Vol. 40 (2009) ACTA PHYSICA POLONICA B No 2

FERMIONS TUNNELING FROM KERR AND KERR–NEWMAN BLACK HOLES

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(Received October 13, 2008)

Black holes radiate not only the scalar particles but also Dirac particles. Extending Kerner and Mann's work (*Class. Quantum Grav.* **25**, 095014 (2008)) to the rotating and charged rotating black holes, we investigate the Hawking radiation of fermions for the Kerr black hole and Kerr–Newman black hole. The Hawking temperatures are recovered when the electromagnetic field effect and rotation effect are taken into account and are same as that computed by other methods.

PACS numbers: 04.70.Dy, 04.62.+v, 11.30.–j

1. Introduction

Ever since Stephen Hawking had proved the black hole radiates thermally, there were several methods presented to derive the Hawking radiation [1–6]. In these methods, the semi-classical tunneling method put forward by Kraus and Wilczek [7,8] and developed by Parikh and Wilczek [9–11] attracted many people's attention [12–16]. In this method, its derivation of the Hawking radiation mainly consists in the computation of the action of the classically forbidden trajectory by considering the unfixed back ground space-time of black holes and particles' self-gravitational interaction. When Hawking radiation was first proved, it was described as a quantum tunneling process triggered by vacuum fluctuations near the horizon. Now that it is a tunneling process, one should find the tunneling potential barrier. In semi-classical tunneling mode, they pointed out the potential was afforded by the radiation particle self, thus the produce mechanism of tunneling potential is overcome. Furthermore the actual radiation spectrum includes not only leading term but also correction term, which makes a correction to the purely thermal one.

In 2005, one ansatz to investigate Hawking radiation by the Hamilton–Jacobi equation to derive the action of radiation particle was put forward by Angheben *et al.* [17]. In fact, this approach is the extension of Padmanabhan's work [18, 19]. In this ansatz, they neglected the back reaction of black holes' geometry and self-gravitational interaction of radiation particles, and therefore derived the leading term of Hawking radiation spectrum. The virtue of this treatment is that the covariant treatment of the horizon singularity through the use of spatial proper distance. Applying this ansatz, people studied the Hawking radiation of diversified space-time and made a great deal of successes [20–23].

However, all of these studies were limited to scalar particle case. As we know, the black hole radiates not only scalar particles but also Dirac particles. Based on this, Kerner and Mann [24] studied the Hawking radiation of spin $1/2$ particles for Rindler space-time and that of the spherically symmetric uncharged black hole by resolving the Dirac equation. However, the case of fermions for the charged black hole and (charged) rotating black hole has not been studied. The difficulty consists in how to consider the effect of electromagnetic field and the rotating effect. Our work in this paper is to extend Kerner and Mann's work to the rotating black hole and charged rotating black hole and investigate the Hawking radiation of fermions by considering the electromagnetic field effect and rotation effect.

The remainders of this paper are outlined as follows. In Sec. 2, taking the rotation effect into account, we investigate the Hawking radiation of fermions of the rotating (Kerr) black hole and recover the Hawking temperature. Extending this work to the charged rotating space-time, the case of the Kerr–Newman black hole is studied by considering rotation effect and electromagnetic effect in Sec. 3. Sec. 4 contains some discussion and conclusion.

2. Hawking radiation from Kerr black hole

In this section, we take the Kerr black hole as model to investigate the Hawking radiation of fermions for rotating black holes. The metric of the Kerr black hole is given by

$$\begin{aligned} ds^2 \;=\; & -\left(1-\frac{2Mr}{\rho^2}\right)dt^2+\frac{\rho^2}{\varDelta}dr^2+\rho^2 d\theta^2 \\ & +\left[\left(r^2+a^2\right)+\frac{2Mra^2\sin^2\theta}{\rho^2}\right]\sin^2\theta d\varphi^2-\frac{4Mra\sin^2\theta}{\rho^2}dtd\varphi\,, \end{aligned} \quad (1)$$

where $\rho^2 = r^2 + a^2\cos^2\theta$, $\Delta = r^2 - 2Mr + a^2 = (r - r_h)(r - r_-)$, M is the physic mass of the black hole, and $r_\mathrm{h} = M + \sqrt{M^2 - a^2}$ ($r_- = M - \sqrt{M^2 - a^2}$) are the outer (inner) horizons. From the metric (1), we find the outer (inner) horizons obtained from $g^{\mu\nu}\partial_\mu f \partial_\nu f = 0$ and the outer (inner) infinite red-shift surfaces obtained from $g_{00} = 0$ are not coincident with each others, which is not convenient for us to view the Hawking radiation of fermions; thereby we should choose one coordinate system to let them be coincident. There are two treatments to approach it: one is treating it in the dragging coordinate system (t, r, θ) and letting $d\varphi = -\frac{g_{03}}{g_{33}}dt$; Another is to introduce a new coordinate system (t, r, θ, χ). We choose the latter one and introduce the new coordinate as $\chi = \varphi - \Omega t$ (where $\Omega = \frac{2Mra}{(r^2+a^2)^2 - \Delta a^2\sin^2\theta}$), on the metric (1) and get

$$\begin{aligned} ds^2 &= -\frac{\Delta\rho^2}{\left(r^2+a^2\right)^2 - \Delta a^2\sin^2\theta}dt^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 \\ &\quad + \frac{\left(r^2+a^2\right)^2 - \Delta a^2\sin^2\theta}{\rho^2}\sin^2\theta d\chi^2 \,. \end{aligned} \tag{2}$$

In this metric, the outer (inner) horizons coincide with the outer (inner) infinite red-shift surfaces. Moreover, the metrics satisfy Landau's condition of the coordinate clock synchronization, which is helpful to investigate the radiation of fermions for the black hole. The Dirac equation in the curved space-time is

$$i\gamma^\mu\left(\partial_\mu + \Omega_\mu\right)\Psi + \frac{m}{\hbar}\Psi = 0\,, \tag{3}$$

where $\Omega_\mu = \frac{i}{2}\Gamma_\mu^{\alpha\beta}\Sigma_{\alpha\beta}$, $\Sigma_{\alpha\beta} = \frac{i}{4}[\gamma^\alpha, \gamma^\beta]$ and γ^μ matrices satisfy $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$. For the convenience of the computation, we let

$$\begin{aligned} F(r) &= \frac{\Delta\rho^2}{\left(r^2+a^2\right)^2 - \Delta a^2\sin^2\theta}\,, \qquad G(r) = \frac{\Delta}{\rho^2}\,, \\ H^2(r) &= \frac{\left(r^2+a^2\right)^2 - \Delta a^2\sin^2\theta}{\rho^2}\sin^2\theta\,, \qquad K^2(r) = \rho^2\,, \end{aligned} \tag{4}$$

and choose the γ^μ matrices as

$$\begin{aligned} \gamma^t &= \frac{1}{\sqrt{F(r)}}\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \qquad \gamma^r = \sqrt{G(r)}\begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix}, \\ \gamma^\theta &= \frac{1}{K(r)}\begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix}, \qquad \gamma^\chi = \frac{1}{H(r)}\begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}. \end{aligned} \tag{5}$$

In the Dirac field, there are two cases which correspond to the particle with spin up and spin down, therefore two wave functions are corresponded respectively and they can be expressed as

$$\Psi_{(\uparrow)} = \begin{pmatrix} A \\ 0 \\ B \\ 0 \end{pmatrix} \exp\left(\frac{i}{\hbar} I_{\uparrow}\right), \qquad \Psi_{(\downarrow)} = \begin{pmatrix} 0 \\ C \\ 0 \\ D \end{pmatrix} \exp\left(\frac{i}{\hbar} I_{\downarrow}\right). \tag{6}$$

In which the actions $I_{\uparrow}$ and $I_{\downarrow}$ are the functions of t, r, θ, χ. In the investigation of Hawking radiation, the key is to compute the action. Considering the number of Dirac particles with spin up is statistically equal to that of spin down case, we only explore the spin up case. To obtain the action, inserting the wave function $\Psi_{(\uparrow)}$ and γ^{μ} matrices into the Dirac equation and yielding

$$-\left(\frac{iA}{\sqrt{F(r)}}\partial_t I_{\uparrow} + B\sqrt{G(r)}\partial_r I_{\uparrow}\right) + mA = 0, \tag{7}$$

$$\left(\frac{iB}{\sqrt{F(r)}}\partial_t I_{\uparrow} - A\sqrt{G(r)}\partial_r I_{\uparrow}\right) + mB = 0, \tag{8}$$

$$-B\left(\frac{1}{K(r)}\partial_\theta I_{\uparrow} + \frac{i}{H(r)}\partial_\chi I_{\uparrow}\right) = 0, \tag{9}$$

$$-A\left(\frac{1}{K(r)}\partial_\theta I_{\uparrow} + \frac{i}{H(r)}\partial_\chi I_{\uparrow}\right) = 0. \tag{10}$$

It is difficult to get the value of the action directly. From the above four equations, we can find the action can be carried out in separation of variables. Considering the properties of the Kerr space-time, we carry out separation of variables as

$$I_{\uparrow} = -\omega t + W(r) + j\varphi + \Theta(\theta) \tag{11}$$

in which ω and j are the energy and magnetic quantum number of the particle, respectively. Although there are four equations, our concern are the first two for that the imaginary part of the action is produced from here. Inserting Eq. (11) into (7) and (8), we have

$$-\left(\frac{iA}{\sqrt{F(r)}}(-\omega + j\Omega) + B\sqrt{G(r)}\partial_r W(r)\right) + mA = 0, \tag{12}$$

$$\left(\frac{iB}{\sqrt{F(r)}}(-\omega + j\Omega) - A\sqrt{G(r)}\partial_r W(r)\right) + mB = 0. \tag{13}$$

When $m = 0$, it means this is investigation of the Hawking radiation of massless particles and Eqs. (12) and (13) decouple. While $m \neq 0$, this is the Hawking radiation of massive particle and the equations couple. Solving $W(r)$ yields

$$W_{\pm}(r) = \pm i\pi \frac{\omega - j\Omega_{\rm h}}{\sqrt{F'(r_{\rm h})G'(r_{\rm h})}}\,, \tag{14}$$

where $\sqrt{F'(r_{\rm h})G'(r_{\rm h})} = \frac{2(r_{\rm h}-M)}{r_{\rm h}^2+a^2}$, $\pm$ correspond to the outgoing/ingoing solutions of the action and $\Omega_{\rm h} = \frac{a}{r_{\rm h}^2+a^2}$ is the angular velocity at the outer horizon. Substituting the result (14) into the separation of variables, we can get the imaginary part of the actions as

$$\mathrm{Im}I_{\pm} = \pm i\pi \frac{r_{\rm h}^2 + a^2}{2\left(r_{\rm h} - M\right)}\left(\omega - j\Omega_{\rm h}\right)\,. \tag{15}$$

So the tunneling probability of the fermions is

$$\begin{aligned} \Gamma &= \frac{P(\text{emission})}{P(\text{absorption})} = \frac{\exp(-2\mathrm{Im}I_{+})}{\exp(-2\mathrm{Im}I_{-})} = \exp\left(-\frac{4\pi(\omega - j\Omega_{\rm h})}{\sqrt{F'(r_{\rm h})G'(r_{\rm h})}}\right) \\ &= \exp\left(-2\pi\frac{r_{\rm h}^2 + a^2}{r_{\rm h} - M}(\omega - j\Omega_{\rm h})\right)\,. \end{aligned} \tag{16}$$

This is the Hawking radiation spectrum of fermions for the Kerr black hole. Thus the Hawking temperature of the Kerr black hole is recovered as

$$T = \frac{\sqrt{F'(r_{\rm h})G'(r_{\rm h})}}{4\pi} = \frac{1}{2\pi}\frac{r_{\rm h} + M}{r_{\rm h}^2 + a^2}\,, \tag{17}$$

which is in full accordance with that obtained by the other method. This implies when black hole radiates massless particle and massive particle, their tunneling probability and Hawking temperature are the same and are not related to the kind of particles. For the spin down case, choosing the corresponding (spin down) wave function and investigating it again as the similar process, we can get the same result. It is not discussed here.

3. Hawking radiation from Kerr–Newman black hole

In this section, we focus our attention on the charged rotating black hole and investigate the Hawking radiation of fermions in the electromagnetic field. Replacing Δ in the metric (1) with $\hat{\Delta} = r^2 - 2Mr + a^2 + Q^2 =$

$(r-r_{\rm h})(r-r_-)$, the Kerr–Newman metric black hole can be given as

$$ds^2 = -\left(1-\frac{2Mr-Q^2}{\rho^2}\right)dt^2+\frac{\rho^2}{\hat{\Delta}}dr^2+\rho^2 d\theta^2-\frac{2(2Mr-Q^2)a\sin^2\theta}{\rho^2}dtd\varphi$$
$$+\left[\left(r^2+a^2\right)+\frac{\left(2Mr-Q^2\right)a^2\sin^2\theta}{\rho^2}\right]\sin^2\theta d\varphi^2\,, \qquad (18)$$

with the electromagnetic potential

$$\hat{A}_\mu=\hat{A}_t dt+\hat{A}_\varphi d\varphi=\frac{Qr}{r^2+a^2\cos^2\theta}dt-\frac{Qra\sin^2\theta}{r^2+a^2\cos^2\theta}d\varphi\,, \qquad (19)$$

where Q is the charge of the black hole, and the outer (inner) horizons $r_{\rm h}=M+\sqrt{M^2-Q^2-a^2}$ $(r_-=M-\sqrt{M^2-Q^2-a^2})$ satisfy $\hat{\Delta}=0$. Following the method given in Sec. 2, we introduce the similar coordinate transformation $\chi=\varphi-\Omega t$ (where $\Omega=\frac{(2Mr-Q^2)a}{(r^2+a^2)^2-\hat{\Delta}a^2\sin^2\theta}$) and get the metric as

$$ds^2 = \frac{\hat{\Delta}\rho^2}{\left(r^2+a^2\right)^2-\hat{\Delta}a^2\sin^2\theta}dt^2$$
$$+\frac{\rho^2}{\hat{\Delta}}dr^2+\rho^2 d\theta^2+\frac{\left(r^2+a^2\right)^2-\hat{\Delta}a^2\sin^2\theta}{\rho^2}\sin^2\theta d\chi^2\,, \qquad (20)$$

with the corresponding potential

$$A_\mu=A_t dt+A_\chi d\chi=\frac{Qr\left(r^2+a^2\right)}{\left(r^2+a^2\right)^2-\hat{\Delta}a\sin^2\theta}dt-\frac{Qra\sin^2\theta}{r^2+a^2\cos^2\theta}d\chi\,. \qquad (21)$$

The metrics (20) and (2) have the similar forms and therefore they have similar properties. Namely the outer (inner) horizons and outer (inner) infinite red-shift surfaces are coincident with each other; and the metrics satisfy Landau's condition of the coordinate clock synchronization. These properties are useful to investigate the Hawking radiation of the Kerr–Newman black hole.

The Dirac equation of the charged particle in the electromagnetic field is

$$i\gamma^\mu\left(\partial_\mu+\Omega_\mu+\frac{i}{\hbar}eA_\mu\right)\Psi+\frac{m}{\hbar}\Psi=0\,, \qquad (22)$$

where e and A_μ are the electronic charge and corresponding potential. Choosing the γ^μ matrices as follows

$$\gamma^t=\frac{1}{\sqrt{\hat{F}(r)}}\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}\,, \qquad \gamma^r=\sqrt{\hat{G}(r)}\begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix}\,,$$

$$\gamma^{\theta} = \frac{1}{\hat{K}(r)} \begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix}, \qquad \gamma^{\chi} = \frac{1}{\hat{H}(r)} \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}, \tag{23}$$

where $\hat{F}(r), \hat{G}(r), \hat{K}(r)$ and $\hat{H}(r)$ are given by replacing Δ in Eq. (4) with $\hat{\Delta} = r^2 - 2Mr + a^2 + Q^2$. In this section, we still detailedly explore the action of radiation particle with spin up case. The spin up wave function in Eq. (6) is still adopted here. Substituting the γ^{μ} matrices and the wave function into the Dirac function, we can get the four equations as

$$-\left(\frac{iA}{\sqrt{\hat{F}(r)}}(\partial_t I_{\uparrow} + eA_t) + B\sqrt{\hat{G}(r)}\partial_r I_{\uparrow}\right) + mA \;=\; 0\,, \tag{24}$$

$$\left(\frac{iB}{\sqrt{\hat{F}(r)}}(\partial_t I_{\uparrow} + eA_t) - A\sqrt{\hat{G}(r)}\partial_r I_{\uparrow}\right) + mB \;=\; 0\,, \tag{25}$$

$$-B\left(\frac{1}{\hat{K}(r)}\partial_{\theta} I_{\uparrow} + \frac{i}{\hat{H}(r)}\left(\partial_{\chi} I_{\uparrow} + eA_{\chi}\right)\right) \;=\; 0\,, \tag{26}$$

$$-A\left(\frac{1}{\hat{K}(r)}\partial_{\theta} I_{\uparrow} + \frac{i}{\hat{H}(r)}\left(\partial_{\chi} I_{\uparrow} + eA_{\chi}\right)\right) \;=\; 0\,. \tag{27}$$

Our interest is still first two equations. Considering the properties of the Kerr–Newman space-time, we carry out the separation of variables as $I_{\uparrow} = -\omega t + W(r) + j\varphi + \Theta(\theta)$. Inserting the action into the above equations yielding

$$-\left(\frac{iA}{\sqrt{\hat{F}(r)}}\left(-\omega + eA_t + j\Omega\right) + B\sqrt{\hat{G}(r)}\partial_r W(r)\right) + mA \;=\; 0\,, \tag{28}$$

$$\Big(\frac{iB}{\sqrt{\hat{F}(r)}}\left(-\omega + +eA_t + j\Omega\right) - A\sqrt{\hat{G}(r)}\partial_r W(r)\Big) + mB \;=\; 0\,. \tag{29}$$

When $m = 0$, this is the Hawking radiation of massless particles and Eqs. (28) and (29) decouple. And then the charge and corresponding potential in the corresponding equations should be zero. While $m \neq 0$, it is the case of massive particle and the equations couple. Solving $W(r)$ produce

$$W_{\pm}(r) = \pm i\pi \frac{\omega - eA_0 - j\Omega_{\mathrm{h}}}{\sqrt{\hat{F}'(r_{\mathrm{h}})\hat{G}'(r_{\mathrm{h}})}}\,, \tag{30}$$

where $\pm$ corresponds to the outgoing/ingoing solution, $\Omega_{\rm h} = \frac{a}{r_{\rm h}^2+a^2}$ and $A_0 = \frac{Qr_{\rm h}}{r_{\rm h}^2+a^2}$ are the angular velocity and electronic charge potential at the outer horizon respectively, and $\sqrt{\hat{F}'(r_{\rm h})\hat{G}'(r_{\rm h})} = \frac{2(r_{\rm h}-M)}{r_{\rm h}^2+a^2}$. Inserting the value of $W_\pm(r)$ into the action, we can obtain the imaginary part of the actions as

$$\mathrm{Im}I_\pm = \pm i\pi \frac{r_{\rm h}^2+a^2}{2(r_{\rm h}-M)}(\omega - eA_0 - j\Omega_{\rm h})\,. \tag{31}$$

So the tunneling probability of the radiation particle is

$$\Gamma = \exp\Big(-2\pi\frac{r_{\rm h}^2+a^2}{r_{\rm h}-M}\,(\omega - eA_0 - j\Omega_{\rm h})\Big)\,. \tag{32}$$

And the Hawking temperature of the Kerr–Newman black hole is recovered as

$$T = \frac{1}{2\pi}\frac{r_{\rm h}+M}{r_{\rm h}^2+a^2}\,, \tag{33}$$

which is fully in consistence with that obtained by other method.

When $Q = 0$ and $a \neq 0$, the metric (18) is reduced to that of the Kerr black hole. Now the electronic charge and corresponding potential are zero, and Eqs. (32) and (33) are distribution of the Dirac particle for the Kerr black hole, which is in accordance with that obtained in Sec. 2.

When $Q \neq 0$ and $a = 0$, the Kerr–Newman metric is reduced to the Reissner–Nordstrom metric. So Eqs. (32) and (33) describe the Hawking radiation of the Dirac particle for the Reissner–Nordstrom black hole.

When $Q = 0$ and $a = 0$, Eqs. (32) and (33) are the distribution of the Dirac particle for the Schwarzschild black hole.

4. Conclusions and discussions

In this paper, extending Kerner and Mann's work to the rotating and charged rotating space-time, we have investigated the Hawking radiation of fermions for the Kerr black hole and Kerr–Newman black hole. The temperatures are recovered and are exactly the same as that obtained by other methods. In the investigation, the back reaction on the black hole geometry and self-gravitational interaction were neglected, the derived Hawking temperature only is a leading term. When these are considered, the correction term would be presented. Moreover, considering the number of particles with the spin up is statistically equal to that of particles with the spin down; we assumed that the angular momentum of the black hole does not change. Anyhow, we have extended Kerner and Mann's work to the general rotating and charged rotating black holes and recovered the Hawking temperatures.

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