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ENTROPY OF EXTREMAL BLACK HOLES IN TWO DIMENSIONS

J. Sadeghi[a†], M.R. Setare[b‡], B. Pourhassan[a]

[a]Sciences Faculty, Department of Physics, Mazandaran University
P.O. Box 47415-416, Babolsar, Iran

[b]Department of Science, Payame Noor University, Bijar, Iran

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In this paper we apply the entropy function formalism to the two-dimensional black hole which come from the compactification of the heterotic string theory with the dilaton coupling function. We find the Bekenstein-Hawking entropy from the value of the entropy function at its saddle point. Also we consider higher derivative terms. After that we apply the entropy function formalism to the Jackiw–Teitelboim (JT) model where we consider the effect of string-loop to this model.

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1. Introduction

The entropy function method is appropriate way to determined the entropy of two-dimensional black hole, since in two-dimensional black hole the horizon is a point, then horizon area simply vanishes and seems entropy must to be zero. However in two-dimensional dilaton gravities [1–4], it has been shown that the entropy is proportional to the value of the dilaton field at the horizon.

Recently, it has been proposed by Sen [5] that the entropy of a specific class of extremal black hole in higher derivative gravity can be calculated using the entropy function formalism. According to this formalism, the entropy function for the black holes which have the near horizon geometry $\mathrm{AdS}_2 \times \mathrm{S}^{D-2}$ is defined as 2π times the Legendre transformation (with respect to the electric charges) of the integration of the Lagrangian over the spherical coordinates on the horizon in the near horizon field configurations. The result is a function of moduli scalar fields as well as the sizes of AdS_2

† pouriya@ipm.ir
‡ rezakord@ipm.ir

and S^{D-2}. The values of moduli fields and the sizes are determined by extremizing the entropy function with respect the moduli fields and the sizes. Moreover, the entropy is given by the value of the entropy function at the extremum. So, the entropy function [5] can derived from Wald's formula [6], for these aim one first rewrite the Lagrangian density in terms of value of fields near horizon, and then taking the Legendre transform of the resulting function with respect to the electric field [6,7]. The near horizon geometry of the extremal black hole is determined by extremizing the entropy function and the black hole entropy is given by the extremum value of the entropy function. This general method is an easier way to calculate the black hole entropy.

Entropy function analysis provides a good understanding of the attractor mechanism for spherically symmetric extremal black holes if:

1. We consider a theory of gravity coupled to Abelian (p-form) gauge fields and neutral scalar fields.
2. The Lagrangian density f is gauge and general coordinate invariant.
3. Define an extremal black hole to be one whose near horizon geometry is $AdS_2 \times S^2$ (in $D = 4$).

In this approach, the theory need not be supersymmetric and f could contain higher derivative terms. For such black holes one can define an 'entropy function' F as follows:

$$F = 2\pi[q_i\epsilon_i - f]\,, \tag{1}$$

where q_i denote electric charges, and ϵ_i are near horizon radial electric field. F is a function of the q_i and various parameters labeling the $SO(2,1)\times SO(3)$ symmetric near horizon background (*e.g.* sizes of AdS_2 and S^2, vacuum expectation value of scalars, radial electric fields, radial magnetic fields). Then for a black hole with given electric charges q and magnetic charges p, all other near horizon parameters are obtained by extremizing F with respect to these parameters. And finally the entropy is given by the value of F at its extremum.

So, in Sec. 2 we calculate the entropy function for two-dimensional effective heterotic string theory. In that case the function of the dilaton Φ appearing as a common factor of the corresponding action is given by some series. By considering this series we account the string tree level contribution [8–11] and string-loop affect to the entropy function and obtained the entropy of system. In Sec. 3 we apply the entropy function formalism to the Jackiw–Teitelboim (JT) model [3].

2. Two-dimensional effective heterotic string theory

First we consider the two-dimensional gravity, which may come from the compactification of the heterotic string theory with the dilaton coupling function $B(\Phi)$, $U(1)$ gauge field F and the two-dimensional cosmological constant λ. The action is given by,

$$S = \frac{1}{2\pi} \int d^2x \sqrt{-g} B(\Phi) \left[R + 4(\nabla\Phi)^2 + 4\lambda^2 - \frac{F^2}{4} \right], \tag{2}$$

where

$$B(\Phi) = e^{-2\Phi} + C_0 + C_1 e^{2\Phi} + C_2 e^{4\Phi} + \cdots . \tag{3}$$

The first term on the right-hand side is the string tree level contribution [9, 10]. The higher terms represent the string-loop effects. Apart from the fact that $B(\Phi)$ is a series in powers of $\exp(2\Phi)$, little is known about the global behavior of the dilaton coupling function $B(\Phi)$. Now we are going to apply this theory to the Callan–Giddings–Harvey–Strominger (CGHS) [2]. In this model the authors investigated theory of quantum gravity coupled to a dilaton and conformal matter in two space-time dimensions, also they have shown that the theory is exactly solvable classically. In this paper the problem of Hawking radiation and back reaction of the metric is analyzed to leading order in a $\frac{1}{N}$ expansion, where N is the number of matter fields. The quantum nature of the black hole is also discussed.

So, the near horizon solutions of the static charged black hole which has SO(2,1) symmetry for the action (2) can be written as

$$\begin{aligned} ds^2 &= v \left(-r^2 dt^2 + \frac{1}{r^2} dr^2 \right), \\ \phi &= u\,, \\ F_{rt} &= \epsilon\,, \end{aligned} \tag{4}$$

where $\phi = e^{-2\Phi}$.

Here u, v and ϵ are constants which can be determined in terms of the charge q and cosmological constant λ. Note that the covariant derivatives of the Riemann tensor, the scalar field and the gauge field strength all vanish in this near horizon geometry. This plays an important role to construct Sen's entropy function from Wald's formula. Therefore, by using Eq. (4) the Lagrangian density becomes,

$$f(u, v, \epsilon) = \frac{v}{2\pi} B(u) \left[-\frac{2}{v} + 4\lambda^2 + \frac{\epsilon^2}{2v^2} \right], \tag{5}$$

where $B(u) = u + C_0 + \frac{C_1}{u} + \cdots$. Here we kept just three terms from dilaton coupling function.

From Eq. (5) one can finds the electric charged as,

$$q = \frac{\partial f}{\partial \epsilon} = \frac{\epsilon B(u)}{2\pi v} \,. \tag{6}$$

Now, the entropy function is defined as the Legendre transformation of the Lagrangian density with respect to the gauge field ϵ,

$$F(u,v,q) = 2\pi[q\epsilon - f] = vB(u)\left[\frac{2}{v} - 4\lambda^2 + \frac{2\pi^2 q^2}{B^2(u)}\right] . \tag{7}$$

The undetermined parameter u and v can be fixed by the equations of motion, which becomes the extremum equations as,

$$\frac{\partial F}{\partial v}(u_e, v_e) = \left[-4\lambda^2 B(u_e) + \frac{2\pi^2 q^2}{B(u_e)}\right] = 0\,, \tag{8}$$

and

$$\frac{\partial F}{\partial u}(u_e, v_e) = \left[2 - 4\lambda^2 v_e - \frac{2\pi^2 q^2 v_e}{B^2(u_e)}\right] = 0\,. \tag{9}$$

Eqs. (8) and (9) yield solutions,

$$\begin{aligned} u_e &= \frac{1}{2}\left(\left(\frac{\pi q}{\sqrt{2}\lambda} - C_0\right) + \sqrt{(\frac{\pi q}{\sqrt{2}\lambda} - C_0)^2 - 4C_1}\right) , \\ v_e &= \frac{1}{4\lambda^2} \,. \end{aligned} \tag{10}$$

The entropy is given by the value of the entropy function at the extremum,

$$S_{\mathrm{BH}}(q) = F(u_e, v_e, q) = 2\left(u_e + C_0 + \frac{C_1}{u_e}\right) . \tag{11}$$

Therefore the string-loop correction give us the entropy as Eq. (11). Our results in case of $C_0 = C_1 = 0$ agree with Refs. [11–12].

Now let us consider the effect of higher derivative terms. Since in two dimensions Riemann tensor and Ricci tensor can be expressed in terms of Ricci scalar, it is sufficient to consider the higher derivative terms of the form R^n. This means we must do replacement $R \to \sum a_n R^n = R + a_2 R^2 + \cdots$. Due to higher derivative terms the corresponding action can be written by

$$S = \frac{1}{2\pi}\int d^2x\sqrt{-g}B(\Phi)\left[R + \sum a_n R^n + 4(\nabla\Phi)^2 + 4\lambda^2 - \frac{F^2}{4}\right], \tag{12}$$

so the entropy function is,

$$F = vB(u)\left[\frac{2}{v} - 4\lambda^2 + \frac{2\pi^2 q^2}{B^2(u)} - \sum a_n \left(\frac{-2}{v}\right)^n\right] . \tag{13}$$

We note that the entropy (11) is modified as,

$$S_{\text{mod}} = B(u_e)\left[2 - \sum n a_n (-2)^n v_e^{1-n}\right] . \tag{14}$$

Also Eqs. (8) and (9) are modified as

$$\frac{\partial F}{\partial v}(u_e, v_e) = \left[-4\lambda^2 B(u_e) + \frac{2\pi^2 q^2}{B(u_e)} - B(u_e)\sum(1-n)a_n(-2)^n v_e^{-n}\right] = 0\,, \tag{15}$$

and

$$\frac{\partial F}{\partial u}(u_e, v_e) = \left[2 - 4\lambda^2 v_e - \frac{2\pi^2 q^2 v_e}{B^2(u_e)} - \sum a_n(-2)^n v_e^{1-n}\right] = 0\,. \tag{16}$$

Using these equations, the entropy (14) can be written in terms of q,

$$S_{\text{mod}} = \frac{4\pi^2 q^2 v_e}{\left(u_e + C_0 + \frac{C_1}{u_e}\right)} . \tag{17}$$

3. Jackiw–Teitelboim (JT) model

Another interesting model in two-dimensional gravity is the Jackiw–Teitelboim (JT) model [3]. We add the effect of string-loop to this model. Moreover in order to study the extremal charged black hole, one can include a gauge field in the model with the following form of the action,

$$S = \frac{1}{2\pi}\int d^2x\sqrt{-g}B(\Phi)\left[R + 4(\nabla\Phi)^2 + 4\lambda^2 - B^2(\Phi)\frac{F^2}{4}\right] . \tag{18}$$

The near horizon solutions are same as Eq. (4). Therefore one can study the black hole entropy in the similar way. As before the entropy function F is the Legendre transform of the Lagrangian density f in terms of the electric field.

$$F(u, v, q) = vB(u)\left[\frac{2}{v} - 4\lambda^2 + \frac{2\pi^2 q^2}{B^4(u)}\right] , \tag{19}$$

where the electric charge is given by,

$$q = \frac{\epsilon B^3(u)}{2\pi v} . \tag{20}$$

Extremizing the entropy function with respect to v and u,

$$\frac{\partial F}{\partial v}(u_e, v_e) = \left[-4\lambda^2 B(u_e) + \frac{2\pi^2 q^2}{B^3(u_e)}\right] = 0\,, \tag{21}$$

and

$$\frac{\partial F}{\partial u}(u_e, v_e) = \left[2 - 4\lambda^2 v_e - \frac{6\pi^2 q^2 v_e}{B^4(u_e)}\right] = 0\,, \tag{22}$$

provides the extremizing solutions,

$$\begin{aligned} u_e &= \frac{1}{2}\left(\left(\sqrt{\frac{\pi q}{\sqrt{2}\lambda}} - C_0\right) + \sqrt{\left(\sqrt{\frac{\pi q}{\sqrt{2}\lambda}} - C_0\right)^2 - 4C_1}\right)\,, \\ v_e &= \frac{1}{8\lambda^2}\,. \end{aligned} \tag{23}$$

By plugging these back into the entropy function, we obtain the black hole entropy as

$$S_{\rm BH}(q) = 2\left(u_e + C_0 + \frac{C_1}{u_e}\right) = \sqrt{\frac{\pi q}{\sqrt{2}\lambda}} \tag{24}$$

which agree with the results of Ref. [13]. We can consider the higher derivative corrections in the similar way. In that case one can find,

$$S_{\rm mod} = \frac{4\pi^2 q^2 v_e}{\left(u_e + C_0 + \frac{C_1}{u_e}\right)^4}. \tag{25}$$

4. Conclusion

The entropy formalism describes the attractor equations and black hole entropy in a general non-supersymmetric and higher derivative gravity theory. In this formalism, the near horizon geometry is determined by extremizing a single entropy function F. The entropy of the black hole is given by the value of F at the extremum. In this paper we use the entropy function to obtain entropy of two-dimensional charged black hole. We study two models in dilaton gravity and also consider the effect of string loop in both models, and generalized it, then find correct entropy. Finally we obtain modified entropy under effect of higher derivative terms. In this work we consider just three terms of dilaton coupling function. If we include higher order terms to this theory we will find same entropy, but in this case the value of dilaton field at horizon will be different.

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