# NEW FORMULATION OF INTERACTING BOSON MODEL AND HEAVY EXOTIC NUCLEI\*

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A novel way of determining the Hamiltonian of the interacting boson model is proposed. The multi-fermion dynamics of surface deformations studied by the mean-field theory, *e.g.*, Skyrme model, can be mapped, in a good approximation, onto a boson system. The method is examined for well-known nuclei, and predictions are presented for unexplored territories of the nuclear chart, namely, Os–W region nuclei with  $A \gtrsim 200$ .

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# 1. Introduction

The quadrupole collectivity is a prominent aspect in the nuclear structure for both stable and exotic nuclei and has been extensively studied in terms of the interacting boson model (IBM) [1]. While the IBM has been successful in describing the experimental data, the parameters of its Hamiltonian are in many cases determined phenomenologically. Besides that, the IBM has its own microscopic foundation, which has been studied by seniority truncation using the shell model and applied only to spherical shapes [2–5]. For general cases, however, the microscopic foundation of the IBM remains an open question. In this study, we propose a novel way of determining the parameters of the IBM Hamiltonian for general cases, starting from the mean-field model [6].

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# 2. Mean-field derivation of an IBM Hamiltonian

While the mean-field model, *e.g.*, the Skyrme model, has been successful in describing the intrinsic properties of all nuclei [7], one has not been able to calculate in general the levels and wave functions of excited states with the exact treatment of the angular momentum and the particle number [8]. Thereby, the Skyrme model seems to be insufficient for nuclear spectroscopy. The IBM is a model of the quadrupole collectivity. Thus, it should be very interesting to construct the IBM Hamiltonian based on the Skyrme model.

We first perform the constrained Skyrme Hartree–Fock+BCS calculations [9] and obtain the potential energy surfaces (PES's). Once we get the PES (HF-PES, for brevity) in the mean-field calculations, we map a point of it, denoted as ( $\beta_{BM}$ ,  $\gamma_{BM}$ ), onto a point of the boson PES (IBM-PES, for brevity) in a good approximation.

In the present study, we discuss the IBM-2 [2], taking the standard Hamiltonian  $\hat{\mathcal{H}} = \epsilon (\hat{n}_{d\pi} + \hat{n}_{d\nu}) + \kappa \hat{\mathcal{Q}}_{\pi} \cdot \hat{\mathcal{Q}}_{\nu}$ . The IBM-PES is calculated as an expectation value of  $\hat{\mathcal{H}}$  in the boson coherent state [10]

$$E(\beta_b, \gamma_b) = \epsilon (n_\pi + n_\nu) \beta_b^2 (1 + \beta_b^2)^{-1} + 2n_\pi n_\nu \kappa \beta_b^2 (1 + \beta_b^2)^{-2} \times [2 - \sqrt{2/7} (\chi_\pi + \chi_\nu) \beta_b \cos 3\gamma_b + \chi_\pi \chi_\nu \beta_b^2/7], \qquad (1)$$

where  $\beta_b$  and  $\gamma_b$  represent the intrinsic variables common to proton and neutron bosons. We also assume  $\beta_b = C_\beta \beta_{BM}$  (with  $C_\beta$  a coefficient) and  $\gamma_b = \gamma_{BM}$  for simplicity. The HF-PES is simulated by the IBM-PES using the parameters in Eq. (1), which are determined so that the latter reproduces the former as well as possible. The overall pattern of the HF-PES reflects the effects of nuclear force and Pauli principle for determining the energy of collective states. By reproducing the HF-PES, the IBM-PES is expected to simulate, to a good extent, these effects in a simple manner.



Fig. 1. Comparisons of PES's calculated by HF and IBM. Contour spacing is 0.1MeV. Minima are identified by solid circles.

Fig. 1 depicts the comparisons of the PES's, where the IBM-PES's reproduce quite nicely the HF-PES's for typical cases, *i.e.*, near spherical (<sup>148</sup>Sm), axially deformed (<sup>154</sup>Sm) and  $\gamma$ -unstable (<sup>132,134</sup>Ba and <sup>208</sup>W) nuclei. Fig. 2(a) exhibits the evolutions of the IBM parameters for Sm isotopes with the neutron number N, reflecting the spherical-deformed shape transition. These variations of the parameters produce levels consistent with experimental tendencies without adjustment to levels.



Fig. 2. (a) Evolutions of the parameters in Eq. (1) with the neutron number N.  $\chi_{\pi}$  is kept constant. (b) Experimental, (c) calculated (IBM from SLy4) levels for Sm isotopes and (d) calculated (IBM from SkM<sup>\*</sup>) levels for W isotopes with N.

#### 3. Results

Figs. 2(b) and 2(c) show the low-lying levels of Sm isotopes as functions of N, computed by the **npbos** code [11]. Around N = 86, the spectra look like those of the U(5) limit in the IBM. As N increases, levels come down consistently with the experimental trends particularly for yrast states. Around N = 90, there seems to be the X(5) critical-point symmetry [12], beyond which  $0_2^+$  and  $2_2^+$  states go up in both experiment and calculation. Finally around N = 92, the levels look like the rotational band (or SU(3)). According to Figs. 1(e)–(h), <sup>134</sup>Ba has larger flat area than <sup>132</sup>Ba: <sup>134</sup>Ba

According to Figs. 1(e)–(h), <sup>134</sup>Ba has larger flat area than <sup>132</sup>Ba: <sup>134</sup>Ba is more like the E(5) critical-point symmetry [13], while <sup>132</sup>Ba is closer to O(6). The calculated levels agree well with the experimental data.

Now we turn to the exotic W isotopes with  $A \gtrsim 200$ . Figs. 1(i) and 1(j) show HF- and IBM-PES's for <sup>208</sup>W. Both PES's have large flat areas like <sup>134</sup>Ba, suggesting the E(5) symmetry. In Fig. 2(d), level evolution in W  $(A \gtrsim 200)$  isotopes is shown. Around N = 128, the levels look like those of U(5) symmetry. As the magnitude of the deformation becomes larger with N, each level comes down, keeping the  $\gamma$ -unstable E(5)–O(6) pattern. Such sustained E(5)–O(6) pattern has never been seen in stable nuclei and may be a characteristic feature of exotic nuclei. Around N = 138, levels look like those of  $\gamma$ -unstable nuclei, or O(6) limit.

## 4. Conclusions

In summary, we propose a novel way to derive the IBM Hamiltonian. The mean-field model and the IBM can be complementary, where the former plays a role to determine the parameters and the latter calculates levels and wave functions precisely. Using this method, low-lying collective states for three dynamical symmetries, as well as recently proposed X(5) and E(5) critical-point symmetries. More importantly, we gain a capability to predict levels and wave functions in unknown territories on the nuclear chart. This can be a great advantage in the era of the third-generation rare-isotope accelerators producing many new *heavy* exotic nuclei.

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