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SUPERALLOWED BETA DECAY: THE ROLE OF NUCLEAR STRUCTURE IN STANDARD-MODEL TESTS*

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Measurements on superallowed $0^+ \to 0^+$ nuclear beta transitions currently provide the most demanding test of the Conserved Vector Current (CVC) hypothesis and the most precise value for the up-down element, V_{ud}, of the Cabibbo–Kobayashi–Maskawa (CKM) matrix. Both are sensitive probes for physics beyond the Standard Model. Analysis of the experimental results depends on small radiative and isospin-symmetry-breaking corrections, some of which depend on the specific structure of the parent and daughter nuclei involved. These calculated corrections affect the precision of the results, and experiments are currently focused on reducing their uncertainties. Although nuclear structure only contributes to rather small corrections, it plays a crucial role in these fundamental tests.

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1. Introduction

Beta decay between nuclear analog states of spin-parity, $J^\pi = 0^+$, and isospin, $T = 1$, has a unique simplicity: it is a pure vector transition and is nearly independent of the nuclear structure of the parent and daughter states. The measured strength of such a transition — expressed as an ft value — can then be related directly to the vector coupling constant, $G_{\rm V}$ with the intervention of only a few small (~1%) calculated terms to account for radiative and nuclear-structure-dependent effects. Once a reliable value has been determined for $G_{\rm V}$, it is only a short step to obtain from it the value for V_{ud}, the up-down mixing element of the Cabibbo–Kobayashi–Maskawa (CKM) matrix; and only another short step to the most demanding available test of the unitarity of that matrix, one of the basic precepts of the electroweak standard model.

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In dealing with these decays, which are referred to as superallowed, it is convenient to combine some of the small correction terms with the measured ft-value and define a "corrected" $\mathcal{F}t$-value. Thus, we write [1]

$$\mathcal{F}t \equiv ft(1+\delta_R')(1+\delta_{NS}-\delta_C) = \frac{K}{2G_V^2(1+\Delta_R^V)}\,, \quad (1)$$

where $K/(\hbar c)^6 = 2\pi^3\hbar \ln 2/(m_e c^2)^5 = 8120.2787(11)\times 10^{-10}$ GeV^{-4}s; δ_C is the isospin-symmetry-breaking correction and Δ_R^V is the transition-independent part of the radiative correction. The terms δ_R' and δ_{NS} comprise the transition-dependent part of the radiative correction, the former being a function only of the electron's energy and the Z of the daughter nucleus, while the latter, like δ_C, depends in its evaluation on the details of nuclear structure. From this equation, it can be seen that a measurement of any one of these superallowed transitions establishes an individual value for G_V; moreover, if the Conserved Vector Current (CVC) assertion is correct that G_V is not renormalized in the nuclear medium, all such values — and all the $\mathcal{F}t$-values themselves — should be identical within uncertainties, regardless of the specific nuclei involved.

This assertion of CVC can be tested and a value for G_V obtained with a precision considerably better than 0.1% if experiment can meet the challenge, since the four small correction terms only contribute to the overall uncertainty at the 0.03% level. As it turns out, experiment has exceeded that goal, leaving theory currently as the dominant contributor to the uncertainty.

2. New survey results

The ft-value that characterizes any β-transition depends on three measured quantities: the total transition energy, Q_{EC}, the half-life, $t_{1/2}$, of the parent state and the branching ratio, R, for the particular transition of interest. The Q_{EC}-value is required to determine the statistical rate function, f, while the half-life and branching ratio combine to yield the partial half-life, t.

We have just completed a new survey of world data on superallowed $0^+ \to 0^+$ beta decays [2], in which all previously published measurements were included, even those that were based on outdated calibrations if enough information was provided that they could be corrected to modern standards. In all, more than 150 independent measurements of comparable precision, spanning nearly five decades, made the cut. A total of ten transitions yielded ft values with 0.1% precision or better, and three more had precision between 0.1 and 0.4%.

Compared with our last review four years ago [1], there have been numerous improvements. First, we have added 27 new measurements published since 2004. Second, we have used the recently improved isospin-symmetry breaking corrections [3]. Third, our calculation of the statistical rate function f now accounts for possible excitation in the daughter atom, a small effect but one which merits inclusion at the present level of experimental precision. Finally, we have re-examined the systematic uncertainty associated with the isospin symmetry-breaking corrections by evaluating the radial-overlap correction using Hartree–Fock radial wave functions and comparing the results with our earlier calculations, which used Saxon–Woods wave functions; the provision for systematic uncertainty has been changed as a consequence.

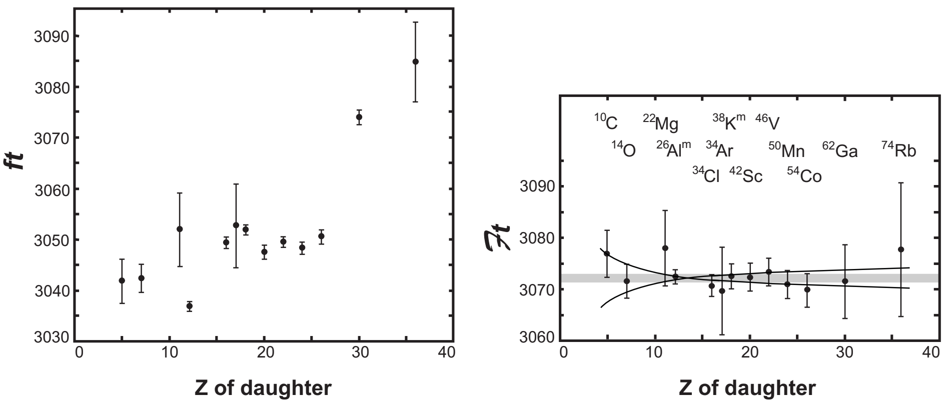

Fig. 1. Results from the 2008 survey [2]. In the left panel the uncorrected ft values are plotted as a function of the charge on the daughter nucleus. In the right panel, the corresponding $\mathcal{F}t$ values appear; they differ from the ft values by the inclusion of the correction terms $\delta'_{\rm R}$, $\delta_{\rm NS}$ and $\delta_{\rm C}$. The gray band in the right panel gives one standard deviation around the average $\overline{\mathcal{F}t}$ value, while the curved lines represent the approximate loci the $\mathcal{F}t$ values would follow if a scalar current existed with $C_{\rm S}/C_{\rm V} = \pm 0.002$.

From those results a number of important conclusions can be drawn. First, as illustrated in Fig. 1, the $\mathcal{F}t$ values for all thirteen transitions, covering the range from $A = 10$ to $A = 74$, form a consistent set with average value $\overline{\mathcal{F}t} = 3072.08(79)$ s and a normalized chi-square of 0.28. This result confirms the constancy of $G_{\rm V}$ to 1.3 parts in 10^4. Second, the survey results set a limit on any possible contribution from scalar currents. The curved lines in the right panel of the figure show that the presence of a scalar current — induced or fundamental — would manifest itself as a Z-dependence in the $\mathcal{F}t$ values, which would be most evident at low Z. There is no hint of any such curvature and a limit can be set on the scalar relative to the vector current of $|C_{\rm S}/C_{\rm V}| \leq 0.0024$ if the scalar current is assumed to violate parity maximally, as does the vector current; or $|C_{\rm S}/C_{\rm V}| \leq 0.065$ if that assumption is not made. Finally, with the test of CVC passed, it is possible to use the average value of G_V to obtain the up-down element of the CKM matrix via the relation $V_{ud} = G_V/G_F$, where G_F is the well known [4] weak-interaction constant for purely leptonic muon decay.

After adjusting the average $\overline{\mathcal{F}t}$ value to include a provision for possible systematic errors in the correction terms, we obtain the result,

$$V_{ud} = 0.97425(22)\,. \tag{2}$$

This number is completely consistent with, but more precise than the numbers we have obtained in previous analyses of superallowed β decay (see Fig. 2). It is also nearly a factor of ten more precise than the result obtained solely from neutron or pion decays, results with which it is also consistent. Evidently, for now the value of V_{ud} is determined entirely by the nuclear measurements, with its precision being limited only by the experimental uncertainties in the nuclear measurements and the theoretical uncertainties attached to their applied correction terms.

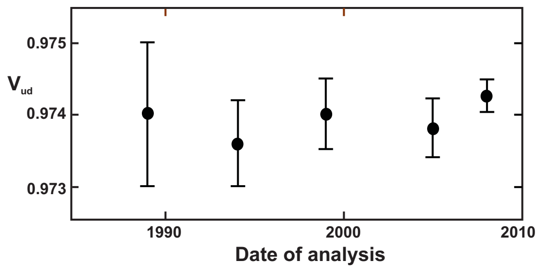

Fig. 2. Values of V_{ud} as determined from superallowed $0^+ \to 0^+$ β decays plotted as a function of analysis date, spanning the past two decades. In order from the earliest date to the most recent, the values are taken from Refs. [5–7], [1] and [2].

With our new value for V_{ud}, we can now test CKM-matrix unitarity by considering the sum of squares of the top-row elements. We take V_{us} from the recent FlaviaNet evaluation [8] and V_{ub} from the current Particle Data Group review [4], and obtain

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.99995(61)\,, \tag{3}$$

a result that shows unitarity to be fully satisfied at the 0.06% level. Only V_{us}, and V_{ud} contribute perceptibly to the uncertainty and their contributions are almost equal to one another. This may seem surprising since V_{ud} is known to much higher precision than V_{us}, but it follows from the fact that $|V_{ud}|^2$ contributes 95% to the unitarity sum.

3. The role of nuclear structure

If we examine the error budget for V_{ud}, we find that the quoted uncertainty, 0.00022, is dominated by theoretical, rather than experimental contributions. By far the largest contribution, 0.00018, arises from the uncertainty in $\Delta_{\rm R}^{\rm V}$; and the next largest, 0.00010, is due to the nuclear-structure-dependent corrections, $\delta_{\rm C}$-$\delta_{\rm NS}$. Experiment ranks third in significance, contributing only 0.00008 to the overall uncertainty. Finally, the radiation-correction term $\delta_{\rm R}'$ contributes a mere 0.00002.

The value we now use for $\Delta_{\rm R}^{\rm V}$ represents a really significant improvement in its calculation — by Marciano and Sirlin [9] — since our last survey was published. Nevertheless, even though its uncertainty has been decreased by a factor of two, this term still remains the largest contributor to the overall error budget. To improve it more must remain an important theoretical goal.

Of more immediate relevance to nuclear physicists is the second most important contributor to the overall uncertainty for V_{ud}: the nuclear-structure-dependent corrections, $\delta_{\rm C}$ and $\delta_{\rm NS}$. To make a manageable calculation, our approach to $\delta_{\rm C}$ has been to divide it into two parts [3]:

$$\delta_{\rm C} = \delta_{\rm C1} + \delta_{\rm C2}\,, \tag{4}$$

where $\delta_{\rm C1}$ is the difference in configuration mixing between the parent and daughter states as calculated with an effective Hamiltonian (including charge-dependent terms) evaluated in a modest-sized shell-model space. Since this space does not allow for nodal mixing, we correct for that limitation by computing the second component, $\delta_{\rm C2}$, to account for the mismatch in the radial wave function between the parent and daughter states. The idea is that $\delta_{\rm C1}$ is the result of a tractable shell-model calculation that does not include any nodal mixing, while $\delta_{\rm C2}$ then corrects for the nodal mixing that would be present if the shell-model space were larger.

Clearly our charge-dependent correction terms are based on the shell model, and require approximations to make their computation possible. This approach brings an important advantage however. It allows us — for most of the nuclei in our superallowed survey — to use well-established shell-model and related parameters, which have been determined from experimental data that are completely independent of the superallowed ft values. As is clearly evident from Fig. 1, these calculated corrections do a remarkable job in converting widely scattered ft values into a consistent set of $\mathcal{F}t$ values. Furthermore, as shown in Ref. [3] they also closely reproduce the measured results for isospin-forbidden $0^+ \to 0^+$ β transitions in all nuclei for which the shell-model calculation is well specified.

These calculations are further supported by a less model-dependent calculation for one of the superallowed transitions. Only for the lightest superallowed emitter, ^{10}C, has it been possible so far even to come close to an exact treatment. Caurier *et al.* [10] have reported a large no-core shell-model calculation for that system but, even though they were able to extend their basis states up to $8\hbar\omega$, their calculated $\delta_{\rm C}$ still had not converged to a stable value. However they used their results together with perturbation theory to estimate that the full value of $\delta_{\rm C}$ should be about 0.19%. This agrees completely with our calculated value of 0.18(2)% (see Table VII in Ref. [3]).

Naturally, the uncertainties attributed to $\delta_{\rm C}$ arise from the input parameters used in their calculation: two-body matrix elements in the shell-model calculations, measured b and c coefficients of the Isobaric Multiplet Mass Equation (IMME), experimental charge radii and single-nucleon transfer-reaction data. The reliability — or even availability — of these parameters depends strongly on the nuclei under consideration. For superallowed transitions between nuclei with $10 \leq A \leq 38$, all required input parameters are well determined; and the situation is almost as favorable for cases with $42 \leq A \leq 54$, although arguably the shell-model matrix elements are somewhat less secure in the $f_{7/2}$ shell. However, for $A \geq 62$ very little relevant information is known and the shell model becomes less and less satisfactory as A increases. Consequently, the uncertainties attributed to the calculated nuclear-structure-dependent correction terms reflect these regional differences.

Future experiments, both those measuring superallowed ft values and those determining other spectroscopic parameters, can play an important role in reducing these uncertainties. The former will rely on a method, which is best described with reference to Fig. 3: it is based on the validity of the CVC hypothesis that the corrected $\mathcal{F}t$ values for the superallowed $0^+ \rightarrow 0^+$ decays should be constant. In the figure we compare the uncorrected measured ft values (points and error bars) with the quantity $\overline{\mathcal{F}t}/((1+\delta'_{\rm R})(1-\delta_{\rm C}+\delta_{\rm NS}))$ shown as a band, the width of which represents the assigned theory error. The band corresponds to the calculated corrections normalized to the data via the measured average $\mathcal{F}t$ value, $\overline{\mathcal{F}t}$. Thus, although this comparison does not test the absolute values of the correction terms, it does test the collective ability of all three calculated correction terms to reproduce the significant variations in ft from one transition to another. In fact, since $\delta'_{\rm R}$ is almost independent of Z when $Z > 10$, this test really probes directly the effectiveness of the calculated values of $(\delta_{\rm C} - \delta_{\rm NS})$.

It can be seen that there is remarkable agreement between theory and experiment. In assessing the significance of this agreement, it is important to recognize again that the origins of the calculated correction terms for

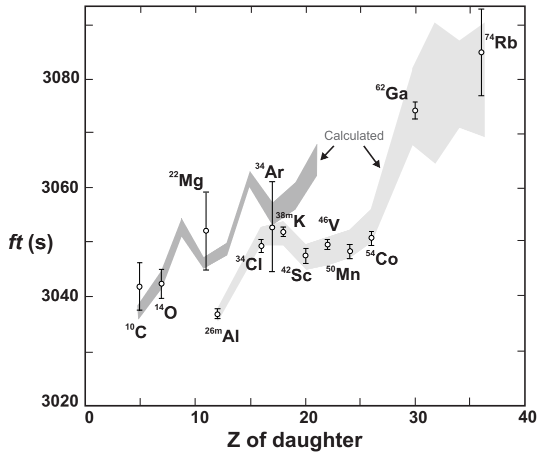

Fig. 3. Experimental ft values plotted as a function of the charge on the daughter nucleus, Z. Both bands represent the quantity $\overline{\mathcal{F}t}/((1+\delta_R')(1-\delta_C+\delta_{NS}))$. The two separate bands distinguish those beta emitters whose parent nuclei have isospin $T_z=-1$ (darker shading) from those with $T_z=0$ (lighter shading).

all cases are completely independent of the superallowed decay data. Thus, the agreement in the figure between the measured superallowed data points and the theoretical band, particularly for $Z \leq 26$ (*i.e.* $A \leq 54$) where the parameters of the calculations are well determined, is already a powerful validation of the calculated corrections themselves. The validation becomes even more convincing when we consider that it would require a pathological fault indeed in the theory to allow the observed nucleus-to-nucleus variations in δ_C and δ_{NS} to be reproduced in such detail while failing to obtain the *absolute* values to comparable precision. As satisfactory as the agreement in Fig. 3 is, though, new experiments can still improve the test, making it even more demanding, and can ultimately serve to reduce the uncertainty in the nuclear-structure-dependent corrections even further.

These new experiments can follow different paths. In the last four years, the biggest impact has come from experiments that focused on the "traditional nine" superallowed transitions, ^{10}C, ^{14}O, ^{26}Al^m, ^{34}Cl, ^{38}K^m, ^{42}Sc, ^{46}V, ^{50}Mn and ^{54}Co. New Penning-trap Q_{EC}-value measurements have been the most significant, but there have been new half-life and branching-ratio measurements as well. More improvements are still possible. If we accept as a goal that experiment should be more than a factor of two more precise than theory, then a close examination of the data in our survey [2] shows that the Q_{EC} values for ^{10}C, ^{14}O and ^{34}Cl, the half-lives of ^{26}Al^m, ^{34}Cl, ^{42}Sc and ^{50}Mn, and the branching ratios for ^{10}C and ^{14}O can all bear improvement. It is also particularly noteworthy that any improvements in the cases of ^{10}C and ^{14}O will lead directly to improvements in the limits on the possible existence of scalar currents. As is evident from the right-hand panel of Fig. 2, it is on these two low-Z superallowed transitions that a scalar current would have the largest effect. Unfortunately the branching ratios for both these transitions offer experimental obstacles that have proved very difficult to surmount.

A second experimental path is to expand the number of precisely measured superallowed emitters to include cases for which the calculated nuclear-structure-dependent corrections are larger, or show larger variations from nuclide to nuclide, than the values applied to the "traditional nine" cases. We argue that if the experimental ft values agree with the calculations where the nucleus-to-nucleus variations are large, then that must surely verify the calculations' reliability for the nine cases whose corrections are considerably smaller. Already four cases of this type have been carefully measured, ^{22}Mg, ^{34}Ar, ^{62}Ga and ^{74}Rb. They appear to agree well with the calculations although, with the exception of ^{62}Ga, their uncertainties are still five times greater than those for the best known transitions. Undoubtedly these uncertainties will be reduced and more cases added in the near future.

These new cases certainly present serious experimental challenges. The parent nuclei are more exotic than the traditional cases, which all have stable daughters, so they are more difficult to produce in pure and statistically significant quantities. They also exhibit more complex branching patterns: Each $T_Z = -1$ parent nucleus decays by Gamow–Teller transitions of comparable strength to the superallowed Fermi one, thus requiring the latter's branching ratio to be measured directly with high precision. For the $T_Z = 0$ parents with $A \geq 62$, each decay includes numerous weak Gamow–Teller transitions, which are very difficult to observe individually but which collectively constitute non negligible branching strength. In both regions, these problems are being, or have been overcome, albeit with very specialized techniques. The recently published branching-ratio measurement [11] for ^{62}Ga is an example of how even meticulously detailed spectroscopic studies must be combined with theory [12] to ensure that missing transitions are properly accounted for in the decays of the heavy $T_Z = 0$ parents.

There is a further important issue that arises for the superallowed emitters with $A \geq 62$: As noted already, the shell-model calculations of the structure-dependent corrections for these nuclei are not solidly based on spectroscopic measurements as they are for the lighter nuclei. Such measurements simply do not exist for most $N \simeq Z$ nuclei in this mass region. Furthermore, charge radii, coefficients for the IMME, and spectroscopic factors for single-particle transfer reactions are not known either and so cannot be used to constrain the radial wave functions, "tune" the charge-dependence embedded in the two-body matrix elements, or identify the parentage of the participating analog states. As a consequence, the uncertainties assigned to the calculated corrections are very large (see the broad band in this mass region in Fig. 3), considerably reducing the usefulness of these nuclei either in testing the corrections or in contributing to the determination of V_{ud}. It would be very valuable in this context for radioactive-beam facilities to direct some attention to determining a wide variety of spectroscopic information in this mass region with a view to obtaining a reasonably effective nuclear model, which, among other things, could lead to much improved calculations for the correction terms.

4. Conclusions

In conclusion, we can assert — as we did in our survey four years ago — that world data for superallowed $0^+ \to 0^+$ β decays strongly support the CVC expectation of an unrenormalized vector coupling constant, and also set a tight limit, consistent with zero, on scalar currents. We can now add, though, that CKM unitarity is satisfied to within an uncertainty of 0.06%. This reconciliation with unitarity has come about as a result of significant changes in V_{us}; the value of V_{ud} determined from nuclear β decay has not varied outside of error bars in twenty years, during which time the size of those error bars has been reduced by a factor of five. We have also noted that the calculated isospin-symmetry-breaking correction terms have recently been improved and continue to stand up favorably to experimental tests, an outcome that must further increase confidence in the nuclear results.

Finally, we have pointed the way to new measurements that can improve the situation even more by refining our understanding of the nuclear-structure-dependent corrections to superallowed decays. This is the path to reduced uncertainties and even more stringent tests of the Standard Model.

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REFERENCES

[1] J.C. Hardy, I.S. Towner, *Phys. Rev.* **C71**, 055501 (2005); *Phys. Rev. Lett.* **94**, 092502 (2005).

[2] J.C. Hardy, I.S. Towner, *Phys. Rev.* **C**, to be published.

[3] I.S. Towner, J.C. Hardy, *Phys. Rev.* **C77**, 025501 (2008).

[4] C. Amsler *et al.*, [Particle Data Group], *Phys. Lett.* **B667**, 1 (2008).

[5] J.C. Hardy, I.S. Towner, V.T. Koslowsky, E. Hagberg, H. Schmeing, *Nucl. Phys.* **A509**, 429 (1990).

[6] I.S. Towner, J.C. Hardy, in *Symmetries and Fundamental Interactions in Nuclei*, Eds. W.C. Haxton and E.M. Henley, World-Scientific, Singapore 1995, pp. 183–249.

[7] I.S. Towner, J.C. Hardy, in *Proceedings of the V International WEIN Symposium: Physics Beyond the Standard Model*, Santa Fe, NM 1998, Eds. P. Herzeg, C.M. Hoffman and H.V. Klapdor-Kleingrothaus, World-Scientific, Singapore 1999, pp. 338-359.

[8] FlaviaNet Kaon working group, `http://www.lnf.infn.it/wg/vus; arXiv:0801.1817 [hep-ph]`.

[9] W.J. Marciano, A. Sirlin, *Phys. Rev. Lett.* **96**, 032002 (2006).

[10] E. Caurier, P. Navrátil, W.E. Ormand, J.P. Vary, *Phys. Rev.* **C66**, 024314 (2002).

[11] P. Finlay *et al., Phys. Rev.* **C78**, 025502 (2008).

[12] J.C. Hardy, I.S. Towner, *Phys. Rev. Lett.* **88**, 252501 (2002).