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STUDY OF MUON CATALYZED FUSION IN DEUTERIUM–TRITIUM FUEL UNDER COMPRESSIVE CONDITIONS

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The sticking probability of the muon to the α-particle produced in the fusion process is the real bottleneck in the muonic three-body fusion. In this work, the stopping power of muonic helium ion in deuterium–tritium target has been obtained for different temperatures and densities as a function of velocity in plasma conditions. By taking all processes which can strip the muonic helium ion into account, the muon regeneration probability is computed. The calculated stopping power decreases with increasing temperature and density at any muonic helium ion velocity. The effective sticking decreases with increasing temperature and density. The results for the regeneration probability and the effective sticking are encouraging, and this investigation makes a contribution towards the goal of finding appropriate conditions which allow to achieve a positive energy balance in the muon catalyzed fusion (μCF) process.

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1. Introduction

The process of using a muon to repeatedly induce the fusion of hydrogen isotopes is known as muon catalyzed fusion (μCF) [1–11]. Transfer of a muon from hydrogen to helium is a loss channel in the μCF induced by hydrogen isotope nuclei. In the μCF cycle, the average number of fusion reactions catalyzed in deuterium–tritium (D–T) fuel for temperatures about 1000°C and liquid hydrogen density (LHD $\equiv 4.25\times10^{22}$ atoms/cm^3) experimentally

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found to be less than about 170 per muon [12]. The muons are produced during the decay of pions that are created in nucleon–nucleon collisions. The energy required to produce a muon was estimated to be about 5 GeV, therefore a fusion reactor based on μCF would only be viable if temperature and density of the fuel increase. Since fusion of deuterium with tritium generates 17.6 MeV, the average number of fusion reactions catalyzed should be about 285 to reach the scientific break-even (1/3 of the commercial break-even). For this purpose, it is usual to inject muons into a solid or liquid D–T target and then compress the fuel using drivers such as a laser or ion beam on a spherical media. As the fuel temperature and density increases it will change to plasma.

2. Theoretical calculations

When a muon beam is shot into an ionized dense plasma, the negative muons stop and thermalize in the medium. In the process of slowing down, a muon in D–T mixture replaces a hydrogen atom electron and a muonic atom is formed. The formation of muonic atoms can proceed via three distinct routes. The first of them involves a muon and hydrogen molecules (D_2, T_2) and therefore occurs at temperatures and densities prior to molecular dissociation. These atoms, formed in exited states, de-excite to the ground-states due to numerous cascade processes. Radiative capture of a muon and three-body transitions are second and third processes that may occur when the medium is ionized. Three-body reactions are less probable at normal target densities but due to the non-linear density dependence of the corresponding reaction rates they become important as density increases. Therefore the formation of muonic atoms in such a plasma is possible through the following processes:

a. Radiative recombination

$$\mu^- + \mathrm{D}^+ \;\rightarrow\; d\mu + \gamma\,, \tag{1}$$

$$\mu^- + \mathrm{T}^+ \;\rightarrow\; t\mu + \gamma\,. \tag{2}$$

b. Three-body recombination

$$\mu^- + \mathrm{D}^+ + X \;\rightarrow\; d\mu + X\,, \tag{3}$$

$$\mu^- + \mathrm{T}^+ + X \;\rightarrow\; t\mu + X\,, \tag{4}$$

$$\mu^- + \mathrm{D}^+ + e \;\rightarrow\; d\mu + e\,, \tag{5}$$

$$\mu^- + \mathrm{T}^+ + e \;\rightarrow\; t\mu + e\,, \tag{6}$$

where X is a ion or neutral particle. The difference between binding energies of $t\mu$ and $d\mu$ is about 48.1 eV. Therefore, in subsequent collisions of $d\mu$ atom, the muon is captured from deuterium to tritium nucleus at density, ϕ for all temperatures with a rate $\lambda_{dt} = 2.8 \times 10^8 \phi$ ($d\mu + t \rightarrow t\mu + d + 48.1$ eV). Muon transfer from $t\mu$ to a deuterium nucleus is possible when the temperature of plasma approaches or exceeds 48.1 eV ($t\mu + d \rightarrow d\mu + t - 48.1$ eV) [13]. In a cold D–T mixture ($< 1000°$C), the $dt\mu$ molecules are formed during a time interval $\tau_{dt\mu} \leq 10^{-8}$ sec [14] through following resonance reactions:

$$t\mu + \mathrm{D}_2 \rightarrow [(dt\mu)_{J\nu} dee](\lambda_{dt\mu-d}) , \tag{7}$$

$$t\mu + \mathrm{DT} \rightarrow [(dt\mu)_{J\nu} tee](\lambda_{dt\mu-t}) , \tag{8}$$

$$\lambda_{dt\mu} = \lambda_{dt\mu-d} C_{\mathrm{d}} + \lambda_{dt\mu-t} C_{\mathrm{t}} , \tag{9}$$

where $J = \nu = 1$ are the quantum numbers of excited rotating-vibrational state and C_{d} and C_{t} are relative concentrations of deuterium and tritium nuclei, respectively. At high temperatures, resonance formation of $dt\mu$ muonic molecule becomes slow and three-body formation mechanisms dominate [15]. There are other possibility for formation mechanisms of $dt\mu$ muonic molecule, such as ionic capture and non-resonance formation which are always dominated by the resonance or three-body formation. The processes of muonic molecules formation are described by the following reactions:

a. by collisions with electrons

$$t\mu + d + e \rightarrow dt\mu + e' , \tag{10}$$

$$t\mu + \mathrm{D} + e \rightarrow (dt\mu)e + e' . \tag{11}$$

b. by neutral-neutral transitions

$$t\mu + \mathrm{D} + X \rightarrow (dt\mu)e + X' . \tag{12}$$

c. by ion-ion or neutral-ion transitions

$$t\mu + \mathrm{D} + X \rightarrow dt\mu + X + e' , \tag{13}$$

$$t\mu + d + X \rightarrow dt\mu + X^+ + e' , \tag{14}$$

where X stands for deuterium or tritium. All of these three-body reactions involve the excitation of an electron to remove the binding energy of $dt\mu_{11}$. The sum of reaction rates of the resonance and the three-body mechanisms yields,

$$\lambda_{dt\mu}(\mathrm{formation}) = \lambda_{dt\mu}(\mathrm{resonance}) + \sum_j \lambda^j_{dt\mu}(\mathrm{three-body}) , \tag{15}$$

where $\lambda_{dt\mu}$(formation) is the total formation rate of $dt\mu_{11}$ muonic molecule. This molecule is an analogue of H_2^+ ion except that it's bond length is smaller by a factor equal to the muon–electron mass ratio (≈ 207). The $dt\mu$ muonic molecule makes deuterium and tritium closer within a distance of 5×10^{-11} cm which allows them to overcome the Coulomb potential barrier and fuse together in a time interval of order 10^{-12} sec with an energy release of 17.6 MeV. The negative muon is not involved in the fusion process (plays a role of a catalyst) and it will be released after the fusion. There is, however, some small fraction of muons which are captured by the recoiling helium nucleus but most of them are freed to start another chain of μCF processes ($\tau_\mu = 2.197 \times 10^{-6}$ sec). The probability of formation of a muonic helium ion (with a kinetic energy $K^{\rm in}_{\alpha\mu} = 3.47$ MeV) is called the initial sticking probability $\omega_{\rm s}^0(= 0.912\%)$ [16]. Once the muonic helium ion is formed, the muon can be stripped from the muonic helium ion by successive collisions with nucleus:

$$\begin{aligned} \alpha\mu + X &\rightarrow \alpha + \mu + X \qquad (16)\\ &\rightarrow \alpha + \mu X\,. \qquad (17) \end{aligned}$$

This process is called muon regeneration, with a corresponding fraction, R. Hence the effective sticking coefficient, $\omega_{\rm s}^{\rm eff}$ become:

$$\omega_{\rm s}^{\rm eff} = \omega_{\rm s}^0 (1 - R)\,. \qquad (18)$$

The effective sticking coefficient ($\omega_{\rm s}^{\rm eff}$) is the most important parameter in the study of D–T μCF. The cycling rate or the number of fusions per muon in the idealistic case (without muon loss), is limited by the sticking probability. The muon regeneration probability depends upon the stopping power of the media and several important cross sections. The kinetics of regeneration described by the various rates can be obtained using a set of coupled differential equations. The total stripping probability, $P_{\rm st}(t)$ is a time-dependent quantity determined by

$$\frac{dP_{\rm st}(t)}{dt} = \sum_i \lambda_{\rm st}^{(i)}\,(v(t))\,P_i(t)\,, \qquad (19)$$

where $\lambda_{\rm st}^{(i)}\,(v(t))$ are velocity-dependent stripping rates from the individual energy levels and $P_i(t)$ are the time-dependent populations for the state i of muonic helium ion and can be determined by

$$\begin{aligned} \frac{dP_i(t)}{dt} = & \sum_{i'(n_{i'}>n_i)} \left(\lambda_{\rm Au}^{(i'\to i)} + \lambda_{\rm ra}^{(i'\to i)} + \lambda_{\rm de-ex}^{(i'\to i)}\right) P_{i'}(t) \\ & + \sum_{i'(n_{i'}<n_i)} \lambda_{\rm ex}^{(i'\to i)} P_{i'}(t) + \sum_{i'(n_{i'}=n_i)} \lambda_{\rm Stark}^{(i'\to i)} P_{i'}(t) \\ & - \lambda_{\rm st}^{(i)} P_i(t) - \sum_{i'(n_{i'}<n_i)} \left(\lambda_{\rm Au}^{(i\to i')} + \lambda_{\rm ra}^{(i\to i')} + \lambda_{\rm de-ex}^{(i\to i')}\right) P_i(t) \\ & - \sum_{i'(n_{i'}>n_i)} \lambda_{\rm ex}^{(i\to i')} P_i(t) - \sum_{i'(n_{i'}=n_i)} \lambda_{\rm Stark}^{(i\to i')} P_i(t)\,, \qquad (20) \end{aligned}$$

where $\lambda_{\rm Au}$, $\lambda_{\rm ra}$, $\lambda_{\rm de-ex}$, $\lambda_{\rm ex}$, $\lambda_{\rm Stark}$ and $\lambda_{\rm st}$ are the Auger de-excitation, radiative, Coulomb de-excitation, Coulomb excitation, Stark mixing and stripping rates, respectively. The excitation rates were obtained using Born approximation given by Bracci and Fiorentini [17]. The de-excitation rates are determined by substitution of $\lambda_{\rm ex}$ in $\lambda_{\rm de-ex}^{n\to 1} = \lambda_{\rm ex}^{1\to n}/n^2$. The ionization rates were obtained by continuum distorted wave-eikonal initial state (CDW-EIS) method given by Igarashi and Shirai [18]. The charge transfer rates are obtained by Bracci and Fiorentini [17]. The Stark mixing rates were calculated using the formulas given by Leon and Bethe [19, 20] and the Auger rates have been replaced by the de-excitation rates in collisions with free electrons. The radiative rates were obtained by scaling the results of hydrogen atom following Bethe and Salpeter [21]. The dependence of velocity on time is calculated using,

$$\frac{dK_{\alpha\mu}}{dt} = -v_{\alpha\mu} S(K_{\alpha\mu}) = -\sqrt{\frac{2K_{\alpha\mu}}{m_{\alpha\mu}}} S(K_{\alpha\mu})\,, \tag{21}$$

where $S(K_{\alpha\mu}) = -dK_{\alpha\mu}/dl$ is the plasma stopping power. The stopping power, which is a measure of energy loss of energetic charged particles in unit length of target medium, is of continuing theoretical and experimental interest in diverse areas such as interaction of charged particles with solids [17–20] and beam-heating of plasma. For high velocity particles or clusters, the energy loss may be mainly due to collective and single-particle excitations in the target medium. The energy loss of a high velocity projectile is formulated, originally following Lindhard [26], by considering the justifiable assumption of a weak coupling between the particle and a target medium which is modelled by a linear response function of the degenerate electron gas (DEG). The total field generated by the muonic helium ion, which travels with velocity $\vec{v}$ at the position $\vec{r}_{\alpha\mu}(t) = \vec{r}_0 + \vec{v}t$ is

$$\vec{E}^{\rm tot}_{\vec{\kappa},\omega} = \frac{\vec{E}^{\alpha\mu}_{\vec{\kappa},\omega}}{\mathrm{D}(\vec{\kappa},\omega)}\,, \tag{22}$$

where $\vec{E}^{\alpha\mu}_{\vec{\kappa},\omega} = (2\pi q)\exp(-i\vec{\kappa}.\vec{r}_0)\delta(\omega - \vec{\kappa}.\vec{v})$ is the Fourier transform of $\vec{E}^{\alpha\mu}(\vec{r}) = q(\vec{r} - \vec{r}_{\alpha\mu}(t))/|\vec{r} - \vec{r}_{\alpha\mu}(t)|^3$. The dielectric function, $\mathrm{D}(\vec{\kappa},\omega)$ describes the response of a degenerate free electron gas to an external (longitudinal) perturbation in terms of the momentum transfer, $\hbar\kappa$ and energy transfer, $\hbar\omega$. A charged particle passing through a plasma will induce an electric field, $\vec{E}^{\rm pol}(\vec{r}_{\alpha\mu}, t)$ by polarizing the medium. The induced electric field can be related to the dielectric function, $\mathrm{D}(\vec{\kappa},\omega)$ of the medium through its Fourier transform. The field generated from the background particles is

$$\vec{E}^{\rm pol}(\vec{r}_{\alpha\mu},t) = \int \vec{E}^{\rm pol}(\vec{\kappa},\omega)\exp(i\vec{\kappa}.\vec{r}_{\alpha\mu}-i\omega t)\frac{d^3\kappa}{(2\pi)^3}\frac{d\omega}{2\pi}$$
$$= q\int \vec{\epsilon}_k\left[\frac{1}{\mathrm{D}(\vec{\kappa},\vec{\kappa}.\vec{v})}-1\right]\frac{d^3\kappa}{(2\pi)^3}\,, \quad (23)$$

where $\vec{\epsilon}_\kappa$ is a unit vector in the direction of $\vec{\kappa}$. The electric field, $\vec{E}^{\rm pol}(\vec{r}_{\alpha\mu},t)$ will then act back on the particle and cause it to lose kinetic energy, K according to the following formula

$$\frac{dK}{dl} = -q\frac{\vec{v}}{v}.\vec{E}^{\rm pol}(\vec{r}_{\alpha\mu},t)\,, \quad (24)$$

which can be rewritten

$$\frac{dK}{dl} = \frac{q^2}{2\pi^2}\int\left[\frac{\vec{\kappa}.\vec{v}}{\kappa^2 v}\frac{Im\mathrm{D}(\vec{\kappa},\vec{\kappa}.\vec{v})}{|\mathrm{D}(\vec{\kappa},\vec{\kappa}.\vec{v})|^2}\right]d^3\kappa\,. \quad (25)$$

Now we apply the drag on the muonic helium ion by the degenerate electrons. The dielectric function, $\mathrm{D}(\vec{\kappa},\omega)$ in a completely degenerate plasma can be calculated using

$$\mathrm{D}(\vec{\kappa},\omega) = 1+\frac{3\omega_{pe}^2}{\kappa^2 v_{\rm F}^2}\left[f_r(u,z)+if_i(u,z)\right], \quad (26)$$
$$f_r(u,z) = \frac{1}{2}+\frac{1}{8z}\left[1-(z-u)^2\right]\log\left(\left|\frac{z-u+1}{z-u-1}\right|\right)$$
$$+\frac{1}{8z}\left[1-(z+u)^2\right]\log\left(\left|\frac{z+u+1}{z+u-1}\right|\right), \quad (27)$$

$$f_i(u,z) = \begin{cases} \frac{\pi}{2}u & \text{if } |z+u|<1\,, \\ \frac{\pi}{8z}\left[1-(z+u)^2\right] & \text{if } |z-u|<1<|z+u|\,, \\ 0 & \text{if } |z-u|>1\,, \end{cases} \quad (28)$$

where $z=\hbar\kappa/(2m_e v_{\rm F})$, $u=\omega/(\kappa v_{\rm F})$ and $\omega_{pe}=\sqrt{4\pi Ne^2/m_e}$ is the plasma frequency and $v_{\rm F}=(\hbar/m_e)(3\pi^2N)^{1/3}=4.16\times10^{-3}\phi^{1/3}c$ is the Fermi velocity.

Finally, substitution of Eqs. (26)–(28) in Eq. (25) leads us to,

$$\frac{dK}{dl} = \frac{4\pi z^2e^4}{m_e v^2}n_e L\,, \quad (29)$$

where the stopping number is

$$L = \frac{6}{\pi}\int_0^{v/v_{\rm F}} u du \int_0^{\infty} \frac{f_i(u,z)}{[z^2+\chi^2 f_r(u,z)]^2+\chi^4 f_i^2(u,z)} z^3 dz\,, \tag{30}$$

where $\chi^2 = e^2/(\pi\hbar v_{\rm F})$ is the density parameter. The stopping number in plasma as a function of velocity have been calculated for different temperatures and densities and have been shown in Figs. 1 and 2. As it is clear from the figures, the stopping number decreases with increasing density in a given temperature and velocity while it decreases with increasing temperature in a given density and velocity.

The muon regeneration probability is obtained using the stripping probability $P_{\rm st}(t)$ at time $t = \infty$,

$$R = \frac{P_{\rm st}(t=\infty)}{\omega_{\rm s}^0}\,. \tag{31}$$

The initial conditions used to calculate muon regeneration probability (R) are: $K_{\alpha\mu}(0) = K_{\alpha\mu}^{\rm in} = 3.47$ MeV, $P_{\rm st}(0) = 0$ and the level populations are equal to the partial sticking fractions,

$$\begin{aligned} P_i(0) &= \omega_{\rm s}^0(i)\,, \\ \sum_i P_i(0) &= \omega_{\rm s}^0\,. \end{aligned} \tag{32}$$

The initial sticking probability, $\omega_{\rm s}^0$ and fractions of the nl states are given in [16, 27]. The populations $P_i(t)$ for $n = 1, 2, \ldots, 6$ and the l sub-levels are considered for $n < 4$. The muon regeneration probability and the effective sticking have been obtained from Eqs. (31) and (18) after a numerical solution of a set of coupled differential Eqs. (19)–(21) along with the initial conditions given by Eq. (32).

The calculated regeneration probability and the effective sticking coefficient as a function of plasma density and temperature are presented in Figs. 3 and 4. These figures show that variation of temperature in the region considered significantly changes the regeneration probability and the effective sticking coefficient at densities $1 < \phi < 100$ LHD. The changes are less pronounced for $\phi > 100$ LHD. Finally, the regeneration probability increases and the effective sticking coefficient decreases rapidly with increasing density at any fuel temperature while these occur slowly in liquid phase ($< 1000^\circ$C) [27]. The results obtained in this work for R are compared with results given in Refs. [27] and [28] in Table I.

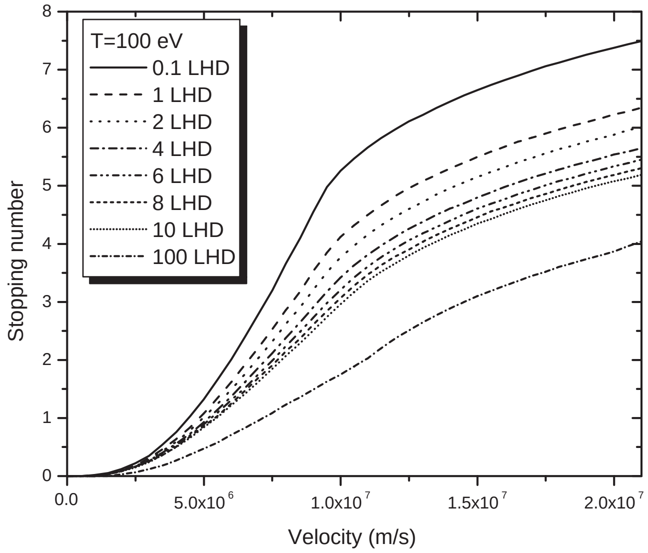

Fig. 1. The stopping number in plasma, as a function of velocity at T = 100 eV and different densities presented in the inset, in units of liquid hydrogen density (LHD).

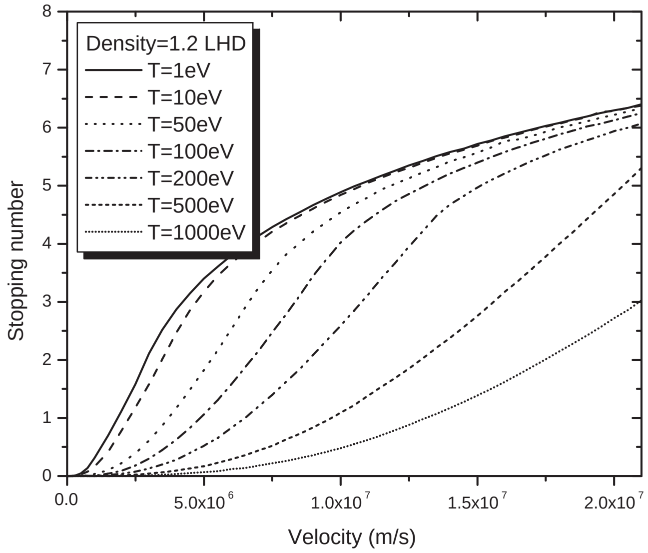

Fig. 2. The stopping number in plasma, as a function of velocity at $\phi = 1.2$ LHD and different temperatures presented in the inset.

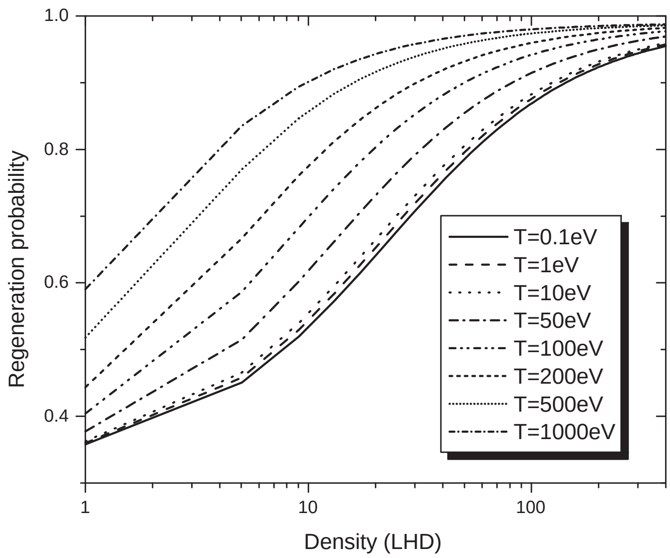

Fig. 3. The regeneration probability, as a function of plasma density at different temperatures presented in the inset.

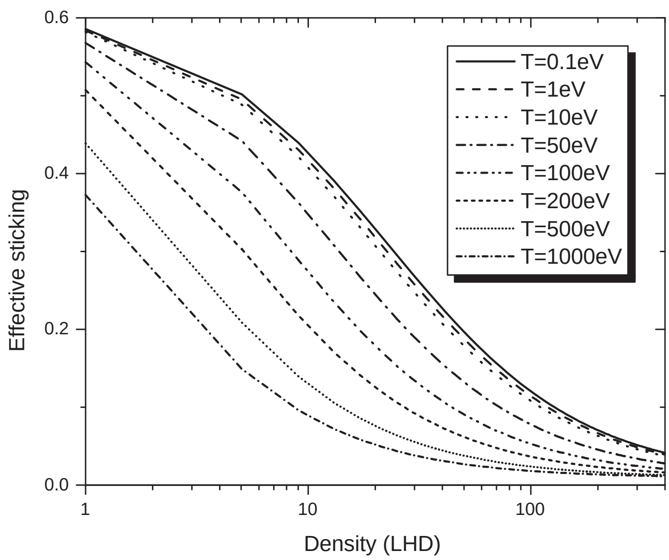

Fig. 4. The effective sticking coefficient, as a function of plasma density at different temperatures presented in the inset.

TABLE I

The regeneration probability, R and the effective sticking coefficient, ω_s^{eff} for the muonic helium ion in liquid and plasma fuel at $\phi = 1.2$ LHD.

Source	R (Ref. [27])	R (Ref. [28])	R (this work)	ω_s^{eff}(this work)
T < 1000°C	0.391	—	—	—
T = 0.1 eV	—	—	0.363	0.580%
T = 1 eV	—	—	0.366	0.578%
T = 10 eV	—	—	0.368	0.576%
T = 50 eV	—	0.352	0.386	0.560%
T = 100 eV	—	0.403	0.417	0.531%
T = 200 eV	—	0.493	0.461	0.491%
T = 500 eV	—	—	0.543	0.416%
T = 1000 eV	—	0.676	0.619	0.347%

3. Conclusion

In the present study, the stopping number in plasma has been obtained for different temperatures and densities as a function of velocity (Figs. 1 and 2). Our calculations in plasma conditions show that the maximal value of stopping power (as function of velocity) decreases with increasing temperature at any density. The maximal value of stopping power for temperatures higher than 18 eV in plasma conditions at 1 LHD is smaller than its value in liquid phase which is comparable with T > 20 eV in Ref. [28]. Calculations for the muon regeneration probability and the effective sticking coefficient have been done by solving a set of coupled differential equations numerically. The results of our numerical method for the regeneration probability is compared with liquid phase [27] and results given by Jändel *et al.* [28] in Table I. Comparison of our results for R-values with the ones obtained by Jändel *et al.* [28] indicates differences that do not exceed 10%. The discrepancies are due to different reaction rates and initial conditions used in both calculations. The regeneration probability increases and the effective sticking coefficient decreases with increasing the plasma temperature and density. But the formation rate of muonic molecule ion, at high temperatures, is very slow (at high temperatures, resonance formation of muonic molecule becomes small and three-body formation mechanisms dominate). Therefore, the average number of fusion reactions catalyzed by one muon can be increased rapidly with increasing density at low temperatures in plasma conditions and the energy gain could be fulfilled while positive energy balance is never achieved in liquid phase.

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