Vol. 40 (2009) ACTA PHYSICA POLONICA B No 3

# A NEW MEAN FIELD APPROACH FOR EXOTIC NUCLEI*

H. Molique[a,b], J. Dudek[b], K. Rybak[b], M.G. Porquet[c]

[a]Institut Universitaire de Formation des Maîtres d'Alsace
141, Avenue de Colmar, 67100 Strasbourg, France
[b]IPHC/DRS Université Louis Pasteur
23, rue du Loess B.P.28, 67037 Strasbourg Cedex 2, France
[c]Centre de Spectrométrie Nucléaire et de Spectrométrie de Masse
91405 Orsay, France

*(Received November 4, 2008)*

We present a new phenomenological mean field approach aiming at the calculation of properties of exotic nuclei. This approach combines the microscopic description of the spin-orbit properties in terms of particle densities, but also vector spin-orbit densities, inspired by results obtained within the Skyrme Hartree–Fock formalism, thus including the contribution of the tensor force. At the same time, the new approach preserves the simplicity of the phenomenological Woods–Saxon calculations and, more importantly, the robustness of the latter towards extrapolations in terms of increasing number of particles and/or isospin.

PACS numbers: 21.60.–n, 21.10.–k

## 1. Introduction

The origin of the nuclear spin-orbit splitting has been a subject of intensive studies during many decades and the present day view-points converge to the idea that both the spin-orbit and the tensor interactions contribute importantly to this observable. In particular, the spin-orbit and the tensor forces respond to different degrees of freedom: while the spin-orbit potential collects the contributions from all active nucleons through the gradients of the densities, the tensor interaction contributes to the mean field mainly due to the presence of the spin unsaturated orbitals.

Historically, it has been noticed very early [1,2] that the spin unsaturated orbitals may contribute to the spin-orbit splitting with the "wrong" sign (thus contributing to a decrease in the finite splitting). During the same period,

* Presented at the Zakopane Conference on Nuclear Physics, September 1–7, 2008, Zakopane, Poland.

the tensor forces within the Skyrme interactions have been investigated, although applications have remained limited [3]. In the framework of the phenomenological realisation of the mean field approach, the effect of the spin-unsaturated orbitals on the spin-orbit splitting has been investigated further on [4]. More recently, there was again a revival of interest in the question of the tensor force in the framework of the shell model [5] as well as Skyrme Hartree–Fock calculations [6].

## 2. Presentation of the method and results

In this article we focus on the phenomenological realisation of the realistic nuclear mean-field; we let ourselves be guided by the Hartree–Fock formalism and consider, for simplicity, only spherically symmetric even-even nuclei. Within the Skyrme Hartree–Fock formalism [3, 7], it can be shown that the operator proportional to the spin-orbit term $\hat{l} \cdot \hat{\sigma}$ for the particle of type $q$ (denoting either protons or neutrons) reads

$$\hat{V}_{\mathrm{SO}}^{q} = \frac{1}{r} W_q(r)\, \hat{l} \cdot \hat{\sigma} \,, \tag{1}$$

where the form-factor $W_q$ is given by

$$W_q = \frac{W_0}{2}\Big(2\frac{d\rho_q}{dr} + \frac{d\rho_{q'}}{dr}\Big) + \Big(\alpha J_q + \beta J_{q'}\Big)\,. \tag{2}$$

Here, $\rho_q$ and $J_q$ represent the particle and vector spin-orbit densities (also called spin-orbit current), respectively, and if $q$ represents the protons, then $q'$ stands for neutrons (and *vice versa*). It is important to notice that the terms depending on the particle densities originate from the spin-orbit part of the Skyrme interaction, whereas the terms containing the spin-orbit currents stem both from the central and the tensor part of the Skyrme Hamiltonian (see [8] for more details).

Guided by this result, we will now use the following form of the effective potential keeping the Woods–Saxon parametrisation only for the central term:

$$\hat{V} = V_{\mathrm{central}}^{\mathrm{WS}}(r) + \frac{1}{r}\left(\lambda^{qq}\frac{d\rho_q}{dr} + \lambda^{qq'}\frac{d\rho_{q'}}{dr}\right)\hat{l} \cdot \hat{\sigma} + \frac{1}{r}\left(\alpha J_q + \beta J_{q'}\right)\hat{l} \cdot \hat{\sigma} \,, \tag{3}$$

where $V_{\mathrm{central}}^{\mathrm{WS}}$ denotes the traditional three-parameter Woods–Saxon potential. In our preliminary tests we use the following constraints [9] on the constants[1]:

$$\lambda^{qq} = \lambda^{qq'} = \lambda > 0 \quad \text{and} \quad \alpha + \beta \simeq 0\,; \qquad \alpha < 0\,; \quad \beta > 0\,. \tag{4}$$

[1] Our constants are expressed in units 50 MeV·fm$^5$; the numerical constant has been introduced for convenience.

From Fig. 1 it is clearly seen that the effect of including the tensor contribution to the spin-orbit potential has almost no influence on the proton spin-orbit splitting of the $\pi\{1d\}$ partners in the spin-saturated nucleus $^{40}$Ca, whereas one observes a reduction of this splitting in the case of the spin unsaturated nucleus $^{48}$Ca.

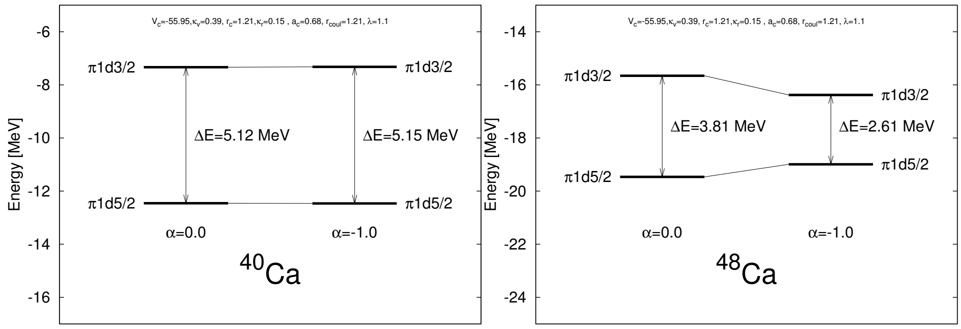


Fig. 1. Energies of the proton spin-orbit partners $\pi 1d_{3/2}$ and $\pi 1d_{5/2}$ without $(\alpha = 0)$ and with $(\alpha = -1.0)$ the tensor force contribution to the spin-orbit form factor. The left pannel shows the case of the spin-saturated nucleus $^{40}$Ca, whereas the right pannel applies for the spin-unsaturated nucleus $^{48}$Ca.

In Fig. 2 one can see that for both *i.e.* $^{40}$Ca and $^{48}$Ca nuclei the radial components of the proton vector spin-orbit densities are relatively small in absolute values. For the neutrons, the situation is quite different in the sense that for the spin-saturated nucleus $^{40}$Ca the vector spin-orbit contribution is also small, but it gets large and essentially positive in the case of the neutron spin-unsaturated nucleus $^{48}$Ca. This result stems from the fact that in the second nucleus only the neutron $1f_{7/2}$ shell is occupied, its spin-orbit partner $1f_{5/2}$ being empty.

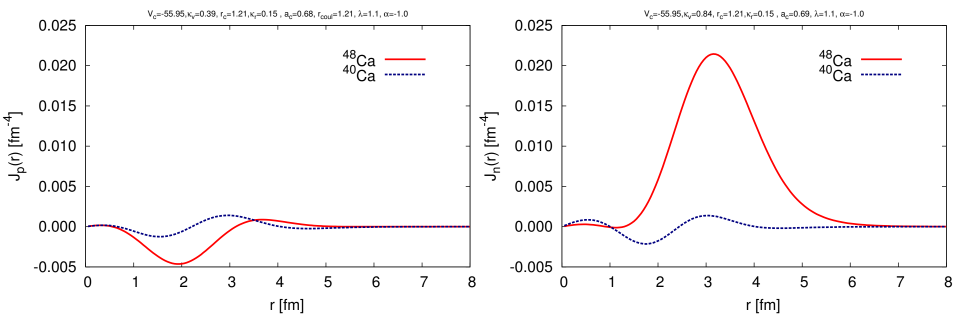


Fig. 2. Radial components for protons (left) and neutrons (right) of the vector spin-orbit densities in the spin-saturated $^{40}$Ca and the spin-unsaturated $^{48}$Ca.

Fig. 3 shows the spin-orbit potentials for protons, constructed out of the particle densities and the vector spin-orbit densities in function of the radial variable $r$:

$$V_\rho^p = r^2(1/r)\lambda[d\rho_p/dr + d\rho_n/dr] \tag{5}$$

and

$$V_\mathrm{T}^p = r^2(1/r)[\alpha J_p + \beta J_n] = r^2(1/r)\alpha[J_p - J_n]\,. \tag{6}$$

It is clear that for the neutrons we would have $V_\rho^n = V_\rho^p$ and $V_\mathrm{T}^n = -V_\mathrm{T}^p$. It can easily be seen from the figure that the proton "tensor" potential is almost zero in the $^{40}$Ca nucleus, but significant and positive in $^{48}$Ca, thus leading to an "abnormal" spin-orbit splitting in this case.

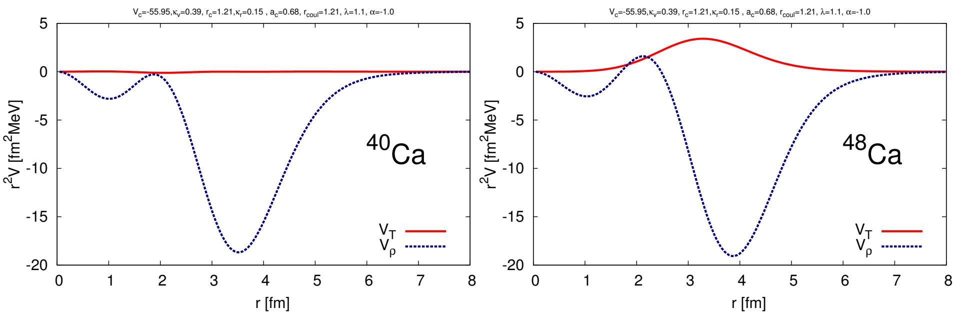


Fig. 3. "Ordinary" and "tensor" spin-orbit potentials for protons in the spin-saturated nucleus $^{40}$Ca (left) and the spin-unsaturated nucleus $^{48}$Ca (right).

## 3. Summary and outlook

We have presented some illustrations of our pilot project aiming at the new formulation of the phenomenological mean-field interaction Hamiltonian. In this formulation the central potential, very robust in terms of extensions into the exotic $Z$ and $N$ combinations is parametrized "traditionally" with the help of the Woods–Saxon form-factor. The "structure sensitive" terms, the spin-orbit contribution depending on the gradients of the densities and the tensor contributions representing the spin unsaturated orbitals are written down with the help of the nucleonic wave functions. It is expected that this construction approximates with a higher level of fidelity the realistic, configuration-dependent structure effects in nuclei. At present the parameters of the potential are optimised to the experimental data on the spherical doubly-magic nuclei.

## REFERENCES


[1] C.W. Wong, *Nucl. Phys.* **A108**, 481 (1968).

[2] R.R. Scheerbaum, *Phys. Lett.* **B63**, 381 (1976).

[3] Fl. Stancu, D.M. Brink, H. Flocard, *Phys. Rev. Lett.* **B68**, 108 (1977).

[4] J. Dudek, W. Nazarewicz, T. Werner, *Nucl. Phys.* **A341**, 253 (1980); M. de Voigt, J. Dudek, Z. Szymański, *Rev. Mod. Phys.* **55**, 949 (1983).

[5] T. Otsuka, T. Suzuki, R. Fujimoto, H. Grawe, Y. Akaishi, *Phys. Rev. Lett.* **95**, 232502 (2005).

[6] J. Dobaczewski, Proceedings of the Third ANL/MSU/JINA/INT RIA Workshop, World Scientific, Singapore 2006.

[7] D. Vautherin, D.M. Brink, *Phys. Rev.* **C5**, 626 (1972).

[8] T. Lesinski, M. Bender, K. Bennaceur, T. Duguet, J. Meyer, *Phys. Rev.* **C76**, 014312 (2007).

[9] D. Brink, Fl. Stancu, *Phys. Rev.* **C75**, 064311 (2007).