RADIATIVE RETURN: A PROGRESS ON FSR TESTS*

Sergiy Ivashyn[a,b], Henryk Czyż[a], Alexandr Korchin[b]

[a]Institute of Physics, University of Silesia
Uniwersytecka 4, Katowice 40-007, Poland
[b]ITP, NSC "Kharkov Institute for Physics and Technology"
Akademicheskaya str., Kharkov 61108, Ukraine

(Received October 29, 2009)

To improve the accuracy of the $e^+e^- \to \pi^+\pi^-\gamma$ Radiative Return Method, one has to control the theoretical uncertainty of the final-state photon emission. It is of particular importance at DAPHNE for the analysis, where cuts are relaxed to cover the threshold region. By means of Monte Carlo generator PHOKHARA we compare several final-state radiation models and present results, relevant for a meson factory running at $\sqrt{s} = 1$ GeV.

PACS numbers: 14.40.Be, 13.40.Ks, 13.60.Le, 13.66.Bc

1. Introduction

The Radiative Return Method [1–3] (RRM) allows an extraction of the hadronic cross-section $\sigma^{\rm had}(Q^2)$ for hadronic invariant mass squared Q^2 from the energy threshold up to the nominal energy of the experiment at the fixed beam energy e^+e^- colliders. High-luminosity meson factories are especially suited for this purpose [4]. Interest in *precise* measurement of $\sigma^{\rm had}(Q^2)$ is motivated, in part, by its relevance to the hadronic contribution to the muon anomalous magnetic moment $a_\mu^{\rm had}$ [5, 6] and the electromagnetic fine structure constant [7]. However, the method can also be applied to extract the meson form factors and other meson properties.

The RRM uses $d\sigma(e^+e^- \to \text{hadrons} + \text{photons})/dQ^2$, a measured differential cross-section, for the extraction of $d\sigma(e^+e^- \to \text{hadrons})$. For the theoretical description, the perturbative QED diagrams at the leading order in QED coupling α (LO) and at the next-to-leading order (NLO) are considered and classified as initial-state radiation (ISR) or final-state radiation (FSR)

* Presented by S. Ivashyn at the XXXIII International Conference of Theoretical Physics, "Matter to the Deepest", Ustroń, Poland, September 11–16, 2009 and at the Topical Workshop: "Low energy constraints on extensions of the Standard Model", Kazimierz, Poland, July 23–27, 2009.

ones. The kinematic cuts are applied to sufficiently suppress the FSR, whenever possible, as the factorization $d\sigma(s,Q^2)\big|_{\rm ISR} = \mathcal{R}(s,Q^2) \times d\sigma_{\rm had}(Q^2)$, which allows for $d\sigma^{\rm had}(Q^2)$ extraction, holds for diagrams with ISR photons only. The function $\mathcal{R}(s,Q^2)$ is given by QED. The FSR part is model-dependent, thus dedicated numerical studies are needed for correct ISR–FSR separation. The Monte Carlo generator PHOKHARA was developed for these and related purposes: FSR at NLO has been included [8] for pion pair production and, in addition to scalar QED (sQED), some particular ingredients (the ϕ radiative decay) were implemented [9]. The FSR was also examined by other Monte Carlo programs, *e.g.*, that with the Resonance Chiral Theory (RχT) motivated framework [10] and phenomenologically-oriented model [11], which was also included into PHOKHARA 6.1 [12].

The reaction $e^+e^- \to \pi^+\pi^-\gamma$ was explored by KLOE [13]: the cross-section $d\sigma^{\rm had}/dQ^2$ and pion form factor $F_\pi(Q^2)$ in the range $0.35\ {\rm GeV}^2 < Q^2 < 0.95\ {\rm GeV}^2$ were extracted [14] from the on-peak ($\sqrt{s}=M_\phi = 1.02$ GeV) data sample by means of the RRM. However, the kinematic cuts, which were applied in order to suppress FSR, did not allow to measure at Q^2 below $0.35\ {\rm GeV}^2$.

One can measure the $F_\pi(Q^2)$ in the threshold region relaxing some of the cuts, but then one has to *subtract the FSR contribution*. In this scenario, one needs to control the description of the final-state emission process and detailed studies are needed to estimate the theoretical uncertainty. To simplify the analysis, it is better to perform the measurement off the ϕ meson peak, because in this case the contributions from the ϕ meson radiative decays are small and the FSR models can be controlled easier.

The investigations presented here are of particular importance for the forthcoming KLOE RRM analysis of pion pair production. We focus on the off-ϕ-peak measurement, at e^+e^- center-of-mass energy $\sqrt{s} = 1$ GeV, for which KLOE collected 230 pb^{-1} of data [15]. Due to the interest in precision at small Q^2 (*i.e.*, below the ρ resonance), the Chiral Perturbation Theory (χPT) [16] can be helpful. The relevant theoretical aspects are sketched in Section 2. We use Monte Carlo generator PHOKHARA to compare several final-sate radiation models. The theoretical heritage of Virtual Compton Scattering (VCS) off the pion in χPT framework [17,18] is used to estimate the rôle of higher order χPT effects.

The numerical results for cross-section and asymmetry are presented in Section 3. All the parameters of implemented models are fixed independently, thus one deals with model *predictions*. In Section 4 we present our conclusions.

2. Theoretical issues of final-state radiation

The transition $\gamma^* \to \pi^+\pi^-\gamma$ is described by the model-dependent FSR tensor $M^{\mu\nu}$[1]. In all realistic models it contains the same Born-level contribution $M^{\mu\nu}_{\rm Born}$, which corresponds to a no-structure approximation for pion (scalar QED or lowest-order χPT). Thus we consider $M^{\mu\nu}_{\rm Born}$ as a model-independent part.

The first correction accounts for the pion structure by means of the pion form factor. It replaces the Born-level amplitude by "Generalized Born" (GB) one [17], $M^{\mu\nu}_{\rm GB}$, which is also called "sQED*VMD" [8]. Generalized Born FSR tensor reads

$$M^{\mu\nu}_{\rm GB} = -ie^2 F_\pi\left(P^2\right)\left(\frac{(k+q_1-q_2)^\mu q_1^\nu}{q_1\cdot k}+\frac{(k+q_2-q_1)^\mu q_2^\nu}{q_2\cdot k}-2\,g^{\mu\nu}\right), \quad (1)$$

where P and k are the virtual and real photon momenta, q_1 and q_2 — pion momenta ($Q = q_1 + q_2$).

Limit $F_\pi(P^2) \to 1$ reproduces the $M^{\mu\nu}_{\rm Born}$ amplitude. Notice, that in $e^+e^- \to \pi^+\pi^-\gamma$ at LO, $P^2 = s$, thus the form factor $F_\pi(P^2) \neq 1$ and its correction is never negligible. Also this part is well established both theoretically and experimentally.

In the ISR amplitude, with $\gamma^* \to \pi^+\pi^-$ transition in the final state, one finds $F_\pi(Q^2)$ factor in the amplitude. Therefore, the $\pi^+\pi^-$ invariant mass distribution is governed by the form factor shape. For consistency, one has to use the same expression for the pion form factor in the ISR and FSR amplitudes. It is important to take the form factor tested experimentally and not to rely only on a particular model assumptions. This will be illustrated in the next section. In order to understand the accuracy of $M^{\mu\nu}_{\rm GB}$ approximation, we study further corrections using the models of Refs [11, 17, 18].

The first model, "VMD*χPT", is based on $\mathcal{O}(p^4)$ χPT SU(2) description of VCS $\gamma^*\pi^\pm \to \gamma\pi^\pm$ [17] and that in SU(3) case [18]. The FSR tensor has the form $M^{\mu\nu} = M^{\mu\nu}_{\rm GB} + M^{\mu\nu}_{\rm NB}$. The first term is given by Eq. (1) and a straightforward improvement beyond χPT is supposed (denoted by prefix "VMD*"): the pion form factor F_π is an external input (*e.g.*, defined by parametrization of the measured F_π). The second term, *the Non-Born correction* to FSR reads: $M^{\mu\nu}_{\rm NB} = -ie^2\,(k^\mu Q^\nu - g^{\mu\nu}\,k\cdot Q)\,f_1^{\rm NB}$, where

$$f_1^{\rm NB} = \frac{-1}{16\,\pi^2\,F^2}\left(\frac{2}{3}\left(\bar{l}_6-\bar{l}_5\right)+\frac{P^2-2\,P\cdot k}{P\cdot k}\times\mathcal{G}_\pi\right), \quad (2)$$

$$f_1^{\rm NB} = \frac{-1}{16\,\pi^2\,F_0^2}\left(128\pi^2\left(L_9^r+L_{10}^r\right)+\frac{P^2-2\,P\cdot k}{P\cdot k}\times\left(\mathcal{G}_\pi+\frac{1}{2}\mathcal{G}_K\right)\right) \quad (3)$$

[1] For example, the matrix element of $e^+e^- \to \gamma^* \to \pi^+\pi^-\gamma$ reads: $M^{\rm (LO)}_{\rm FSR} = s^{-1}e\,\bar{v}\,\gamma_\mu u\,\epsilon^*_\nu\,M^{\mu\nu}$, where $e = \sqrt{4\pi\alpha}$.

in SU(2) and SU(3) framework, correspondingly; see original papers [17,18] for the explicit form of the loop functions $\mathcal{G}_\pi$ and $\mathcal{G}_K$. Numerical values of the low energy constants are $F = 92.4$ MeV, $(\bar{l}_6 - \bar{l}_5) = 3.0 \pm 0.3$ and $F_0 = 87.7$ MeV, as cited in [21], and $(L_9^r + L_{10}^r) = (1.32 \pm 0.14) \times 10^{-3}$ at scale $\mu = M_\rho$, as estimated in [22].

The second model [11], called the "main model" further in the text, can be considered as *a parametrization* of $\pi^0\pi^0\gamma$ KLOE data, transformed to $\pi^+\pi^-\gamma$ via isospin symmetry [23]. It was implemented in FASTERD Monte Carlo generator [11] and in PHOKHARA 6.1 recently [12]. The FSR tensor contains $M_{\mathrm{GB}}^{\mu\nu}$ given by Eq. (1) and the Non-Born corrections due to important vector-resonance and double-vector-resonance contributions.

3. Numerical results

We use Monte Carlo generator PHOKHARA to compare the model-dependent effects in $e^+e^- \to \pi^+\pi^-\gamma$ cross-section and asymmetry for the off-peak case at ϕ-meson factory, $\sqrt{s} = 1$ GeV.

First of all, we stress that any simplification of the pion form factor F_π can drastically affect the model results. Figure 1 shows that rigorous $\mathcal{O}(p^4)$ χPT form factor [16] gives completely wrong estimate for differential cross-section even in the region of Q^2 below the ρ meson peak. The theoretical explanation of the form factor rôle was given above. The form factor used in VMD*χPT and "main model" is the parametrization of available data given by Gounaris–Sakurai version of Ref. [20].

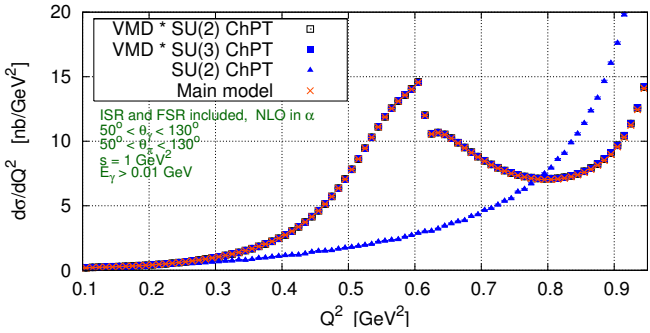


Fig. 1. Differential cross-section for $e^+e^- \to \pi^+\pi^-\gamma$. All the listed models, but the one with the $\mathcal{O}(p^4)$ χPT form factor, give very close predictions (overlapping curves in the plot).

In Fig. 1, one can see the very close cross-section predictions, despite the fact, that the models have *completely different* Non-Born corrections. This is due to the fact that the GB contribution dominate for the given event

selection. Taking the GB approximation, Eq. (1), as a reference, we plot $(d\sigma[\text{model}]-d\sigma[\text{GB}])/d\sigma[\text{GB}]$. To show the relative contribution of loop and "constant" terms in χPT we consider also the case of $(\bar{l}_6-\bar{l}_5)$ and $(L_9^{\text{r}}+L_{10}^{\text{r}})$ being artificially set to zero. Corresponding results are marked as "loop only" in the pictures. Figure 2 shows that the Non-Born corrections are at a few per cent level. From Fig. 3 one concludes that the FSR contribution to the cross-section is significant in the whole range of Q^2, especially at low Q^2.

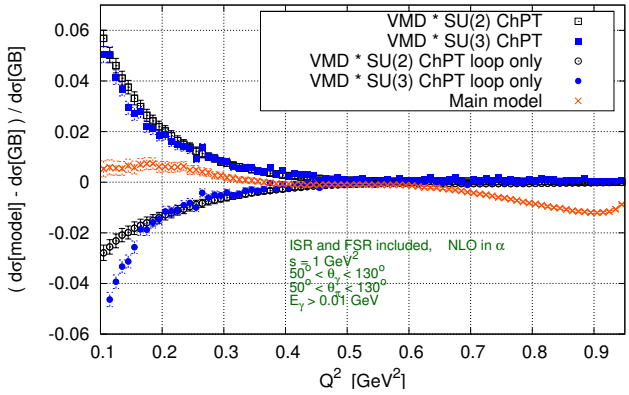


Fig. 2. Non-Born corrections to $d\sigma/dQ^2$. Note, that at NLO the interference is neglected. At $Q^2 > 0.5$ GeV^2 all the curves but "main model" overlap.

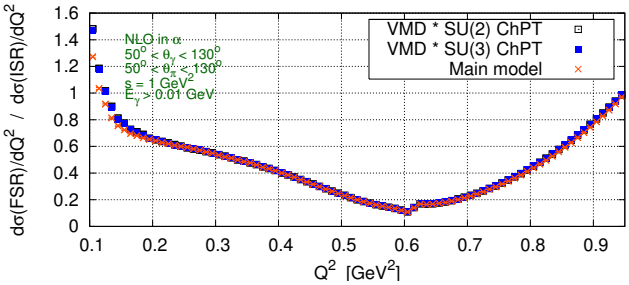


Fig. 3. The rôle of FSR in $d\sigma/dQ^2$. All the listed models give very close predictions (overlapping curves in the plot). Note, that at NLO the interference is neglected.

Pion forward–backward asymmetry (FBA) as a function of Q^2 reads

$$A_{\text{FB}}(Q^2) = \frac{N(\theta_{\pi^+} > 90^\circ) - N(\theta_{\pi^+} < 90^\circ)}{N(\theta_{\pi^+} > 90^\circ) + N(\theta_{\pi^+} < 90^\circ)}\,(Q^2) \tag{4}$$

in terms of numbers of events. Origin of the non-zero FBA is the interference of C-odd and C-even amplitudes, *e.g.*, that of ISR and FSR at LO. Thus,

FBA is sensitive to the relative phase, which may differ among the models even if they predict the same cross-section. Notice, that the experimental data on asymmetry and cross-section are to large extent independent. Therefore the FBA is a good test for models. Aspects of using the FBA in $e^+e^- \to \pi^+\pi^-\gamma$ were discussed in [3, 8–10, 19].

Figure 4 shows that FBA is sizable and relatively easy measurable. From Fig. 5 we conclude that the Non-Born corrections to the FBA are of few per cent order and will not have a big influence on the theoretical uncertainty.

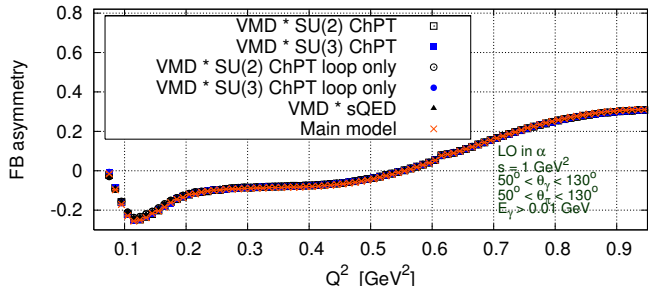


Fig. 4. Forward–backward asymmetry for pion. All the listed models give very close predictions (overlapping curves in the plot).

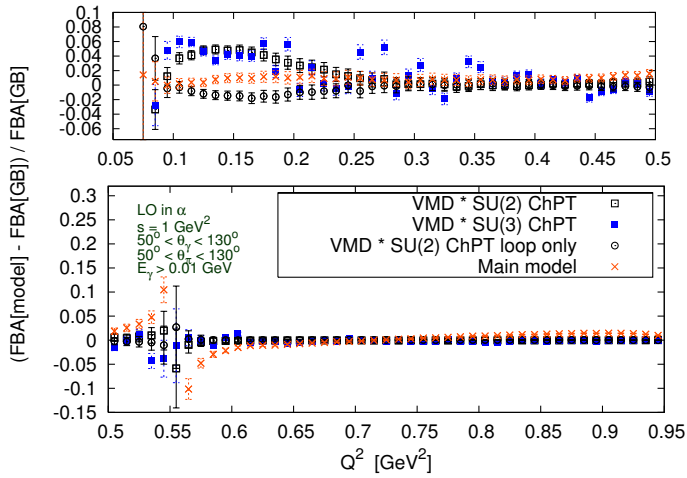


Fig. 5. Relative non-born corrections to the forward–backward asymmetry for pion.

It has to be stressed that if the χPT corrections were not accounted for in the formulae used to measure the $|F_\pi|$, they are partly accounted for in the experimental parameters of F_π and other model parameters. In other words, one model should be used in all experimental analyzes and adding *ad hoc* additional corrections is not appropriate.

4. Conclusions

Using PHOKHARA, we studied the corrections given by χPT [17, 18], and by a phenomenological model including miscellaneous hadronic resonance effects [11]. Corrections due to a_1 resonance [19] are to be considered elsewhere. The rôle of the pion form factor is seen to be very important. Final-state radiation is significant in the whole range of Q^2, especially at low Q^2. We have found the NLO corrections to be non-negligible, even if the Generalized Born contribution is dominant. Non-Born corrections are of the order of few per cent. They differ among the models, but it will be difficult to distinguish them with the present KLOE off-peak statistics. The results presented here show that one should include the χPT corrections in the analysis when the accuracy of the experiment reaches a per cent level.

We would like to thank Achim Denig, Stefan Scherer, Stefan Müller and Roman Zwicky for discussions. Partial support from MRTN-CT-2006-035482 "FLAVIAnet", MRTN-CT-2006-035505 "HEPTOOLS" and Contract No. 227431 "TARI" is acknowledged.

REFERENCES

[1] M.S. Chen, P.M. Zerwas, *Phys. Rev.* **D11**, 58 (1975).

[2] A.B. Arbuzov, E.A. Kuraev, N.P. Merenkov, L. Trentadue, *J. High Energy Phys.* **9812**, 009 (1998).

[3] S. Binner, J.H. Kühn, K. Melnikov, *Phys. Lett.* **B459**, 279 (1999).

[4] A. Grzelińska, H. Czyż, A. Wapienik, *Nucl. Phys. Proc. Suppl.* **189**, 216 (2009).

[5] F. Jegerlehner, A. Nyffeler, *Phys. Rep.* **477**, 1 (2009).

[6] J. Prades, `arXiv:0909.2546 [hep-ph]`.

[7] F. Jegerlehner, *Nucl. Phys. Proc. Suppl.* **181–182**, 135 (2008).

[8] H. Czyż, A. Grzelińska, J.H. Kühn, G. Rodrigo, *Eur. Phys. J.* **C33**, 333 (2004).

[9] H. Czyż, A. Grzelińska, J.H. Kühn, *Phys. Lett.* **B611**, 116 (2005).

[10] G. Pancheri, O. Shekhovtsova, G. Venanzoni, *Phys. Lett.* **B642**, 342 (2006).

[11] O. Shekhovtsova, G. Venanzoni, G. Pancheri, *Comput. Phys. Commun.* **180**, 1206 (2009).

[12] PHOKHARA 6.1, http://ific.uv.es/~rodrigo/phokhara/

[13] A. Aloisio *et al.* [KLOE Collaboration], *Phys. Lett.* **B606**, 12 (2005).

[14] F. Ambrosino *et al.* [KLOE Collaboration], *Phys. Lett.* **B670**, 285 (2009).

[15] P. Beltrame, "Measurement of the Pion Form Factor via Radiative Return for Data Taken at $\sqrt{s} = 1$ GeV with the KLOE Detector", PhD thesis, IEKP-KA/2009-8 (2009).

[16] J. Gasser, H. Leutwyler, *Ann. Phys.* **158**, 142 (1984).

[17] C. Unkmeir, S. Scherer, A.I. L'vov, D. Drechsel, *Phys. Rev.* **D61**, 034002 (2000).

[18] T. Fuchs, B. Pasquini, C. Unkmeir, S. Scherer, *Czech. J. Phys.* **52**, B135 (2002).

[19] S. Dubinsky, A. Korchin, N. Merenkov, G. Pancheri, O. Shekhovtsova, *Eur. Phys. J.* **C40**, 41 (2005).

[20] C. Bruch, A. Khodjamirian, J. H. Kühn, *Eur. Phys. J.* **C39**, 41 (2005).

[21] J. Bijnens, arXiv:0904.3713 [hep-ph].

[22] R. Unterdorfer, H. Pichl, *Eur. Phys. J.* **C55**, 273 (2008).

[23] O. Shekhovtsova, private communication.