

ISOMER SPECTROSCOPY AND SHELL STRUCTURE  
AROUND DOUBLY-MAGIC  $^{132}\text{Sn}^*$ 

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Isomeric decays in the heavy even-even Cd isotopes populated in the fragmentation of  $^{136}\text{Xe}$  as well as projectile fission of  $^{238}\text{U}$  have been studied within the RISING project at GSI. The new experimental results suggest an energy of 1325 keV for the first excited  $2^+$  state in semi-magic  $^{130}\text{Cd}$  and confirm the previously established positions of this state in  $^{126}\text{Cd}$  and  $^{128}\text{Cd}$  (652 keV resp. 645 keV). The origin of the unexpectedly low  $2^+$  excitation energies in the  $N = 78, 80$  isotopes has been investigated in detail performing modern beyond mean field calculations employing the Gogny force.

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**1. Introduction**

The region around  $^{132}\text{Sn}$ , the heaviest doubly-magic nucleus far-off stability that can be studied experimentally using currently available instrumentation, has provided us in the past with a series of surprises. Examples in the region above  $^{132}\text{Sn}$  are the unexpectedly low  $2^+$  excitation energies in the  $N = 84$  isotones  $^{134}\text{Sn}$  and  $^{136}\text{Te}$  (which are much smaller as the ones observed in the  $N = 80$  isotones), as well as a reduced  $B(E2)$  strengths

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in  $^{136}\text{Te}$  [1], indicating an unexpected asymmetry with respect to  $N = 82$ . Below  $^{132}\text{Sn}$  the probably most intriguing experimental finding has been the anomalous behaviour of the  $2^+$  energies in the Cd isotopic chain towards the  $N = 82$  shell closure. Whereas in all neighbouring isotopic chains the  $2^+$  energy smoothly increases when approaching  $N = 82$ , it has been found in beta decay studies that in the  $N = 80$  isotope  $^{128}\text{Cd}$   $E(2^+)$  is even lower by 7 keV as compared to  $N = 78$   $^{126}\text{Cd}$  [2]. In addition, a  $2^+$  state in semi-magic  $^{130}\text{Cd}$  had been tentatively assigned at 957 keV in that work, much lower than in all neighbouring  $N = 82$  isotones. These observations have been interpreted as indirect evidence for a quenching of the  $N = 82$  shell closure in the Cd isotopes [2]. Due to the importance of this question, not only for nuclear structure but also in view of its astrophysical implications, a new experimental study has been performed using isomer spectroscopy within the RISING project at GSI to obtain additional and independent information about the excitation schemes of the heavy Cd isotopes as well as neighbouring nuclei.

## 2. Experiment and results

The heavy Cd isotopes were produced both in the fragmentation of a  $^{136}\text{Xe}$  beam at 750 MeV/ $u$  and projectile fission of a  $^{238}\text{U}$  beam at 650 MeV/ $u$  on Be targets (with thicknesses of 4 g/cm<sup>2</sup> respectively 1 g/cm<sup>2</sup>). They were separated from other reaction products and identified ion by ion in the GSI fragment separator FRS via the measurement of the energy loss, the magnetic rigidity, the positions in the intermediate and the final focal plane, and the time of flight in the second half of the FRS. Fig. 1 illustrates the identification of the different Cd isotopes. The ions which have been produced in an isomeric state and which survived in this excited state the flight through

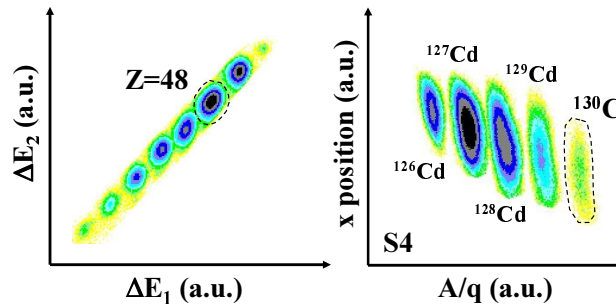


Fig. 1. Example of particle identification plots from the fragmentation of  $^{136}\text{Xe}$ . Left:  $Z$  identification from the energy losses measured in two multiple sampling ionization chambers. Right: Isotope identification from the positions of the Cd ions in the final focal plane S4 of the FRS shown as a function of  $A/Q$ .

the FRS and the implantation in a passive stopper then decay to the ground state by  $\gamma$ -ray emission. These  $\gamma$ -rays were then detected by the RISING spectrometer [3] in delayed coincidence with the implanted ions. For all three studied even-even Cd isotopes, namely  $^{126}\text{Cd}$ ,  $^{128}\text{Cd}$  and  $^{130}\text{Cd}$ , a detailed analysis of singles and coincidence  $\gamma$ -ray spectra as well as decay curves has been performed leading to the construction of the excitation schemes and the determination of isomeric lifetimes in these three nuclei. Details about the analysis are given in Ref. [4–7]. In the following, we will concentrate on one aspect of the obtained information, namely the  $2^+$  energies in these nuclei. Fig. 2 shows the  $\gamma$ -ray spectra obtained in our experiment in delayed coincidence with identified and implanted  $^{126}\text{Cd}$ ,  $^{128}\text{Cd}$  and  $^{130}\text{Cd}$  ions, respectively. In  $^{130}\text{Cd}$  four  $\gamma$  transitions are observed forming the E2 cascade from the isomeric  $8^+$  state (with a  $\pi g_{9/2}^2$  configuration) down to the ground state [4]. In analogy to the other semi-magic Cd isotope  $^{98}\text{Cd}$ , in which an  $8^+$  isomer of the same structure has been observed [8], the observed 1325 keV transition has been assigned as the  $2^+ \rightarrow 0^+$  transition in  $^{130}\text{Cd}$ , placing the  $2^+$  state at 1325 keV in contrast to the tentative assignment of 957 keV in

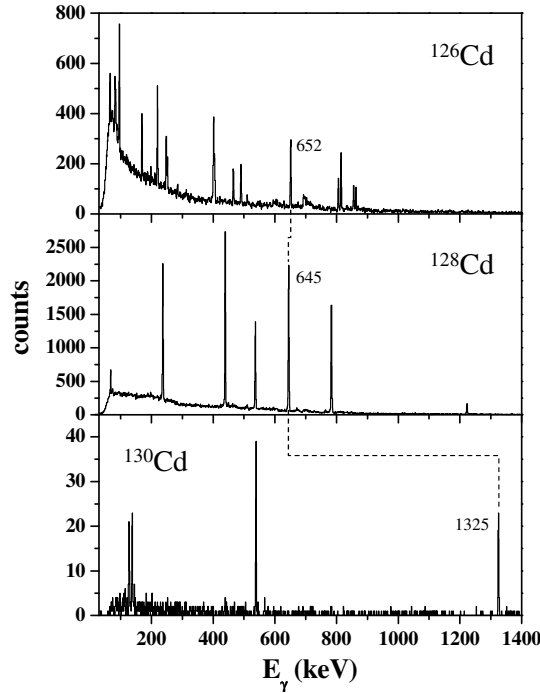


Fig. 2. Gamma-ray spectra obtained in delayed coincidence with identified and implanted  $^{126}\text{Cd}$  (top),  $^{128}\text{Cd}$  (middle) and  $^{130}\text{Cd}$  (bottom) ions.

Ref. [2]. A full analysis of the new experimental information obtained with respect to the excitation schemes of  $^{126,128}\text{Cd}$ , on the other hand, confirmed the positions of the  $2^+$  states in these nuclei as established in beta-decay work [2]. While the  $2^+$  energy of the semi-magic  $^{130}\text{Cd}$  isotope is now in agreement with the values in the neighbouring  $N = 82$  isotones, the anomalous flattening of the  $2^+$  energies in the Cd isotopes above  $N = 72$ , which is in strong contrast to the parabolic behaviour towards  $N = 82$  observed in all other isotopic chains, still persists as illustrated in Fig. 3. To unveil the origin of this flattening, which is not reproduced by current large-scale shell model calculations [5,7], we performed extensive state-of-the-art beyond mean field calculations using the Gogny force comparing the Cd isotopes with their Te isotones.

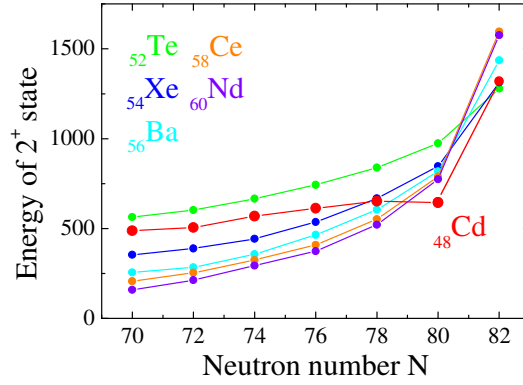


Fig. 3. Experimental excitation energies of the first excited  $2^+$  states as a function of the neutron number  $N$  for the even-even Cd, Te, Xe, Ba, Ce and Nd isotopes in the range  $N = 70$ –82.

### 3. Beyond mean field calculations

The recently proposed symmetry conserving configuration mixing approach [9] with particle number and angular momentum projection and the finite range density dependent Gogny interaction with all its terms has been used to calculate  $2^+$  energies and  $B(E2)$  transition strengths in the even-even Cd and Te isotopes (for more details see [10]). This interaction has been adjusted to reproduce global properties of nuclear matter more than twenty five years ago and does not invoke any parameter adjustment to certain regions of the nuclear chart. Its strength is the ability to describe in a consistent way a large variety of phenomena. It is important to understand that the intention of these calculations is to describe general trends and evolutions of nuclear properties and to help to unveil the basic underlying origins rather than to obtain an as detailed as possible description of a single nucleus of interest.

In a first step the collective subspace of HFB wave functions is generated using quadrupole constrained particle number projection *before the variation*, *i.e.*, the energy

$$E^{N,Z}(q) = \frac{\langle \Phi^{N,Z}(q) | \hat{H} | \Phi^{N,Z}(q) \rangle}{\langle \Phi^{N,Z}(q) | \Phi^{N,Z}(q) \rangle}, \quad (1)$$

with  $|\Phi^{N,Z}(q)\rangle = \hat{P}^N \hat{P}^Z |\phi(q)\rangle$  is minimized, where  $\hat{P}^{N(Z)}$  is the projector onto neutron (proton) number,  $|\phi(q)\rangle$  are HFB-type wave functions and  $q$  is a set of intrinsic constraints that allows the definition of potential energy surfaces along the most relevant degrees of freedom. In the present calculations we restrict this set to the quadrupole deformation  $\beta$ , triaxial shapes, *i.e.* the  $\gamma$  degree of freedom, are not considered. The particle number projected energy curves as a function of the intrinsic quadrupole deformation  $\beta$  for  $^{128}\text{Cd}$  and its  $N = 80$  isotone  $^{132}\text{Te}$  are shown as dotted black curves labeled “PNP” in Fig. 4.

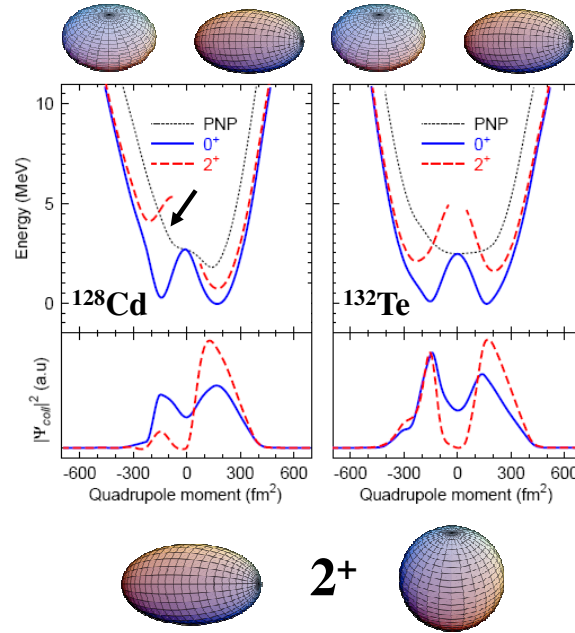


Fig. 4. Top: Potential energy curves as a function of the intrinsic quadrupole deformation  $\beta$ , with particle number projection (dotted lines) and after additional angular momentum projection,  $J = 0\hbar$  (continuous lines) and  $J = 2\hbar$  (dashed lines); Bottom: squared amplitudes of the wave functions of the collective states with  $J = 0\hbar$  (continuous lines) and  $J = 2\hbar$  (dashed lines). The average shapes of the resulting  $2^+$  states are illustrated in the bottom of the figure.

Already after this first step, a difference between the two nuclei is apparent. On the oblate side ( $\beta < 0$ ), the energy increases much faster for  $^{128}\text{Cd}$  compared to  $^{132}\text{Te}$  ( $E(\beta = -0.1) - E(\beta = 0) = 2.5$  MeV and 1.0 MeV, respectively). This difference can be understood on the basis of the single particle energies (SPE), represented in Fig. 5 for  $^{128}\text{Cd}$ . The SPE for  $^{132}\text{Te}$  are not significantly different from these ones. The proton Fermi levels for both nuclei are included in Fig. 5, while for the neutrons we assume the Fermi level to be the same since both nuclei have the same number of neutrons,  $N = 80$ . At an oblate deformation of about  $\beta = -0.1$  the neutron  $1s_{1/2}$  level

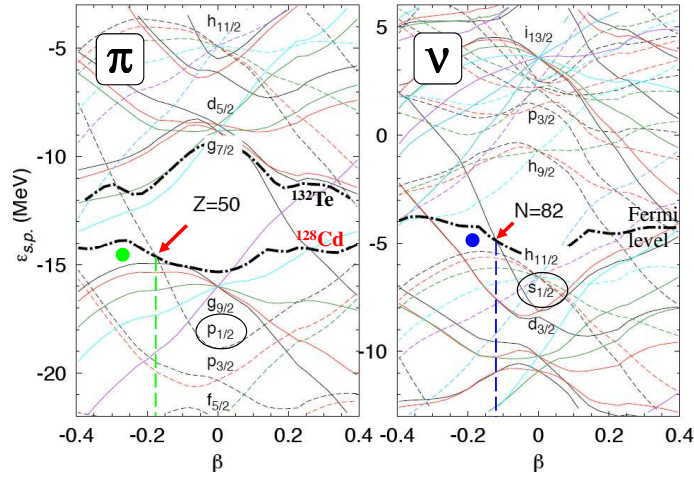


Fig. 5. Proton (left) and neutron (right) single particle energies for  $^{128}\text{Cd}$ . The energy scale for neutrons has been shifted by 9 MeV. The thick dotted-dashed lines represent the Fermi levels for  $^{128}\text{Cd}$  and the thick dashed line the proton Fermi level for  $^{132}\text{Te}$ .

is emerging from the Fermi surface (arrow and blue dashed line in Fig. 5). From here on both  $N = 80$  isotones have a completely filled  $\nu h_{11/2}$  shell. On the proton side, a similar situation occurs around  $\beta = -0.15$  where the proton  $1p_{1/2}$  level is crossing the Fermi surface for  $^{128}\text{Cd}$  (arrow and green dashed line in Fig. 5). That means that for oblate deformations larger than  $|\beta| \approx 0.15$  in addition to the full  $h_{11/2}$  neutron shell the nucleus  $^{128}\text{Cd}$  also has a completely filled large intruder proton orbit, namely the  $\pi g_{9/2}$  shell. To deform this nucleus with closed large intruder shells for both protons and neutrons a lot of energy is required resulting in the very steep PNP curve for  $^{128}\text{Cd}$  in the upper part of Fig. 4. Furthermore, the generation of two units of angular momentum to form a  $2^+$  state is very costly for such a stiff configuration since it requires the admixing of orbitals above the  $Z = 50$  or  $N = 82$  shell closures. The situation in  $^{132}\text{Te}$  is very different. In this case, the proton Fermi level lies above the  $Z = 50$  shell closure and the

oblate branch of the proton system can easily be deformed (see Fig. 5) due to the positive slope of the relevant  $g_{7/2}$  orbitals. This deformation will be directly transferred to the neutron system due to the strong proton–neutron interaction. Even more important, a  $J = 2^+$  state can easily be build by coupling two  $g_{7/2}$  protons.

Although severe differences between the two isotones  $^{128}\text{Cd}$  and  $^{132}\text{Te}$  there are already observed at the PNP level, the calculations have to be refined in order to be able to quantitatively compare the systematics of  $2^+$  energies in the Cd and Te isotopes to the experimental data. Therefore, in the next step angular momentum projection is performed, *i.e.* the energy expectation values for  $J = 0\hbar$  and  $J = 2\hbar$  are calculated with the wave functions  $|\Phi_J^{N,Z}(q)\rangle = \hat{P}^J \hat{P}^N \hat{P}^Z |\phi(q)\rangle$ , where  $\hat{P}^J$  is the projector onto angular momentum. These potential energy curves after angular momentum projection are shown as continuous and dashed lines for the  $0^+$  and  $2^+$  states, respectively, in the upper part of Fig. 4. They are rather similar for  $^{128}\text{Cd}$  and  $^{132}\text{Te}$  in the case of the  $0^+$  state presenting coexisting prolate and oblate minima. For the  $2^+$  state, on the other hand, the results are quite different: for  $^{128}\text{Cd}$  we obtain a deep prolate deformed minimum about 3.5 MeV lower than the oblate one, whereas in  $^{132}\text{Te}$  we observe again two symmetric minima as for the  $0^+$  state. The observation of a prolate minimum for the  $2^+$  state in  $^{128}\text{Cd}$  is clearly a consequence of the “blocking” of the oblate side discussed above by means of the single-particle energies.

Finally, in the last step of the calculations, configuration mixing is performed within the generator coordinate method (GCM) framework taking linear combinations of the particle number and angular momentum projected wave functions:

$$|\Psi_{J,\sigma}^{N,Z}\rangle = \int f_{J,\sigma}^{N,Z}(q) \hat{P}^J |\Phi^{N,Z}(q)\rangle dq. \quad (2)$$

Then, the variational principle applied to the weights  $f_{J,\sigma}^{N,Z}(q)$  gives the generalized eigenvalue problem (Hill–Wheeler equation):

$$\int \left( \mathcal{H}_J^{N,Z}(q, q') - E_{J,\sigma}^{N,Z} \mathcal{N}_J^{N,Z}(q, q') \right) f_{J,\sigma}^{N,Z}(q') dq' = 0 \quad (3)$$

with  $\mathcal{H}_J^{N,Z}$  and  $\mathcal{N}_J^{N,Z}$  the Hamiltonian and norm overlaps, respectively (see [11] for further details). Solving Eq. (3) both the energies of the states,  $E_{J,\sigma}^{N,Z}$ , and the collective wave functions are obtained. In the lower part of Fig. 4, the wave functions of the  $0^+$  and  $2^+$  collective states, *i.e.* the states obtained after configuration mixing, are shown. Once more, pronounced differences are observed for the  $2^+$  state: In  $^{132}\text{Te}$  this state represents on average a quadrupole oscillation around a spherical shape. In  $^{128}\text{Cd}$ , on the other

hand, the  $2^+$  state is essentially an oscillation around a prolate deformed shape and therefore strongly coupled to rotation. The resulting  $2^+$  energies are too high on an absolute scale. The origin for that is supposedly the absence of  $K$ -mixing in the calculations (see detailed discussion in Ref. [10]). However, the ratio between calculated and experimental  $2^+$  energies is nearly constant [10]. Taking into account this constant factor (0.6 for Cd and 0.5 for Te), *i.e.* renormalizing the calculated  $2^+$  energies, the calculations nicely reproduce both the flattening of the  $2^+$  energies in Cd as well as the parabolic behaviour in Te in the range  $N = 70$ –82 as shown in Fig. 6.

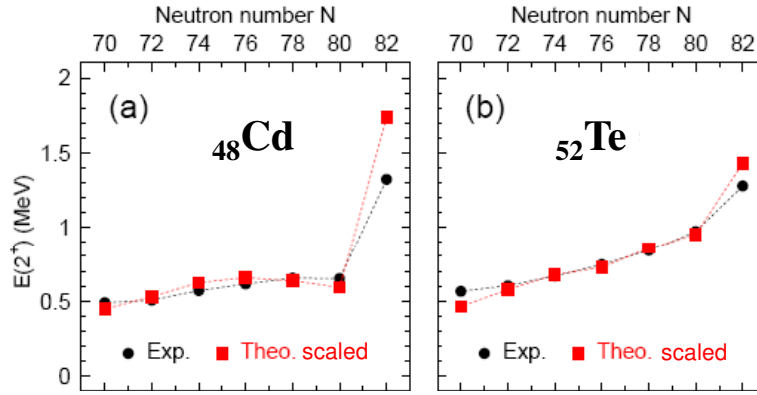


Fig. 6. Normalized calculated and experimental  $2^+$  energies in the Cd and Te isotopes as a function of the neutron number.

To summarize this discussion, we have shown that the prolate deformation of the  $2^+$  state in  $^{128}\text{Cd}$  has its origin in a very peculiar coincidence of closed shell proton and neutron configurations for  $\beta < -0.15$  leading to a “blocking” of the oblate branch of the energy curve, *i.e.*, in this branch the nucleus behaves as a double magic one. Since the proton oblate blocking persists also in the nearby  $N = 76$  and  $78$  Cd isotopes it is obvious that these isotopes will be on average more deformed than the corresponding Te isotopes. This means that the anomalous behaviour of the heavy Cd isotopes towards the  $N = 82$  shell closure can be explained by “standard” nuclear structure effects without invoking shell quenching.

#### 4. Summary and conclusions

In conclusion we presented a theoretical study of the origin of the unusual behaviour of  $2^+$  excitation energies in neutron-rich Cadmium isotopes towards the  $N = 82$  shell closure using beyond mean field techniques and the Gogny force. Our calculations show that the anomalous behaviour of the  $2^+$  energies in the heavy Cd isotopes is caused by the very special char-

acteristics of the Cd isotopes which favor prolate configurations close to the  $N = 82$  shell closure. This example demonstrates again that modern beyond mean field calculations with the Gogny force, which do not dispose of any adjustable parameters, are not only able to describe general features of atomic nuclei over wide ranges of isospin, but also to reproduce local features of nuclear structure such as the low  $2^+$  energies in  $^{126,128}\text{Cd}$  and unravel their origin.

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#### REFERENCES

- [1] D. Radford *et al.*, *Nucl. Phys.* **A752**, 264c (2005); *Phys. Rev. Lett.* **88**, 222501 (2002).
- [2] T. Kautzsch *et al.*, *Eur. Phys. J.* **A9**, 201 (2000).
- [3] S. Pietri *et al.*, *Nucl. Instr. Methods* **A261**, 1079 (2007).
- [4] A. Jungclaus *et al.*, *Phys. Rev. Lett.* **99**, 132501 (2007).
- [5] L. Caceres *et al.*, *Phys. Rev. C Rapid Comm.*, in press.
- [6] L. Caceres, Ph.D. thesis, Universidad Autónoma de Madrid, 2008, unpublished.
- [7] L. Caceres *et al.*, in preparation.
- [8] M. Gorska *et al.*, *Phys. Rev. Lett.* **79**, 2415 (1997).
- [9] T.R. Rodríguez, J.L. Egido, *Phys. Rev. Lett.* **99**, 062501 (2007).
- [10] T.R. Rodríguez, J.L. Egido, A. Jungclaus, *Phys. Lett.* **B668**, 410 (2008).
- [11] R. Rodríguez-Guzman, J.L. Egido, L. Robledo, *Nucl. Phys.* **A709**, 201 (2002).