

TOROIDAL SUPER-HEAVY NUCLEI
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Within the self-consistent constraint Skyrme–Hartree–Fock+BCS model (SHF+BCS), we found equilibrium toroidal nuclear density distributions in the region of super-heavy elements. For nuclei with a sufficient oblate deformation ($Q_{20} \leq -200$ b), it becomes energetically favourable to change the genus of nuclear surface from 0 to 1, *i.e.*, to switch the shape from a biconcave disc to a torus. The energy of the toroidal (genus = 1) SHF+BCS solution relative to the compact (genus = 0) ground state energy is strongly dependent both on the atomic number Z and the mass number A . We discuss the region of Z and A where the toroidal SHF+BCS total energy begins to be a global minimum.

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1. Introduction

The term *doughnut nuclei* was coined by Wheeler [1, 2] many years ago. In 1970's and 80's the idea of a toroidal geometry in nuclear physics was investigated in a framework of the liquid drop model (LDM) and shell corrections [3–7]. In 1990's [8–13] and more recently [14, 15] the toroidal breakup configurations were found in the simulations of heavy-ion collisions using the Boltzmann–Nordheim–Vlasov (BNV) or the Boltzmann–Uehling–Uhlenbeck (BUU) kinetic transport models. The stability of toroidal nuclei against a change of the quadrupole moment was studied recently in the Hartree–Fock–Bogoliubov (HFB) theory [16] with the Gogny D1S force [17] and in the semiclassical extended Thomas–Fermi (ETF) method [18] with the Skyrme SkM* force [19].

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The purpose of this work is to study the stability of toroidal super-heavy nuclei against collective deformations within the self-consistent constraint Skyrme–Hartree–Fock+BCS framework (SHF+BCS) with the Skyrme SkM* force [19]. Since the classical LDM and semiclassical ETF approaches do not include shell effects, we would like to examine quantitatively the role of the shell effects in the description of toroidal nuclei.

The HF+BCS equations were solved using the code HFODD [20,21] that uses the basis expansion method in a three-dimensional Cartesian deformed HO basis. The details of our model are the same as in Ref. [22].

2. Results

In topology, the genus of a closed surface is equal to the number of handles on the surface. For instance, a torus is a surface with genus = 1 while a sphere or disc has genus = 0. The shape of a toroidal nucleus can be characterized by an aspect ratio R/d , where the major radius R is measured from the center of the toroid to the center of the circular meridian and the minor radius d is the radius of the meridian. Assuming axial symmetry and a uniform mass distribution we can calculate a quadrupole moment Q_{20} for a given aspect ratio R/d as

$$\begin{aligned} Q_{20} &= \sqrt{\frac{16\pi}{5}} \int d^3r \varrho(\vec{r}) r^2 Y_{20}(\cos \vartheta) \\ &= \frac{A}{4} \bar{R}_0^2 \left(\frac{2}{3\pi \cosh \eta_0} \right)^{2/3} [\sinh^2 \eta_0 - 5 \cosh^2 \eta_0] , \end{aligned} \quad (1)$$

where in the toroidal coordinates [4] $\varrho(r) = \frac{A}{V} \Theta(\eta - \eta_0)$, $\cosh \eta_0 = R/d$ and $\bar{R}_0$ is a spherical radius of the same volume $V = (4\pi/3) \bar{R}_0^3 = 2\pi^2 R d^2$.

Fig. 1 shows the potential energy curves for toroidal nuclei with $114 \leq Z \leq 138$ calculated within the LDM [4,5]. The potential energy curves are plotted as a function of R/d and Q_{20} in the panel (a) and (b), respectively. As one can see all toroidal super-heavy nuclei (genus = 1) show an oblate deformation with $Q_{20} \leq -200$ b. In addition, as the atomic number Z exceeds 138, the genus = 1 equilibrium lies at an energy even lower than that for the spherical shape.

In Fig. 2 we display the total binding energy (E^{tot}) calculated within SHF+BSC approach as a function of the quadrupole moment Q_{20} for $^{316}_{122}$, $^{340}_{130}$, $^{352}_{134}$ and $^{364}_{138}$ super-heavy nuclei. The two branches of E^{tot} corresponding to a compact (genus = 0) and a toroidal (genus = 1) SHF+BCS solutions have been found. Both solutions coexist in the vicinity of $Q_{20} \simeq -200$ b. But with the further increase of the oblate deformation it becomes

energetically favourable to change the genus of nuclear surface from 0 to 1, *i.e.*, to switch the shape from a biconcave disc to a torus. For $Q_{20} < -250$ b only genus = 1 self-consistent solutions exist.

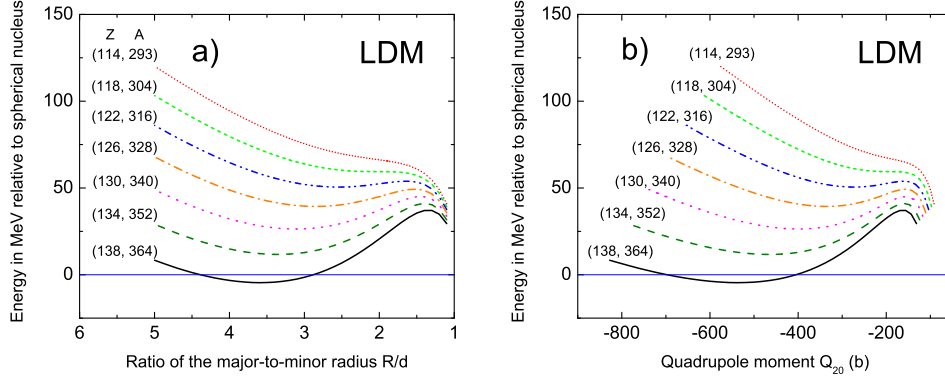


Fig. 1. (a) Potential energy curves for toroidal (genus = 1) nuclei of different atomic numbers Z and mass numbers A as a function of the aspect ratio R/d , where R is the major radius and d the minor radius of the toroid. (b) The same as a function of the quadrupole moment Q_{20} defined in Eq. (1).

We also show the self-consistent density distributions obtained for both branches with the compact and toroidal topology. Since the SHF+BCS model is not restricted to the axial symmetry, the toroidal density distributions show the characteristic sausage deformations [4]. The sausage instability is responsible for a multifragmentation of toroidal nuclei and in the case of $^{316}_{122}$ and $^{340}_{130}$ the toroidal solutions cease to exist at $Q_{20} \simeq -450$ b.

Similarly to the LDM results showed in Fig. 1, the genus = 1 and genus = 0 total energy minima become closer in energy with increasing atomic number Z and mass number A . For $Z = 138$ and $A = 364$ the toroidal equilibrium again begins to be the global minimum.

The neutron Fermi energies (λ^n) oscillate around -6 MeV with a change of quadrupole deformation for all nuclei from Fig. 2. The proton Fermi energies (λ^p) stronger depend on Q_{20} . For the prolate deformations λ^p oscillate between -2 and -1 MeV but in the region of oblate deformation λ^p decreases to the value of λ^n at $Q_{20} \simeq -400$ b. Thus, the toroidal SHF+BCS solutions are more stable against β -decay than the compact (genus = 0) solutions.

In conclusion, it appears that our self-consistent SHF+BCS calculations for the toroidal nuclei are consistent with the classical LDM [5] and semi-classical ETF [18] models as well as with the HFB theory [16].

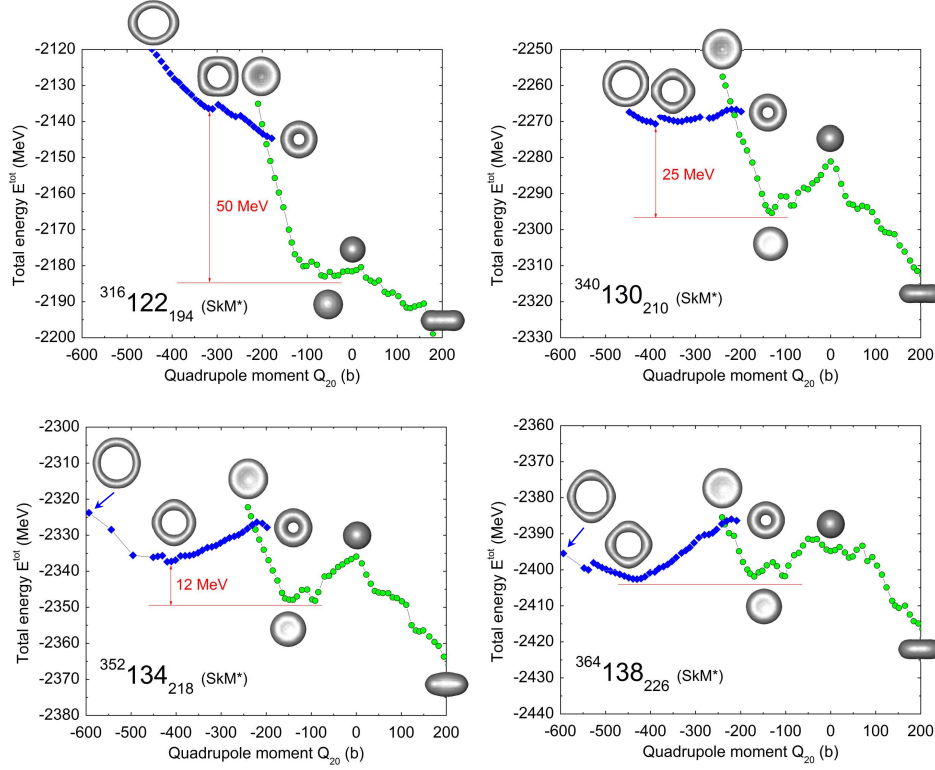


Fig. 2. The total binding energy (E^{tot}) of the toroidal (genus = 1) and compact (genus = 0) SHF+BCS solutions as a function of the quadrupole moment Q_{20} for $^{316}_{122}$, $^{340}_{130}$, $^{352}_{134}$ and $^{364}_{138}$. The self-consistent density distributions with the compact and toroidal topologies are also shown, where a z -axis lies on the surface of the page for a prolate deformation and is perpendicular to the page in the case of an oblate deformation.

The toroidal nuclear density distribution is not a surprising phenomenon, but a regular characteristic of the strongly oblate deformed heavy and super-heavy nuclei. The doughnut shaped nuclei are not the exception but rather the norm in this region of deformations!

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