

MICROSCOPIC CALCULATION AND LOCAL
APPROXIMATION OF THE SPATIAL DEPENDENCE
OF THE PAIRING FIELD WITH BARE AND INDUCED
INTERACTION*

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(Received December 2, 2008)

The pairing gap obtained adding the bare nucleon–nucleon potential and the interaction induced from the exchange of collective vibrations is strongly peaked at the nuclear surface. It is possible to mock up the detailed spatial dependence of this field using a contact interaction, with parameters which are quite different from those commonly used in more phenomenological approaches.

PACS numbers: 21.10.–k, 21.30.Fe

1. Introduction

The coupling with collective vibrations plays a key role in finite nuclei, both in the particle–hole and in the particle–particle channel. On the one hand, it renormalizes in an important way the level densities at the Fermi energy, modifying the quasiparticle strength and the occupation factors [1]. On the other hand, the exchange of vibrations between nucleons moving in time reversal states lying close to the Fermi energy induces an interaction which has been shown to account for an important fraction of the pairing gap Δ both in stable and in halo nuclei [2]. In this work we study the

* Presented at the Zakopane Conference on Nuclear Physics, September 1–7, 2008, Zakopane, Poland.

spatial dependence of Δ in the nucleus ^{120}Sn , solving the gap equation with a pairing interaction that is the sum of a bare nucleon–nucleon force and of the induced interaction, taking into account self-energy effects.

2. Theory

We start by performing a Hartree–Fock calculation with the two-body interaction SLy4 (associated with a k -mass $m_k \approx 0.7m$ at saturation density), obtaining a set of single-particle energy levels e_{nlj} . Using different pairing interactions, that will be discussed below, we then solve in the calculated HF basis the Hartree–Fock–Bogoliubov (HFB) equations in the pairing channel. The calculation are performed in a spherical box of radius $R_{\text{box}} = 15$ fm. For more details we refer to Ref. [4]. We adopt as a pairing interaction the sum of the bare nucleon–nucleon interaction, here taken to be the Argonne v_{14} interaction v_{Arg} , and of the interaction v_{ind} induced by the exchange of density and spin vibrations. The total matrix elements are given by

$$\langle \nu'_1 m' \nu'_2 \bar{m}' | v_{\text{Arg}+\text{ind}} | \nu_1 m \nu_2 \bar{m} \rangle = Z \langle \nu'_1 m' \nu'_2 \bar{m}' | v_{\text{Arg}} + v_{\text{ind}} | \nu_1 m \nu_2 \bar{m} \rangle, \quad (1)$$

where Z denote the quasi-particle strength around the Fermi energy, taken to be constant with the typical value $Z = 0.7$. In this way we take approximately into account the reduction of particle-coupling vertex by a factor Z^2 , which is partially compensated by an increase in the single-particle density by a factor Z [3].

The diagonal pairing matrix elements $\Delta_{nn'lj}$ obtained with the Argonne interaction (and $Z = 1$) and with the the matrix elements (1) are compared in Fig. 1(a). From them we can construct the pairing field, that is

$$\Delta(\vec{r}_1, \vec{r}_2) = \sum_{nn'lj} \Delta_{nn'lj} \psi_{nn'lj}(\vec{r}_1, \vec{r}_2), \quad (2)$$

which is shown in Fig. 1(b) for the total interaction, as a function of the center of mass $R_{\text{c.m.}}$ and of the relative distance r_{12} of the Cooper pair. The strong repulsive core associated with the bare interaction leads to negative values of the pairing field for small values of r_{12} . It is more useful to consider the Fourier transform of the pairing field, shown in Fig. 1(c), which displays a strong peak at the nuclear surface, $R_{\text{c.m.}} \sim 6$ fm. While the pairing field (2) is a non-local quantity, we can define a local pairing field Δ_{loc} through the Thomas–Fermi approximation, calculating the Fourier transform at the local Fermi momentum: $\Delta_{\text{loc}} = [\Delta(R_{\text{c.m.}}, k_F(R_{\text{c.m.}}))]$. In Fig. 1(d) we show the local fields obtained using the bare Argonne force and the $v_{\text{Arg}+\text{ind}}$ interaction (1). The induced interaction enhances the surface character of the bare interaction. The negative values in the interior of the nucleus are

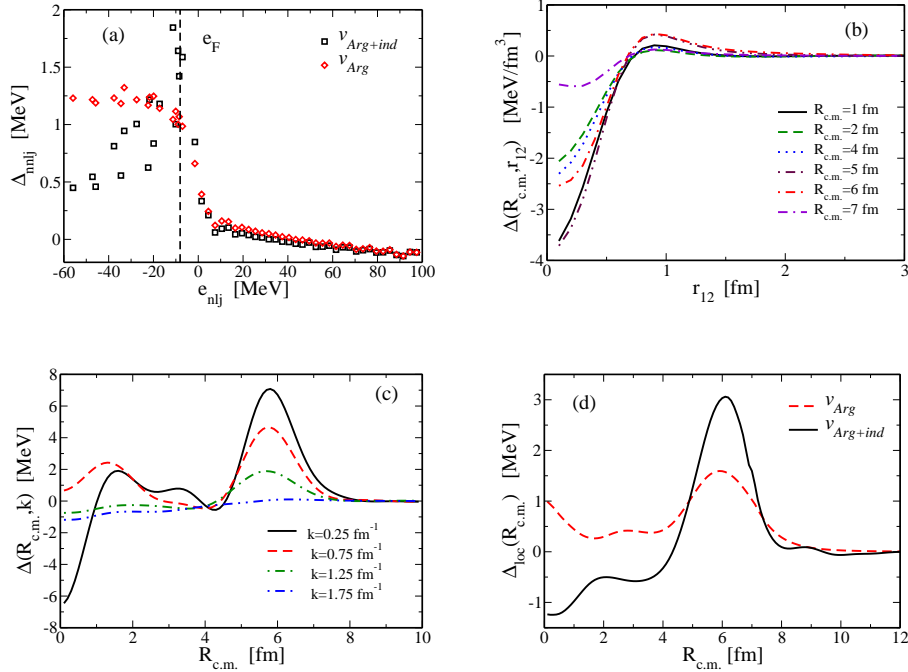


Fig. 1. Different pairing gaps and Cooper pair wavefunctions obtained with the $v_{Arg+ind}$ interaction. See text for details.

due to the coupling with the spin-modes (prevalently non natural parity phonons). In more phenomenological approaches, one often employs a zero-range, density-dependent interaction of the type

$$v^\delta(\vec{r}_1, \vec{r}_2) = v_0 \left[1 - \eta \left(\frac{\rho \left(\frac{\vec{r}_1 + \vec{r}_2}{2} \right)}{\rho_0} \right)^\alpha \right] \delta(\vec{r}_1 - \vec{r}_2), \quad (3)$$

together with a cut-off. We have found that we can reproduce rather accurately the local pairing gap resulting from our microscopic calculations using the interaction (3). The values of the cutoff and of the strength v_0 are taken from Ref. [5], while we fix the parameters α, η so as to reproduce the spatial dependence of the pairing gap shown in Fig. 1(d). We obtain the values $\alpha = 0.66$, $\eta = 0.84$ for the bare interaction, and $\alpha = 2.00$, $\eta = 1.31$. The latter are quite different from those commonly adopted in phenomenological interactions. In Table I we show the pairing gaps and the pairing energies obtained with Eq. (3) and with the full calculation. Extending our study

of the bare Argonne interaction to even tin isotopes (from ^{102}Sn to ^{130}Sn) and calcium isotopes (from ^{36}Ca to ^{46}Ca) we found an average value of the parameters $\bar{\alpha} = 0.67$ and $\bar{\eta} = 0.82$, in good agreement with those obtained for the case of ^{120}Sn .

TABLE I

Pairing gaps at the Fermi energy and pairing energies (in MeV) obtained with the full calculation, $\Delta_{\text{F}}^{\text{full}}$ and $E_{\text{pair}}^{\text{full}}$, and with the corresponding density dependent interaction, $\Delta_{\text{F}}^{\delta}$ and E_{pair}^{δ} .

Interaction	$\Delta_{\text{F}}^{\text{full}}$	$\Delta_{\text{F}}^{\delta}$	$E_{\text{pair}}^{\text{full}}$	E_{pair}^{δ}
v_{Arg}	1.04	1.03	-13.2	-8.9
$v_{\text{Arg+ind}}$	1.47	1.28	-15.78	-14.47

3. Summary and conclusions

The coupling of quasiparticles with collective surface vibrations gives rise to an induced pairing interaction which renormalizes the bare nucleon-nucleon interaction in an important way, leading to a total pairing field which is strongly peaked at the surface of the nucleus. Although the pairing induced interaction is non-local and energy dependent, it is possible to adopt a semiclassical approximation, which yields a local pairing field that reproduces to a good accuracy the features of the full quantal solution. This local field can also be obtained adopting the widely used zero-range, density-dependent interaction, with an appropriate choice of the parameters, which turn out to be quite different from those usually employed in more phenomenological approaches.

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