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CORRELATIONS OF ENERGY RATIOS FOR COLLECTIVE NUCLEAR BANDS*

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It is shown that the Mallmann's energy correlations, introduced a long time ago for the ground state bands of the even–even nuclei are, in fact, universal. Various bands in all collective nuclei (even–even, odd–even, and odd–odd) obey the same systematics. This unique, universal behaviour indicates the same spin dependence of the energy of the levels in all bands in all collective nuclei. Based on a second-order anharmonic vibrator description, parameter-free recurrence relations between energy ratios are deduced. These relations can be used to predict levels of higher spins in various bands.

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1. Introduction

Search for systematics in collective bands is as old as nuclear structure and it was the phenomenological driving force in establishing the best nuclear collective models. Fifty years ago, Mallmann pointed out [1] that correlations between the ratios of energies of the members of the ground state bands (g.s.b.) in even–even nuclei define some universal curves. We will show in this paper that these types of systematics are universal in the sense that they are obeyed by all collective bands in even–even, odd-mass and odd–odd nuclei (before back/up-bending). We consider all collective $[R(4/2) = E(4^+)/E(2^+) > 2.00]$ even–even nuclei with $Z > 20$ and odd-mass and odd–odd nuclei (with a collective even–even core) with $Z = 33-80$ [2]. This type of correlations can be explained by the description of the band states as maximum phonon aligned states of an anharmonic vibrator [3].

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Using a third degree polynomial in spin for the energies, recurrence relations for various members of the band are obtained which can be used to predict energy of levels with higher spin.

2. Mallmann-type correlations

Mallmann showed [1] that the data for the $R(6/2) = E(6^+)/E(2^+)$ and $R(8/2) = E(8^+)/E(2^+)$ energy ratios for the g.s.b. of even–even nuclei represented as functions of $R(4/2) = E(4^+)/E(2^+)$ lie on two universal curves, respectively. Fig. 1 (left) shows the updated empirical Mallmann-type correlations for g.s.b. in all collective nuclei ($R(4/2) = E(4^+)/E(2^+) > 2.00$) with $Z > 20$ [2]. Mallmann interpreted the correlations for some range of $R(4/2)$ (> 3.27, *i.e.*, for axially symmetric rotor) as a consequence of the rotational formula for axially symmetric nuclei perturbed by rotation-vibration interaction $E(J) = AJ(J+1) + B[J(J+1)]^2$. In this case the correlations are given by the straight lines $R(6/2) = 27/8R(4/2) - 11$ and $R(8/2) = 594/35R(4/2) - 312/7$. It is worth noting that the two straight lines are independent of the coefficients A and B. In fact it can be easily shown that a linear relation between the energy ratios automatically results from any two-parameter energy–spin relation.

As it can be easily seen, these formulas completely fail to reproduce the empirical situation for $R(4/2) < 3.2$ and the excitation energy in yrast bands has another spin dependence. Das *et al.* [3] developed a description of the g.s.b. of all collective nuclei, based on the picture of anharmonic vibrations, as fully aligned phonons. It was shown [4,5] that the Anharmonic Vibrator (AHV) relations:

$$E(J) = nE(2_1^+) + \frac{n(n-1)}{2}\varepsilon_4\,, \tag{1}$$

where n is the number of phonons, fit very well all the energies in the ground-state band of all even–even nuclei, *i.e.* those states with maximum alignment of the angular momentum $J = 2n$. The parameter ε_4 is the anharmonicity for 2-phonon state with $J = 4^+$ and is almost the same for all transitional nuclei [4]. The expression (1) is equivalent with the two-parameter formula proposed on purely empirical grounds by Ejiri [6], $E(J) = aJ + bJ(J+1)$ (a, b are parameters) and, in fact, is a second degree polynomial in J:

$$E(J) = \alpha J + \beta J^2\,. \tag{2}$$

In this case the Mallmann-type relations are:

$$R(J/2) = \frac{n(n-1)}{2}R(4/2) - n(n-2)\,. \tag{3}$$

In particular $R(6/2) = 3R(4/2) - 3$, $R(8/2) = 6R(4/2) - 8$, $R(10/2) = 10R(4/2) - 15$, and $R(12/2) = 15R(4/2) - 24$. As can be seen in Fig. 1 (left) the relations (3) reproduce the empirical situation rather well but start to deviate for higher spins, as seen in Fig. 1 (right). Although the

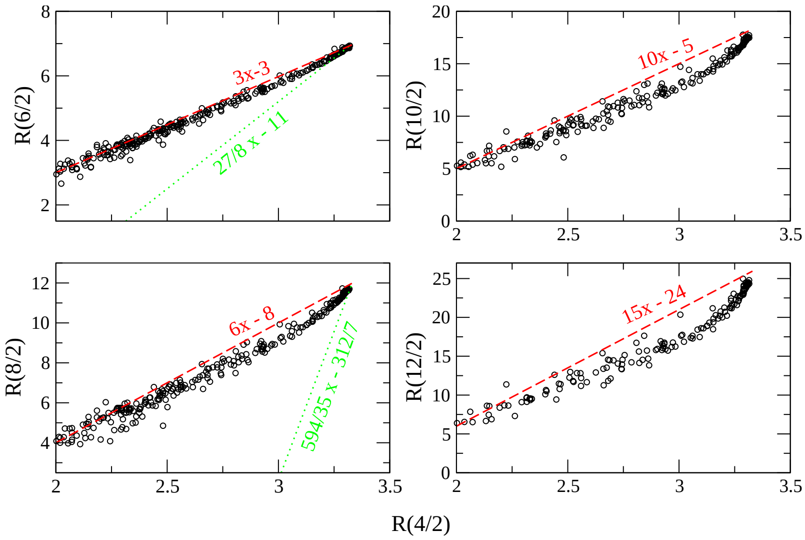

Fig. 1. Correlations of the energy ratios for all even–even collective nuclei with $Z > 20$ [2]. Dotted lines correspond to the rotational formula $E(J) = aJ(J+1) + b[j(J+1)]^2$, and the dashed lines are the AHV predictions corresponding to the phonon formula $E(J) = \alpha J + \beta J^2$ $[R(J/2) = (J(J-2))/8R(4/2) - (J(J-4))/4]$.

general trend of the data is reproduced, there are systematic deviations which increase with spin. This was somehow expected since it was shown [5] that, in order to reproduce the energy of the high spin members of the ground state bands with AHV type formulas, it is necessary to introduce higher order anharmonicities. The next (second) order AHV expression

$$E(J) = nE(2_1^+) + \frac{n(n-1)}{2}\,\varepsilon_4 + \frac{n(n-1)(n-2)}{6}\,\varepsilon_6 \tag{4}$$

describes quite well the experimental gsb's of all collective even–even nuclei, including the good rotor ones ($R(4/2) > 3.15$), which is a surprising, empirical finding. This formula, in fact, is equivalent to the third order polynomial in J:

$$E(J) = \alpha J + \beta J^2 + \gamma J^3\,. \tag{5}$$

It was shown [7–10] that the AHV type relations work also very well for the band members in odd-mass and odd–odd nuclei. Fig. 2 shows the Mallmann type correlations for all bands j, $j+2$, $j+4$, ... in odd-mass nuclei and in odd–odd nuclei, respectively. All nuclei with a collective even–even core ($R(4/2) > 2.00$) are included. The energy ratios are defined as $R(j{+}2n/j{+}2) = E(j+2n)/E(j+2)$ where $E(j+2n)$ is the energy of the n-th state in the band relative to the bandhead energy $E(j)$. The energy ratio correlations are identical to those for even–even nuclei, the data points following the spline curves obtained from the correlations in the even–even nuclei.

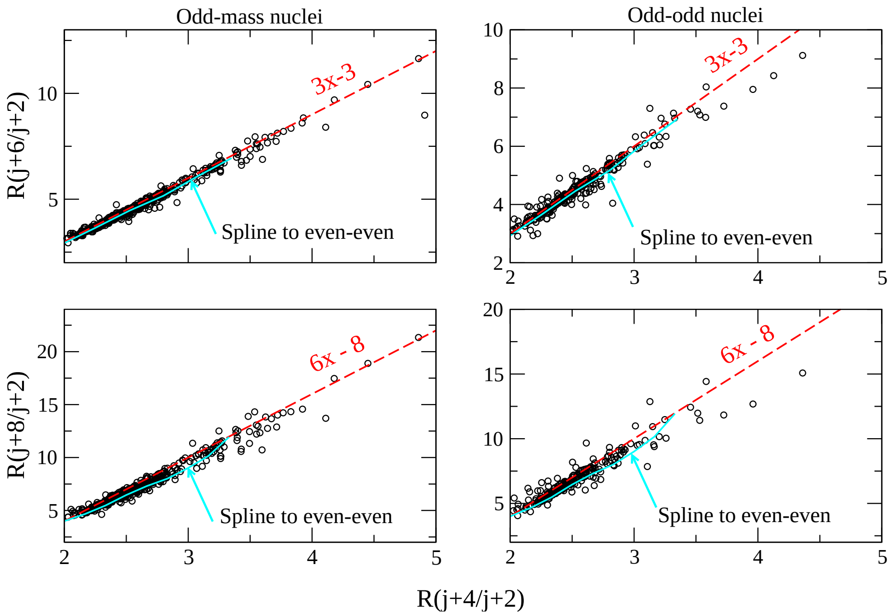

Fig. 2. Correlations of the energy ratios for all odd-mass and odd–odd nuclei (with collective even–even core) with $Z = 33$–80 [2]. The dashed lines correspond to Eq. (3), while the solid line is a spline interpolation to the corresponding even–even nuclei correlations.

In Fig. 3 we show separately Mallmann-type plots for the superdeformed (SD) bands. The experimental data from this figure are from Ref. [11]: 234 bands for which at least 7 transitions are known. It is seen that in this case the experimental data closely follow the straight lines (3) predicted by the AHV formula (1), which means a J dependence of the type (2). Indeed, the SD bands are good rotational bands, with very regularly spaced gamma transition energies, therefore they are equivalent to the good rotational bands ($R(4/2) \approx 3.33$ in even–even nuclei) with normal deformation, for which the experimental energy ratio correlations approach the prediction of Eq. (3) — see Fig. 1. In this particular case, formula (1) applies very well, with an ε_4 value related to the (constant) value of the moment of inertia.

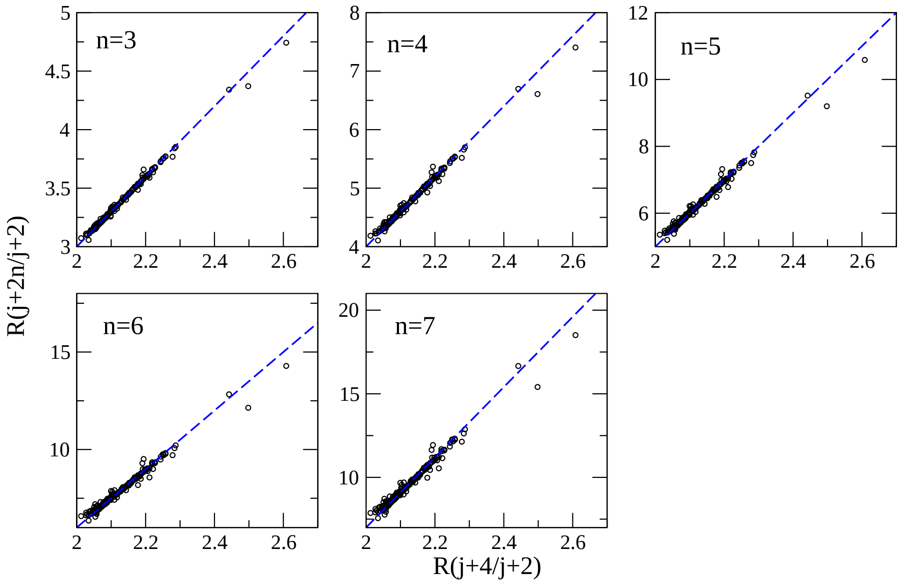

Fig. 3. Correlations of energy ratios for 234 superdeformed bands from Ref. [11]. The dashed lines correspond to the prediction (3) of the AHV formula (1).

3. Recurrence relations

Eq. (4) gives a good description of all the bands in even–even, odd-mass and odd–odd nuclei [8,9], and, by using it, the Mallmann-type energy ratios within a band can be written as

$$R(j+2n/j+2) = n + \frac{n(n-1)}{2}\frac{\varepsilon_4}{E(j+2)} + \frac{n(n-1)(n-2)}{6}\frac{\varepsilon_6}{E(j+2)}\,. \quad (6)$$

This relation can be applied to different states (n) in the band, and, by eliminating the parameters $\varepsilon_4/E(j+2)$ and $\varepsilon_6/E(j+2)$, one gets recurrence relations which express the energy ratio of a state as a function of other two lower ratios in the band [12]. For example:

$$R(j+2n/j+2) = n + \frac{n}{n-3}R(j+2n-2/j+2) - \frac{n(n-1)}{2(n-3)}R(j+4/j+2) \quad (7)$$

gives the Mallmann ratio for the n-th state in the band as a function of those of the $(n-1)$-th and 2-nd excited state in the band. Similarly,

$$R(j+2n/j+2) = \frac{n}{n-2} \times \left[\frac{2}{n-3} + R(j+2n-2/j+2) - \frac{n-1}{n-3}R(j+2n-4/j+2)\right] \quad (8)$$

gives the Mallmann ratio for the state n in the band, as a function of those of the states $(n-1)$ and $(n-2)$. Such type of relations, giving the energy ratio for the n-th state as functions of energy ratios of any two states with spins lower than $j+2n$ can be established. We note that these recurrence relations can also be obtained from the empirical energy recurrence relations of Buck *et al.* [13], which were shown to be satisfied by a general solution for the band energies as a function of spin similar to that given by Eq. (5). Fig. 4 shows how relation (8) works for our collection of bands by comparing the calculated ratios using the data for lower spins with the empirical ones.

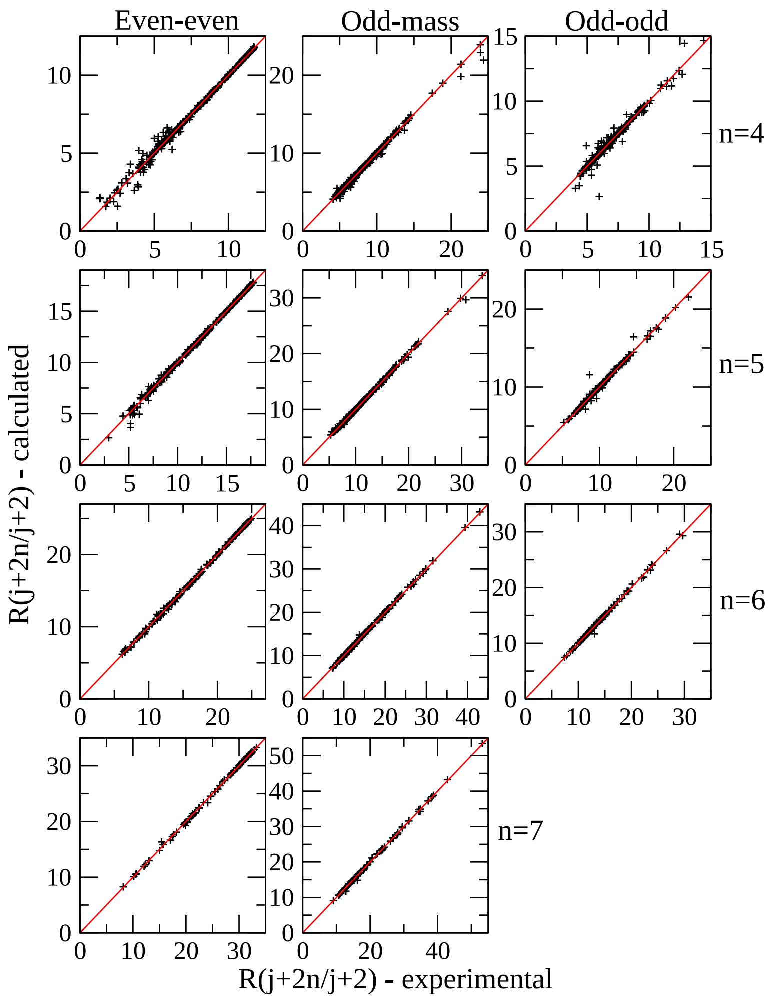

Fig. 4. Comparison of the experimental $R(j+2n/j+2)$ ratios with those calculated using formula (8). The continuous lines show the equality of the two quantities.

One can see from the distributions of the ratio $R(j+2n/j+2)_{\rm exp}/R(j+2n/j+2)_{\rm calc}$ (which is 1.0 for perfect agreement), showed in Fig. 5, that for the overwhelming majority of the cases the deviation between the experimental and calculated values is below 1%. In a small number of cases (compared to the total number), mostly in transitional nuclei ($R(4/2) < 2.5$ in the even–even nuclei) one can observe larger deviations. An inspection of, *e.g.*, the even–even nuclei shows that most of these cases correspond to nuclei with numbers of nucleons differing by 2 from a magic number (Zn, Cd, Te, Hg), or shape coexistence regions (Ge, Se, Kr), therefore g.s.b. with non-collective effects or perturbations. A more strict collectivity criterion would eliminate these cases. Among all possible recurrence relations for the ratio of the n-th state, relation (8) is the most accurate, since the highest states "known" in the band, $(n-1)$ and $(n-2)$, collect the maximum information on the anharmonicity of the band.

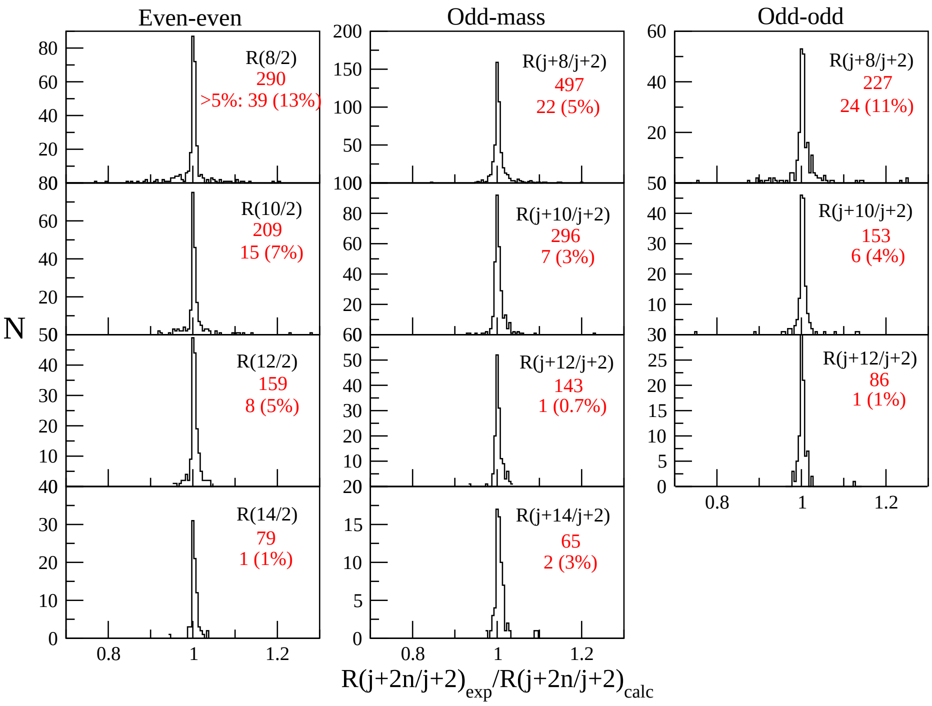

Fig. 5. Similar with Fig. 4, but showing the distributions of the ratio between the experimental and the calculated [formula (8)] energy ratios (this ratio is 1.0 for perfect agreement). In each case, the numbers indicate: the total number of cases, the number of cases for which the discrepancy is larger than 5%, and (within parentheses) the percentage of the later from the total number of cases.

Fig. 6 shows a detailed comparison of the experimental energy ratios for the SD bands data shown in Fig. 3, with those calculated with formula (8). One can observe an excellent agreement between the two quantities, similar with that obtained for the good rotational bands ($R_{4/2} \geq 3.30$) with normal deformation (Figs. 4, 5). The eight $n = 4$ cases where the two quantities differ by more than 1% are actually expected, because they correspond to bands perturbed in the region of the lowest states, as it could be immediately seen from the irregularities in the plot of the dynamical moment of inertia [11]. For example, one of these cases corresponds to a highly deformed band in ^{133}Nd, which presents a strong mixing of the lowest levels with those of another band [14]. The other (smaller) deviations can be also related to band perturbations [11].

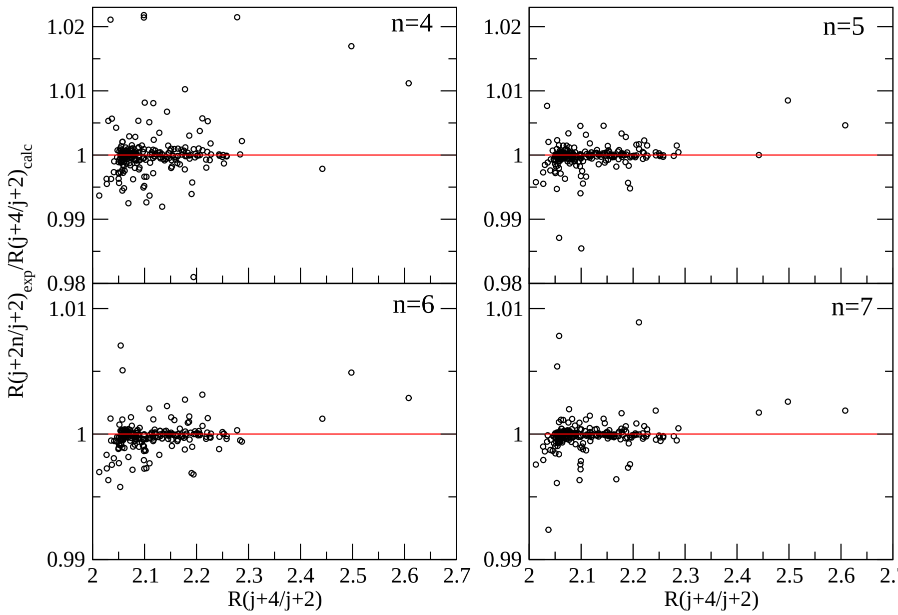

Fig. 6. Comparison of the experimental $R(j+2n/j+2)$ ratios with those calculated using formula (8), for the collection of SD bands (Fig. 3).

The fact that relation (8) works very well for all nuclei, from vibrational to rotational, implies that relation (4), from which it is deduced, represents a universal evolution of the excitation energies with the relative spin of the band states. The parameter-free recurrence relations that can be deduced from Eq. (4) can be used to predict higher spin members, based on the energy of lower spin members of the band.

4. Conclusions

The correlations between energy ratios within bands have a universal character: various bands in all collective nuclei (even–even, odd–even, and odd–odd), and even the superdeformed bands, display a similar behavior. The energies of the band members follow very well the spin dependence $E(J) = \alpha J + \beta J^2 + \gamma J^3$, which is the same with that predicted by the AHV model. At the same time, based on this spin dependence of the energy of the states, recurrence relations between energy ratios can be obtained. These relations can be used to predict with high accuracy (if there is no perturbation of the band) the energy of higher members of the band.

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