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PARTICLE FREEZE-OUT WITHIN THE SELF-CONSISTENT HYDRODYNAMICS*

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Here I discuss some implicit assumptions of modern hydrodynamic models and argue that their accuracy cannot be better then 10–15%. Then I formulate the correct conservation laws for the fluid emitting particles from an arbitrary freeze-out (FO) hypersurface (HS) and show that the derived momentum distribution function of emitted particles does not contain negative contributions which appear in the famous Cooper–Frey formula. Further I analyze the typical pitfalls of some hydro models trying to alternatively resolve the FO problem.

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1. Introduction

Relativistic hydrodynamics is one of the most powerful theoretical tools to study the dynamics of phase transitions in nucleus nucleus collisions at high energies. During last 20 years it was successfully used to model the phase transition between the quark gluon plasma (QGP) and hadronic matter [1, 2]. So far, only within hydro approach and hydro inspired models it was possible to find the three major signals of the deconfinement transition seen at SPS energies, *i.e.* the Kink [3], the Strangeness Horn [4] and the Step [5]. Nevertheless, from its birth the hydro modeling of relativistic heavy ion collisions suffers from a few severe difficulties which I discuss in this work along with the self-consistent formulation of relativistic hydro equations.

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2. Explicit and implicit hydro assumptions

Relativistic hydrodynamics is a set of partial differential equations which describe the local energy-momentum and charge conservation [6]

$$\partial_\mu T_{\rm f}^{\mu\nu}(x,t) \; = \; 0\,, \qquad T_{\rm f}^{\mu\nu}(x,t) = (\varepsilon_{\rm f} + p_{\rm f})\, u_{\rm f}^\mu u_{\rm f}^\nu - p_{\rm f} g^{\mu\nu}\,, \tag{1}$$

$$\partial_\mu N_{\rm f}^{\mu}(x,t) \; = \; 0\,, \qquad N_{\rm f}^{\nu}(x,t) = n_{\rm f} u_{\rm f}^\nu\,. \tag{2}$$

Here the components of the energy-momentum tensor $T_{\rm f}^{\mu\nu}$ of the perfect fluid and its (baryonic) charge 4-current $N_{\rm f}^{\mu}$ are given in terms of energy density $\varepsilon_{\rm f}$, pressure $p_{\rm f}$, charge density $n_{\rm f}$ and 4-velocity of the fluid $u_{\rm f}^\nu$. This is a simple indication that hydrodynamic description directly probes the equation of state of the matter under investigation.

As usual to complete the system (1) and (2) it is necessary to provide

(A) *the initial conditions* at some hypersurface and

(B) *equation of state* (EOS).

The tremendous complexity of (A) and (B) transformed each of them into a specialized direction of research of relativistic heavy ion community. However, there are several specific features of relativistic hydrodynamics which have to be mentioned. In contrast to nonrelativistic hydrodynamics which is an exact science, the relativistic one, while applied to collisions of hadrons or/and heavy nuclei, faces a few problems from the very beginning. Since the system created during the collision process is small and short living there were always the questions whether the hydro description is good and accurate, and whether the created system thermalizes sufficiently fast in order that hydro description can be used.

Clearly, these two questions cannot be answered within the framework of hydrodynamics. One has to study these problems in a wider frame, and there was some progress achieved on this way. However, there are several implicit assumptions which are difficult to verify for the heavy ion collisions (HIC). Thus, we implicitly assume that the EOS of infinite system may successfully describe the phase transformations in a finite system created in collisions. The exact solutions of several statistical models both with a phase transition [7] and without it [8] found for finite volumes teach us that in this case the analog of mixed phase consist of several metastable states which may transform into each other. Clearly, such a process cannot be described by the usual hydro which is dealing with the stable states.

Furthermore, usually it is implicitly assumed that the matter created during the HIC is homogeneous. However, the realistic statistical models of strongly interacting matter [9, 10] tell us that at and above the cross-over this matter consists of QGP bags with the mean volume of several cubic fm. Moreover, the model of QGP bags with surface tension [9] predicts an existence of very complicated shapes of such bags above the cross-over due to negative surface tension. Note that the existence of QGP bags of such a volume is supported by the model of QGP droplets [11] which successfully resolved the HBT puzzles at RHIC.

Also the assumption that the heavy QGP bags (resonances) are stable compared to the typical life-time of the matter created in the HIC is, perhaps, too strong. The recent results obtained within the finite width model [12] show that in a vacuum the mean width of a resonance of mass M behaves as $\Gamma(M) \approx 600 \left[\frac{M}{M_0}\right]^{1/2}$ MeV (with $M_0 \approx 2$ GeV), whereas in a media it grows with the temperature. At the moment it is unclear how the finite width of QGP bags and other implicit assumptions affect the accuracy of hydrodynamic simulations, but from the discussion above it is clear that their *a priori* accuracy cannot be better than 10–15% [13, 14]. In fact, from the hydro estimates of the HBT radii at RHIC one concludes that, depending on the model, the real accuracy could be between 30% to 50%. Clearly, the same is true for the hydro-cascade [13, 14] and hydro-kinetic [15] approaches. Thus, at present there are no strong reasons to believe that these approaches are qualitatively better than the usual hydrodynamics.

3. Boundary conditions

In addition to the assumptions discussed above, to complete relativistic hydrodynamics it is necessary to know *the boundary conditions* which must be consistent with the conservation laws (1) and (2). The latter is known as the *freeze-out problem*, and it has two basic aspects [6]: (C1) the hydro equations should be terminated at the FOHS $\Sigma_{\rm fr}(x,t)$ beyond which the hydro description is not valid; (C2) at the FOHS $\Sigma_{\rm fr}(x,t)$ all interacting particles should be converted into the free-streaming particles which go into detector without collisions.

The complications come from the fact that the FOHS cannot be found *a priori* without solving the hydro equations (1) and (2). This is a consequence of relativistic causality on the time-like (t.l.) parts of the FOHS[1].

Therefore, the *freeze-out criterion* is usually formulated as an additional equation (constraint) $F(x,t^*) = 0$ with the solution $t = t^*(x)$ which has to be inserted into the conservation laws and solved simultaneously with them.

There were many unsuccessful attempts to resolve this problem (for their incomplete list see [16]) by *a priori* imposing the form of the FOHS, but all

[1] In this work I analyze the two dimensional hydro to which the four dimensional one can be always reduced. Then the t.l. HS is defined by the positive element square $ds^2 = dt^2 - dx^2 > 0$, whereas the space-like HS is defined by $ds^2 < 0$.

of them led to severe difficulties — either to negative number of particles or break up of conservation laws. The major difficulty is that the hydro equations should be terminated in such a way, that their solution remains unmodified by this very fact. In addition, this problem cannot be postponed to later times because at the boundary with vacuum the particles start to evaporate from the very beginning of hydro expansion, and this fact should be accounted by equations as well.

The hydrodynamic solution of the FO problem was found in [16] and developed further in [17]. This problem was solved after a realization of a fact that at the t.l. parts of the FOHS there is a fundamental difference between the particles of fluid and the particles emitted from its surface: the EOS of the fluid can be anything, but it implies a zero value of the mean free path, whereas, according to Landau [6], the emitted particles cannot interact at all because they have an infinite mean free path. Therefore, it was necessary to extend the conservation laws (1) and (2) from a fluid alone to a system consisting of a fluid and the particles of gas emitted (gas of free particle) from the FOHS. The resulting energy-momentum tensor and baryonic current (for a single particle species) of the system can be, respectively, cast as

$$T^{\mu\nu}_{\rm tot}(x,t) \; = \; \Theta^*_{\rm f}\; T^{\mu\nu}_{\rm f}(x,t) + \; \Theta^*_{\rm g}\; T^{\mu\nu}_{\rm g}(x,t)\,, \qquad (3)$$

$$N^{\mu}_{\rm tot}(x,t) \; = \; \Theta^*_{\rm f}\; N^{\mu}_{\rm f}(x,t) \; + \; \Theta^*_{\rm g}\; N^{\mu}_{\rm g}(x,t)\,, \qquad (4)$$

where at the FOHS the energy-momentum tensor of the gas $T^{\mu\nu}_{\rm g}$ and its baryonic current $N^{\mu}_{\rm g}$ are given in terms of *the cut-off distribution function* [16] of particles that have the 4-momentum p^μ

$$\phi_{\rm g} \; = \; \phi_{\rm eq}\,(x, t^*, p) \;\; \Theta\,(p_\rho d\sigma^\rho)\;, \qquad (5)$$

$$T^{\mu\nu}_{\rm g}\,(x,t^*) \; = \; \int \frac{d^3p}{p_0}\, p^\mu p^\nu\, \phi_{\rm eq}\,(x,t^*,p)\; \Theta\,(p^\rho d\sigma_\rho)\;, \qquad (6)$$

$$N^{\mu}_{\rm g}\,(x,t^*) \; = \; \int \frac{d^3p}{p_0}\, p^\mu \;\; \phi_{\rm eq}\,(x,t^*,p)\; \Theta\,(p^\rho d\sigma_\rho)\;. \qquad (7)$$

Here $\phi_{\rm eq}\,(x,t^*,p)$ denotes the equilibrium distribution function of particles and $d\sigma_\rho$ are the components of the external normal 4-vector to the FOHS $\Sigma_{\rm fr}(x,t^*)$ [16, 17].

The important feature of equations (3)–(5) is the presence of several Θ-functions. The $\Theta^*_{\rm g} = \Theta(F(x,t))$ function of the gas and $\Theta^*_{\rm f} = 1 - \Theta^*_{\rm g}$ function of the fluid can be explicitly expressed in terms of the FO criterion and can automatically ensure that the energy-momentum tensor of the gas (liquid) is not vanishing only in the domain where the gas (liquid) exists. On the other hand $\Theta\,(p^\mu d\sigma_\mu)$ function ensures that only the outgoing particles leave the fluid domain and go to the detector. Such a form of the distribution function (5) not only resolves the negative particles paradox of the famous Cooper–Frye formula [18] at the t.l. parts of the FOHS, but it allows one to express the hydrodynamic quantities of the gas of free particles in terms of the invariant momentum spectrum measured by detector. I would like to stress that the cut-off distribution (5) was *rigorously derived* [16] within the simple kinetic model, suggested in [19].

4. The self-consistent hydro equations

The analysis of Refs. [16, 17] shows that the equations of motion for the full system

$$\partial_\mu T^{\mu\nu}_{\rm tot}(x,t) = 0\,, \qquad \partial_\mu N^{\mu}_{\rm tot}(x,t) = 0 \tag{8}$$

are split into two subsystems

$$\Theta^*_{\rm f}\, \partial_\mu T^{\mu\nu}_{\rm f}(x,t) \;=\; 0\,, \qquad \Theta^*_{\rm f}\, \partial_\mu N^{\mu}_{\rm f}(x,t) = 0\,, \tag{9}$$

$$d\sigma_\mu T^{\mu\nu}_{\rm f}(x,t^*) = d\sigma_\mu T^{\mu\nu}_{\rm g}(x,t^*)\,, \qquad d\sigma_\mu N^{\mu}_{\rm f}(x,t^*) = d\sigma_\mu N^{\mu}_{\rm g}(x,t^*)\,, \tag{10}$$

since equations for the gas of free particles, $\partial_\mu T^{\mu\nu}_{\rm g} \equiv 0$ and $\partial_\mu N^{\mu}_{\rm g} \equiv 0$, are identities due the fact that the trajectories of free particles are straight lines.

Here Eqs. (9) are the equations of motion of the fluid, whereas Eqs. (10) are the boundary conditions for the fluid at the FOHS. On the other hand (10) is a system of the nonlinear partial differential equations to find the FOHS $\Sigma_{\rm fr}(x,t^*)$ for a given FO criterion. To find the FOHS $\Sigma_{\rm fr}(x,t^*)$ the solution of the fluid equations (9) should be used as an input for (10).

There is a fundamental difference between the equations of motion (1) of traditional hydrodynamics and the corresponding equations (9) of hydrodynamics with particle emission: if the FOHS is found, then, in contrast to the usual hydrodynamics, the equations (9) automatically vanish in the domain where the fluid is absent. In this way the equations (3)–(10) resolve the FO problem in relativistic hydrodynamics.

In addition, as shown in [17] for a wide class of hadronic EOS these equations resolve the usual paradox of relativistic hydrodynamics of finite systems which is known as *a recoil problem* due to the emission of particles. The latter means that a substantial emission of particles from the t.l. parts of the FOHS is expected to inevitably modify the hydrodynamic solution interior the fluid. However, this is not the case for a wide class of realistic EOS of hadronic matter because at the t.l. parts of the FOHS there appears a new kind of hydro discontinuity, *the freeze-out shock* [16]. The FO shock is a generalization of the usual hydrodynamic shock waves [20, 21] which for the nonrelativistic flows transforms into the usual hydrodynamic shock. As shown in [17] the supersonic FO shock is not only thermodynamically stable, *i.e.* in such a shock the entropy increases, but also it propagates interior the fluid faster than the information about the possible change of hydrodynamic solution.

5. Concluding remarks

The hydrodynamic solution of the FO problem required an insertion of the boundary conditions into the conservation laws for the fluid and emitted particles. The subsequent transport simulations [22] showed that the assumptions of thermal equilibrium at the FOHS and small width of the FO front at the t.l. parts of the FOHS are quite reasonable, whereas the main problem appears at the s.l. FOHS where the decay of shortly living resonances may essentially modify the equilibrium distribution function. This problem, however, requires more complicated hydro-kinetic models [15] or even the kinetic approach with specific boundary conditions [23].

Further attempts of the Bergen group [24] to improve the suggested hydro solution of the FO problem were based on the hand waiving arguments and, hence, they did not lead to any new discovery. Note also that from time to time the erroneous attempts to resolve the FO problem appear [25], but as usual they are running into severe troubles. Thus, in [25] (and subsequent works) the artificial δ-like drains in relativistic hydrodynamic equations were inserted, which, besides other pitfalls, in principle cannot reproduce the nonrelativistic hydro equations even for weak flows.

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