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# NEW NUCLEAR STABILITY ISLANDS OF OCTAHEDRAL AND TETRAHEDRAL SHAPES*

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Large scale calculations based on the microscopic–macroscopic method with Woods–Saxon single particle potential guided by the use of the discrete point group symmetries allow us to find the new islands of nuclear stability. These new stability regions are the consequence of particularly strong shell effects which are obtained in the calculations when the nuclear mean field is allowed to deform by respecting some special symmetries related to the so called high-rank point groups. The underlying mechanism is illustrated together with the full chain of the symmetry-associated magic numbers.

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## 1. Introduction

Description and understanding of nuclear stability in terms of nucleonic degrees of freedom is one of the most fundamental problems in sub-atomic physics. It parallels the problem of understanding the stability of elementary particles in terms of fundamental constituents: the quarks. On the experimental level it relates to measuring the nuclear masses and, since the majority of the nuclei studied contemporarily are unstable — to measuring the associated life-times. On the conceptual level, the nuclear stability can conveniently be studied within the framework of the realistic mean-field

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theory. There exist in the literature well known qualitative ways of obtaining a global view of nuclear stability. For instance, Copenhagen School introduced the model of the tri-axial harmonic oscillator and associates the presence of big gaps in the single particle spectra at certain deformations with the particularly stable nucleonic configurations, Ref. [1]. The underlying mathematical argument uses the fact that the systematic appearance of degeneracies in the single particle spectra must be accompanied by lowering of the nucleonic densities (appearance of gaps) and thus signifies increasing the nuclear stability.

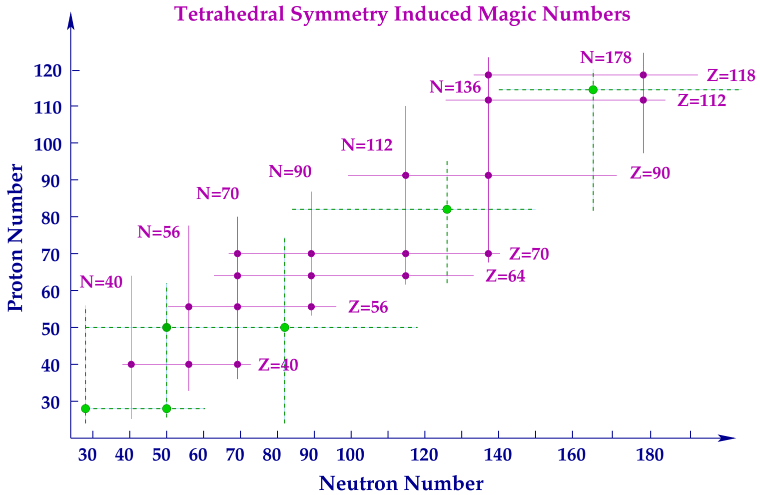


Fig. 1. Each mean-field symmetry generates its 'magic' numbers corresponding to an increased nuclear stability. Points connected with the dashed lines correspond to the spherical-symmetry well-known magic numbers (including the super-heavy nuclei). The points connected with full lines represent the result of the systematic mean-field calculations for the tetrahedral symmetry as discussed in the text.

Recently a general scheme of optimizing the conditions for finding big gaps in the mean-field single-particle spectra, based on group representation theory, has been proposed, *cf.* Ref. [2]. The criterion formulated there states that the symmetry point groups with relatively big number of irreducible representations and/or with irreducible representations with the highest possible dimensions (or both of these factors simultaneously) generate, on the average, relatively large single particle gaps in the spectra and thus lead to increased stability. The latter criterion is the most general in the context of nuclear stability and englobes, as a particular case, the argument with the harmonic oscillator mentioned above. Among symmetry groups that satisfy both of the above group-theoretical criteria we find octahedral-, $O_h$ and its tetrahedral sub-group, $T_d$. Each symmetry generates its own series of magic numbers, *cf.* Ref. [3]; the magic numbers obtained for tetrahedral group with a realistic nuclear Woods–Saxon mean field are illustrated in Fig. 1.

## 2. Importance of the symmetry-driven shell effects

It is impossible to present here the results obtained for hundreds of nuclei throughout the periodic table. Instead we will give some illustrations of the systematically strong shell effects that accompany deformations of a certain symmetry. Here we focus on the lowest-order tetrahedral-symmetry deformation that can be modeled with the help of spherical harmonics $Y_{3\pm2}$ (tetrahedral deformation $\alpha_{32}$ is denoted alternatively with the symbol $t_3$).

The point-group symmetry-driven shell-effects are strong, occasionally very strong, and may compete with the 'traditional' spherical shell-gaps. The shell effects are often illustrated in the literature by plotting the sum of the Strutinsky shell energy and pairing energy defined as the difference between the system's energy with pairing calculated using the empirical pairing constants $G$ — minus the same energy expression obtained with the pairing constant $G = 0$.

Such an illustration is given in Fig. 2 for the shell energy of the neutrons in function of the increasing neutron number $N$. One can see from this 'academic' illustration that the shell energies of a *tetrahedral-deformed* nucleus can become <u>lower</u> than the shell energies of the neighbouring spherical nuclei (almost $-10$ MeV for the tetrahedral symmetry as compared to $\approx -8$ MeV for the spherical one). One can see from the Fig. 2 that the strongest deformation-driving effects can be expected at the neutron

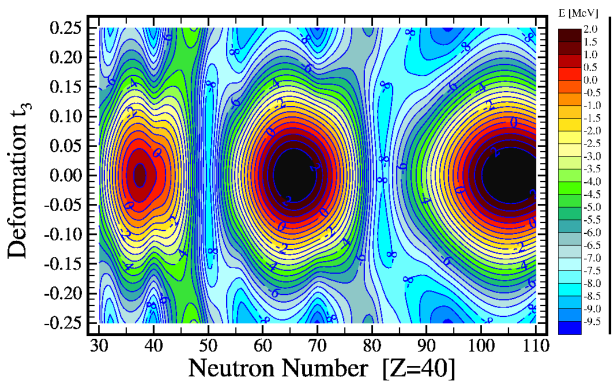


Fig. 2. Illustration of the dependence of the sum of the shell- and particle-number projected pairing energies on the tetrahedral-symmetry deformation ($t_3 \equiv \alpha_{32}$) and neutron number for fixed $Z = 40$ (zirconium nuclei).

numbers at which the energy difference between the null and non-null deformations are the strongest: in the illustrated example for $N \approx 36$–40, around 60–70 and around 100–110, the latter case possibly of interest in the astrophysical conditions.

The results illustrated in Fig. 2 are represented in an alternative, more global form in Fig. 3 for a big portion of the Periodic Table. Each $(Z, N)$ square gives the difference of the shell energies: the one obtained by minimising the shell energy over the quadrupole and hexadecapole axial deformations — minus the one obtained by minimising over tetrahedral and octahedral deformations. Thus the positive energies (inside black circles) represent the tetrahedral and octahedral symmetry shell effect stronger as compared to the traditional ones. Dotted lines indicate the borders of all known nuclei. As can be seen, many of the new islands of stability are formed outside or at the borders of the regions of known nuclei, predominantly for very neutron rich nuclei, which are impossible to reach nowadays. However, the near future new facilities with intense radioactive beams (SPIRAL2, FAIR, RIKEN) might allow the access to these new stability islands of tetrahedral and octahedral shapes.

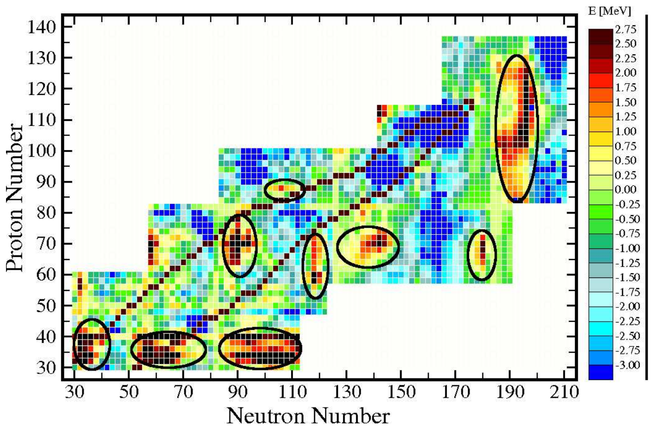


Fig. 3. Differences of the shell-energies as discussed in the text.

## 3. Summary and conclusions

Systematic calculations of the total nuclear energies show the presence of the shell effects driven by the tetrahedral symmetry; they are comparable with the shell effects known from the spherical nuclei, the latter intensively studied in the past. New stability islands of tetrahedral and octahedral shapes are predicted.

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## REFERENCES

[1] A. Bohr, B.R. Mottelson, *Nuclear Structure*, Vol. 2, W.A. Benjamin, New York 1975.

[2] J. Dudek, J. Dobaczewski, N. Dubray, A. Góźdź, V. Pangon, N. Schunck, *Int. J. Mod. Phys.* **E16**, 516 (2007).

[3] J. Dudek *et al.*, *Acta Phys. Pol. B* **40**, 713 (2009) these proceedings.