IR regime the RDF can be obtained using the AdS/CFT correspondence [5] which gives

$$g_{\mathcal{U}} = (\pi^5/8)(LM_{\rm Pl})^2 \gtrsim 100 \tag{7}$$

since L, the AdS radius is $> 2/M_{\rm Pl}$. It follows that for all available models $g_{\rm NP} \gtrsim 100$, and in the IR we expect this number to be even larger.

3. SM–NP interactions; equilibrium, freeze-out and thaw-in

In order to understand the NP effects on cosmic evolution it is important to determine when and if this sector was in equilibrium with the SM sector. The standard approach for investigating this makes heavy use of the Boltzmann Equation (BE) [6], for our purposes the idea is to *(i)* calculate $\dot\rho_{\rm SM}$ and $\dot\rho_{\rm NP}$ in terms of $\vartheta = T_{\rm NP} - T_{\rm SM}$ using the BE and then *(ii)* use $\rho \propto T^4$ to obtain an evolution equation of the form

$$\dot\vartheta + 4H\vartheta = -\Gamma\vartheta \tag{8}$$

together with an explicit and calculable expression for the reaction rate Γ. H denotes the Hubble parameter, $H^2 = [(8\pi/(3M_{\rm Pl}^2)](\rho_{\rm SM} + \rho_{\rm NP})$, where we assumed a flat universe with zero cosmological constant. We will assume that the SM–NP interactions are of the form

$$\mathcal{L}_{\rm int} = \epsilon \mathcal{O}_{\rm SM} \mathcal{O}_{\rm NP} \,. \tag{9}$$

The BE approach [6] requires at intermediate steps the introduction of the unparticle distribution function, which might pose conceptual problems. This can be avoided by using instead the Kubo formalism [7], though it lacks the intuitive appeal of the BE. Both formalisms yield identical expressions for $\mathit{\Gamma}$ (the momenta in the BE expression are defined in Fig. 2):

$$\begin{aligned} \mathit{\Gamma} &= \frac{\pi^2}{12T^4}\left(\frac{1}{g_{\rm SM}}+\frac{1}{g_{\rm NP}}\right)\mathcal{I}\,, \\ \mathcal{I}_{\rm Kubo} &= \epsilon^2 \Re \int_0^\beta ds \int_0^\infty dt \int d^3\boldsymbol{x} \\ &\quad \times \left\langle \mathcal{O}_{\rm SM}(-is,\boldsymbol{x})\dot{\mathcal{O}}_{\rm SM}(t,\boldsymbol{0})\right\rangle \left\langle \mathcal{O}_{\rm NP}(-is,\boldsymbol{x})\dot{\mathcal{O}}_{\rm NP}(t,\boldsymbol{0})\right\rangle \,, \\ \mathcal{I}_{\rm BE} &= \tfrac{1}{2}\sum\!\int\! d\mathit{\Phi}_{\rm NP} d\mathit{\Phi}_{\rm SM} \beta (E_{\rm SM}-E'_{\rm SM})^2 e^{-\beta E} \left|\mathcal{M}\right|^2 (2\pi)^4 \delta(K-K') \end{aligned} \tag{10}$$

and one can show $\mathcal{I}_{\rm BE} = \mathcal{I}_{\rm Kubo}$.

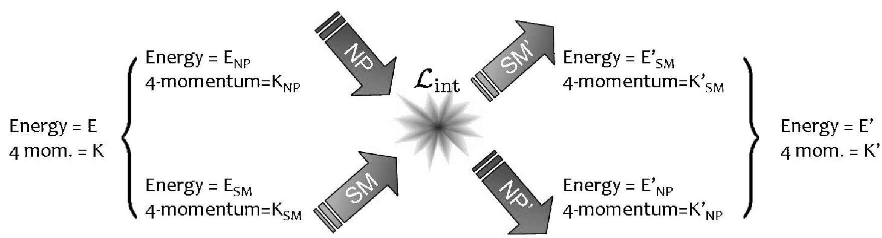

Fig. 2. Definition of momenta involved in the BE calculation of $\mathit{\Gamma}$.

For the arguments below the detailed structure of $\mathit{\Gamma}$ will not be needed and the following order of magnitude estimate suffices:

$$\mathit{\Gamma} \simeq \frac{\epsilon^2 \lambda (g_{\rm SM}+g_{\rm NP})}{(4\pi)^{n_{\rm SM}+n_{\rm NP}-1}} T^{2d_{\rm SM}+2d_{\rm NP}-7}\,, \qquad \lambda = \frac{g'_{\rm SM}}{g_{\rm SM}}\frac{g'_{\rm NP}}{g_{\rm NP}}\,, \tag{11}$$

where $n_{\rm SM}$, $n_{\rm NP}$ are the number of SM and NP fields in $\mathcal{L}_{\rm int}$, and $g'_{\rm SM}$, $g'_{\rm NP}$ are the RDF involved in the interaction (in the unparticle phase we take $n_{\rm NP} = 2d_{\mathcal{U}} - 2$, $g'_{\rm NP} = d_{\mathcal{U}}$). It then follows that

$$\frac{\mathit{\Gamma}}{H} \propto T^{2d_{\rm SM}+2d_{\rm NP}-9}\,, \tag{12}$$

where the SM and NP sectors will be (de)coupled as long as $\varGamma > H$ ($\varGamma < H$), the transition temperature $T_{\rm f}$ is determined by the condition $\varGamma = H$. Note that when $d_{\rm SM} + d_{\rm NP} < 4.5$, the sectors are coupled for T *below* $T_{\rm f}$, what we call a "thaw-in" scenario; in the complementary case $d_{\rm SM} + d_{\rm NP} > 4.5$ decoupling occurs for $T < T_{\rm f}$ (standard freeze-out scenario).

For the calculation we will use

$$\mathcal{L}_{\rm int} = \begin{cases} (\phi^\dagger\phi)\sum_Q \bar{Q}Q/M_{\mathcal{U}} & \mathcal{BZ} \\ (\phi^\dagger\phi)\mathcal{O}_{\mathcal{U}}\left(\varLambda_{\mathcal{U}}^{3-d_{\mathcal{U}}}/M_{\mathcal{U}}\right) & \mathcal{U} \end{cases}, \tag{13}$$

where Q denotes a quark in the $\mathcal{BZ}$ phase of the NP sector and $\mathcal{O}_{\mathcal{U}}$ is the unparticle operator corresponding to $\sum_Q \bar{Q}Q$. For consistency we require $M_{\mathcal{U}} > T_{\rm f} > \varLambda_{\mathcal{U}}$ in the $\mathcal{BZ}$ sector, and $\varLambda_{\mathcal{U}} > T_{\rm f} > v$ in the $\mathcal{U}$ sector (the lower value of v is needed because below this scale the SM operator $\phi^\dagger\phi$ is no longer relevant). With these preliminaries one can determine the regions in the $M_{\mathcal{U}}$–$\varLambda_{\mathcal{U}}$ for which the SM and NP are coupled in each of the NP phases, the results are presented in Fig. 3.

4. Unparticle effects in Big Bang Nucleosynthesis (BBN)

Let us first consider the case where SM and NP were in equilibrium down to a temperature $T_{\rm f} > v$, and decoupled thereafter. The relationship between the NP and SM temperatures are thereafter determined by entropy conservation [6], specifically,

$$\begin{aligned} (T_{\rm f}R_{\rm f})^3 g^\star_{\rm NP}(T_{\rm f}) &= (T_{\rm NP}R)^3 g^\star_{\rm NP}(T_{\rm NP})\,, \\ (T_{\rm f}R_{\rm f})^3 g^\star_{\rm SM}(T_{\rm f}) &= (T_\gamma R)^3 g^\star_{\rm SM}(T_\gamma)\,, \end{aligned} \tag{14}$$

where $g^\star_{\rm NP}$ and $g^\star_{\rm SM}$ stand for the NP and SM effective numbers of RDF conventionally [6] adopted for the entropy density, $R_{\rm f}$, (R) denote the scale factor at $T = T_{\rm f}$ (T_γ), and T_γ, $T_{\rm NP}$ denote the SM and NP temperatures at the BBN epoch.

Using this we find

$$T_{\rm NP} = T_\gamma \left[\frac{g_\gamma}{g_\gamma + g_e}\frac{g_\gamma + g_e + g_\nu}{g_{\rm SM}(v)}\right]^{1/3}, \tag{15}$$

where $g_\gamma = 2$, $g_e = (7/8)\times 4$, $g_\nu = (7/8)\times 3\times 2$, $g_{\rm SM}(v) = 106.75$. The NP contribution to the total energy density can be expressed in terms of an additional number of sterile neutrinos ΔN_ν defined by the expression

$$\rho_{\rm NP} = \frac{3}{\pi^2} g_{\rm IR} T^4_{\rm NP} \equiv \frac{3}{\pi^2}\frac{7}{4}\left(\frac{4}{11}\right)^{4/3}\Delta N_\nu\, T^4_\gamma\,. \tag{16}$$

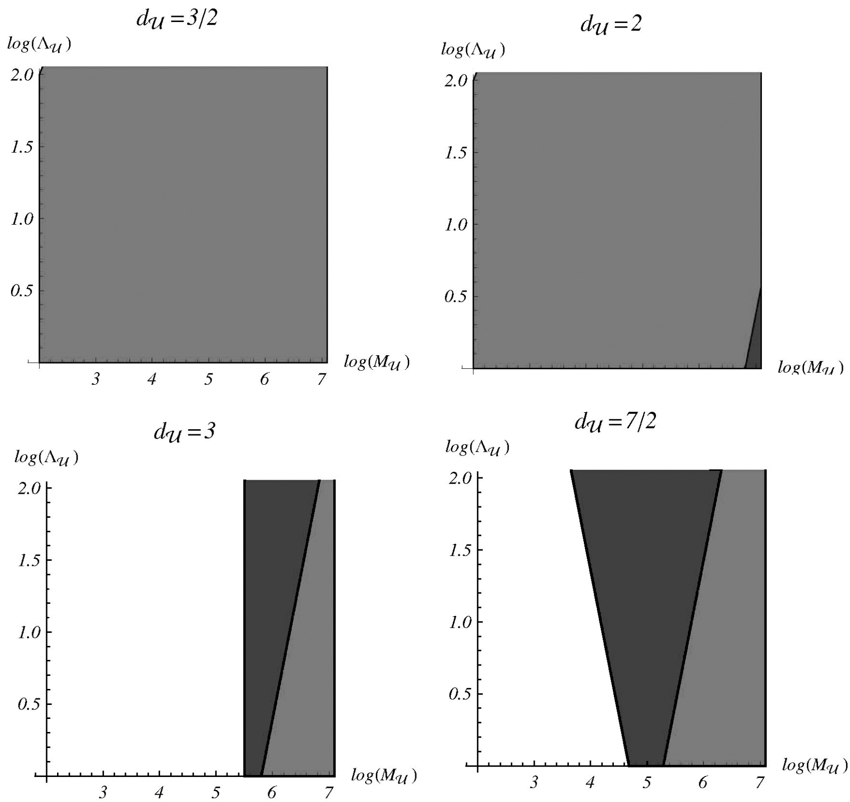

Fig. 3. Regions in the $\Lambda_{\mathcal{U}}$–$M_{\mathcal{U}}$ plane corresponding to various freeze-out and thaw-in scenarios for $d_{\mathcal{U}} = 3/2,\ 2,\ 3,\ 7/2$. Dark grey: SM–NP decoupling in the unparticle phase only; light gray: no SM–NP decoupling; in the white regions $T_{\mathcal{U}\text{-f}} < v$ ($\Lambda_{\mathcal{U}}$, $M_{\mathcal{U}}$ are in TeV units). We assumed $g_{\rm SM} = g_{\mathcal{BZ}} = g_{\mathcal{U}} = 100$, $g'_{\rm SM} = 4$, $g'_{\mathcal{BZ}} = 50$ and $g'_{\mathcal{U}} = d_{\mathcal{U}}$. $\mathcal{BZ}$ phase: $n_{\rm SM} = n_{\rm NP} = 2$, $d_{\rm SM} = 2$ and $d_{\rm NP} = 3$; $\mathcal{U}$ phase: $n_{\rm SM} = 2$, $n_{\rm NP} = 2(d_{\mathcal{U}} - 1)$, $d_{\rm SM} = 2$ and $d_{\rm NP} = d_{\mathcal{U}}$.

Then,

$$g_{\rm IR} = \frac{7}{4}\left[\frac{g_{SM}(v)}{g_\gamma + g_e + g_\nu}\right]^{4/3}\Delta N_\nu\,. \tag{17}$$

Current data [8] provides the limits $\Delta N_\nu = 0 \pm 0.3 \pm 0.3$, leading to

$$g_{\rm IR} < 20 \tag{18}$$

at the 95% C.L. In the extreme case where the SM and NP remain in equilibrium at the BBN epoch this bound is considerably strengthened: $g_{\rm IR} < 0.3$.

Viable unparticle models should exhibit conformal invariance with a small number of RDF in the IR. We are unaware of any model with these characteristics. In fact the AdS/CFT correspondence suggests this constraint is very strongly violated.

5. Comments

We have shown that strongly coupled NP can lead to $\Gamma/H \sim T^n$ with n positive or negative, and that this results in a variety o freeze-out and thaw-in scenarios. Current BBN data generates strong constraints on the properties of the NP. Even for "normal" decoupling scenarios ($n > 0$) the BBN constraint is significant leading to a bound $g_{\rm IR} < 20$ to be compared to $g_{\rm IR} > 100$ for the available models.

Unparticle models also suffer from potential theoretical problems: the coupling to the SM necessarily breaks conformal invariance, but the scale at which this occurs lies below the BBN temperature for a range of $d_{\mathcal{U}}$, so our conclusions still apply in this case.

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